Efficient Parallel Analysis of Shell-fluid Interaction Problem by Using Monolithic Method Based on Consistent Pressure Poisson Equation*

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Abstract
In this paper, a parallel monolithic method for shell-fluid interaction based on the consistent Pressure Poisson Equation (PPE) is developed and its parallel computational efficiency is demonstrated. The Conjugate Gradient (CG) method without any preconditioner works well to solve the consistent PPE, even though the coefficient matrix of the original coupled equation system becomes ill-conditioned due to (a) the inhomogeneity of submatrix elements between the fluid and the structure and (b) the ill-conditioned submatrix of shell structure. Thus our parallel monolithic method using the consistent PPE and the CG method without any preconditioner is efficient for parallel analyses of shell-fluid interaction problems. The present parallel solution procedure is based on the mesh decomposition. To demonstrate the performances of the developed method, it is applied to simulate the vibration of an elastic plate situated in the wake of a rectangular prism and a flapping elastic wing in quiescent fluid.

Key words: Parallel Analysis, Fluid-Structure Interaction, Finite Element Method, Shell, Incompressible Viscous Fluid, Monolithic Method, Pressure Poisson Equation, Conjugate Gradient Method

1. Introduction

The analysis methods for Fluid-Structure Interaction (FSI) problems are roughly classified into partitioned or monolithic methods. In the monolithic methods, the fluid and structure governing equations and the equilibrium condition on the fluid-structure interface are simultaneously solved by formulating the coupled equation system. Although the development of completely new computer programs is required, the monolithic methods are regarded as effective for the FSI problems with strong dependence between the fluid and structure (1, 2). However the formulation of the single equation system may lead to its ill-conditioned coefficient matrix, especially if the structure is very stiff or heavy compared to the fluid or the ill-conditioned structural submatrix is generated due to the shell elements. Straightforward approach to overcome this drawback is to develop sophisticated preconditioners to improve the matrix condition (3-5). However in this approach it is required to develop the preconditioners with high parallelization efficiency to archive efficient
parallel computations. On the other hand, we have proposed the monolithic method based on the Pressure Poisson Equation (PPE) (6-8) which is formulated so as to be consistent with the original coupled equation system. The coefficient matrix of the consistent PPE is well-conditioned compared to that of the original coupled equation system, thus it is expected that the efficient parallel analysis by using the proposed monolithic method is realized by combining with the iterative solver without any preconditioner or with the simple preconditioner. In this study, we develop a parallel monolithic method for the shell-fluid interaction problem based on the consistent PPE, and it is applied to simulate the vibration of an elastic plate situated in the wake of a rectangular prism and a flapping elastic wing in fluid to demonstrate its performances.

2. Monolithic Method for FSI problems

In this section, we briefly describe the formulation of the coupled equation system for the FSI and the conventional monolithic method. The details of them are described in Ref. (6). The finite element discretized equation systems for the incompressible viscous fluid and the structure are combined using the continuity and equilibrium conditions on the fluid-structure interface to derive the coupled equation system. The linearized coupled equation system can be expressed as

\[
\begin{bmatrix}
\mathbf{M}_s^f & \mathbf{M}_c^f & 0 & -\mathbf{G}_s \\
\mathbf{M}_s^c & \mathbf{M}_c^f + \mathbf{M}_c^c & -\mathbf{G}_c \\
0 & \mathbf{M}_s^c & \mathbf{M}_c^c & 0 \\
\gamma \Delta t \mathbf{G}_s & \gamma \Delta t \mathbf{G}_c & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \mathbf{a}_s \\
\Delta \mathbf{a}_c \\
\Delta \mathbf{g}_s \\
\Delta \mathbf{h}_c
\end{bmatrix}
= \begin{bmatrix}
\Delta \mathbf{a}_f \\
\Delta \mathbf{g}_c \\
\Delta \mathbf{p} \\
\Delta \mathbf{h}
\end{bmatrix},
\]

(1)

where \(\Delta \mathbf{a}\) denotes the acceleration increment, \(\Delta \mathbf{p}\) the fluid pressure increment, \(\Delta \mathbf{g}\) the residual force of the equilibrium equations, \(\Delta \mathbf{h}\) the residual of the incompressibility constraint, \(\mathbf{M}\) the generalized mass matrix, \(\mathbf{G}\) the divergence operator matrix, \(\gamma\) the control parameter of the generalized trapezoidal method, and \(\Delta t\) the time increment. The subscript \(T\) indicates the transpose of the matrix, the subscripts \(i\) and \(c\) denote the degree of freedom (DOF) independent and dependent, respectively, on the fluid-structure interface. Further, the superscripts \(f\), \(s\) and \(fs\) indicate the quantities of the fluid, structure, and the FSI, respectively. Note that the linear relations among \(\Delta \mathbf{a}\), \(\Delta \mathbf{v}\) and \(\Delta \mathbf{u}\) given by the generalized trapezoidal method and the Newmark's method are used to reduce the unknown variables.

In this study, the fluid convective and diffusive terms are evaluated using the known velocity field, while the structural internal force is evaluated using the unknown displacement field. Thus the fluid and structural generalized mass matrices become \(\mathbf{M} + \beta \Delta \mathbf{r} \mathbf{K}\) and \(\gamma \mathbf{M}_c\), respectively, where \(\mathbf{M}\) denotes the mass matrix, \(\mathbf{K}\) the (tangential) stiffness matrix, the subscript \(L\) lumping of the matrix, and \(\beta\) the control parameter of the Newmark's method, respectively. The DOFs of the shell structure independent and dependent on the fluid-structure interface are defined as the rotational and translational DOFs of the shell structure, respectively (2). The coefficient matrix of Eq. (1) becomes ill-conditioned due to (a) the inhomogeneity of submatrix elements between the fluid and the structure, and (b) the ill-conditioned submatrix of shell structure. Since Eq. (1) is directly solved in the conventional monolithic method, the preconditioners to improve its matrix condition are required to use iterative solvers (3-5).

3. Parallel Analysis by Monolithic Method based on Consistent PPE

3.1 Monolithic Method based on Consistent PPE

In this section, we briefly describe the monolithic method based on the consistent PPE. Its detail is described in Ref. (6). First Eq. (1) is reduced to the equation system with \(\Delta \mathbf{a}_f\)
and Δafc by eliminating Δai. Next Δai and Δafc obtained from the reduced equation system are substituted into the fourth line of Eq. (1), which corresponds to the incompressibility constraint. Finally the following consistent PPE is derived:

\[
\mathbf{M}_f^c \Delta \mathbf{p} = \mathbf{M}_f^c \Delta \mathbf{a}_f + \mathbf{M}_f^c \Delta \mathbf{a}_c,
\]

\[
\mathbf{M}_f^c = \mathbf{M}_f^c + \left( \mathbf{M}_f^c - \mathbf{M}_f^c \mathbf{M}_f^c \right)^{-1} \mathbf{M}_f^c \mathbf{M}_f^c.
\]

Note that Eq. (2) replaces the continuity equation with the equilibrium equation. Instead of solving Eq. (1) directly, we can obtain its solution using the following procedure: (Step1) solve Eq. (2) to obtain Δp, (Step2) substitute Δp into the second and third lines of Eq. (1) and solve them to obtain Δaf and Δafc, (Step3) substitute Δaf and Δafc into the first line of Eq. (1) and solve it to obtain Δai.

3.2 Characteristics of Consistent PPE

In this section, we present characteristics of the coefficient matrix of the consistent PPE (2) theoretically. For simplicity, we define the generalized mass matrices of the fluid and the structure as the lumped mass matrices, and employ the simplified finite element model shown in Fig. 1. In this model, ○ indicates the position of fluid velocity nodes, and × indicates the position of fluid pressure nodes. The above setup reduces the coefficient matrix of Eq. (2) to

\[
\begin{pmatrix}
4 f(R) & 1 & 1/2 \\
1 & f(R) & 1/2 & 1 \\
1/2 & 1 & 1 & 6 \\
\end{pmatrix}
\]

\[
f(R) = 3 + 3/(1 + R), \quad R = (\rho' h) / (\rho' \Delta x),
\]

where \(\rho'\) denotes the structural mass density, \(\rho'\) the fluid mass density, \(\Delta x\) the reference length of elements, and \(h\) the thickness of shell elements. Note that the nondimensional parameter \(R\) represents the ratio of the generalized mass matrix elements between the structure and the fluid, and measures the inhomogeneity of them. The row or column index of the matrix (4) corresponds to the fluid element number, which is indicated by the integer with round brackets in Fig. 1. The characteristic function \(f(R)\) in (5) is the monotonically decreasing function, and takes the value 6 or 3 for \(R=0\) or \(\infty\), respectively, therefore the matrix (4) is always diagonally dominant.

Fig. 1 The employed finite element model. Each integer denotes the node number and each integer with round brackets denotes the element number.
The condition number of the matrix (4) monotonically increases and takes the value between 1.9 and 4.0 approximately as shown in Fig. 2. In addition, the matrix (4) for \( R=0 \) or \( \infty \) is equivalent to the coefficient matrix of the fluid PPE to which the fluid equations are reduced, i.e. the matrix (4) for \( R=0 \) becomes the coefficient matrix of the fluid PPE in the case that the shell structure is replaced with the completely fixed wall, and the matrix (4) for \( R=\infty \) becomes the coefficient matrix of the fluid PPE in the case of no shell structure. These theoretical results suggest that the coefficient matrix of the consistent PPE (2) is as well-conditioned as that of the fluid PPE, and its condition is little affected by the inhomogeneity of the fluid and structural submatrix elements. These characteristics are also examined numerically in Section 4.1.

![Fig. 2 The relation between the nondimensional parameter \( R \) and the condition number of the coefficient matrix of the consistent PPE.](image)

### 3.3 Parallel Solution Procedure of Consistent PPE

When we solve the consistent PPE (2) using iterative solvers, the following matrix-vector products provide the most expensive computations:

\[
\gamma \mathbf{G}_i \mathbf{M}_g^{-1} \mathbf{G}_i \mathbf{A} \mathbf{p}, \quad \gamma \mathbf{G}_i \mathbf{M}_s^{-1} \mathbf{G}_i \mathbf{A} \mathbf{p}. \tag{6a, b}
\]

Note that the expression (6a) is equivalent to the coefficient matrix of the fluid PPE. Assuming that the number of DOFs of the shell structure is far smaller than that of the fluid, we employ the following parallel solution procedure based on the mesh decomposition. The mesh is decomposed as shown in Fig. 3. The symbols \( \Omega_S \) and \( \Omega_F \) denote the structural and fluid meshes, respectively, and \( \Omega_{Fi} (i=1, 2, \ldots, N_d) \) denotes the \( i \)-th submesh of \( \Omega_F \). Note that \( \Omega_{F1} \) surrounds \( \Omega_S \) so that the matrix-vector product (6b) can be computed at one computational node. The computations concerning \( \Omega_{Fi} \) are executed at the computational node \( P_i \) (\( i=1, 2, \ldots, N_d \)), while the computations concerning \( \Omega_S \) are executed at \( P_1 \). Under the above setup, the matrix-vector product (6b) is computed at \( P_1 \) using the data concerning \( \Omega_i \) and \( \Omega_{S1} \), while the parallel computation of (6a) is executed using the following steps.

(Step1) the matrix-vector product (6a) is computed at \( P_i \) using the element-by-element method:

\[
\mathbf{A} \mathbf{p} = \sum_e \mathbf{A}^{(e)} \mathbf{p}^{(e)} = \sum_e \mathbf{q}^{(e)} = \mathbf{q}, \tag{7}
\]

where \( \mathbf{A} \) denotes the global matrix, \( \mathbf{p} \) the global vector, \( \mathbf{q} \) the matrix-vector product, and \( e \) the element number,

(Step2) the nodal data of the matrix-vector product (6a) on the interface between \( \Omega_{Fi} \) and \( \Omega_{Fj} \) (\( j \neq i \)) computed at \( P_j \) is transferred to \( P_i \) to complete the corresponding nodal data computed in Step 1.
4. Basic Performances of the Parallel Analysis

4.1 Characteristic of Consistent PPE

4.4.1 Problem Setup

We consider a two-dimensional problem of an elastic cantilever plate of length $D$ and thickness $h$ situated in the wake of a rectangular prism in the uniform flow as shown in Fig. 4. The cross section of the prism is square with the side length $D = 0.01$ m. The velocity of inlet uniform flow in the $x$-direction is $V = 0.61695$ m/sec and the fluid is air ($\rho = 1.18$ kg/m$^3$ and $\mu = 0.0000182$ kg/(m sec)). The Reynolds number becomes $Re = 400$ under the definition of $Re = VD/\nu$, where $\nu$ denotes the dynamic viscosity of the fluid. The unit extent in the $z$-direction (out-of-plane) is assumed with no variation in this direction. The $xy$-plane view of the employed finite element mesh for the fluid is shown in Fig.5, where the fluid domain is discretized into 23,352 P1P1 elements (9). The plate is equally discretized into 32 MITC shell elements (10, 11) so as to have the matching interface between the fluid and structural meshes. The time increment is given $\Delta t = 2.0 \times 10^{-4}$ sec. We set the structural mass density as $\rho_s = 1000$ kg/m$^3$, the Young's modulus as $E = 5.067 \times 10^7$ Pa and the thickness of the plate as $h = 4.0 \times 10^{-4}$ m. Under the assumption of the Euler-Bernoulli beam, the present setup for the plate gives 9.091 Hz to the natural frequency of the plate bending mode $f_s$, which is equal to the oscillation frequency of the lift force acting on the plate given by the preliminary CFD analysis. The following FSI computations start with the flow, obtained by the preliminary CFD analysis, that is sufficiently developed with the elastic cantilever plate at rest. The Conjugate Gradient (CG) and BiCGSTAB methods without any preconditioner are employed to solve the proposed PPE (2), while the BiCGSTAB method with the diagonal scaling is employed to solve the coupled equation system (1). The iterations in the iterative solvers are terminated when the residual norms become $10^{-6}$. Fig. 6 shows the time history of the $y$-displacement at the free end of the plate. As shown in Fig. 6, the history given by the proposed method agrees well with that given by the conventional monolithic method.
After the initial stabilization phase, the displacement and the lift force oscillate regularly as the result of the alternate vortex shedding, and their amplitudes gradually keep growing to convergence. In the regular oscillation, the frequency of the displacement and lift force and the amplitudes of the displacement and the lift force become 9.143 Hz, 3.236×10⁻³ m and 2.292×10⁻⁵ N, respectively. When the maximum lift acting on the plate in the present result is applied to the plate statically, its free end results in the y-displacement 1.531×10⁻⁵ m, which is approximately 0.5 % of the maximum amplitude of the y-displacement in the present result. Thus the bending mode of the plate motion is synchronized with the vortex shedding in the present result.

4.1.2 Comparison of Coefficient Matrices between Consistent PPE and Coupled Equation System

In this section, we compare the conditions of the coefficient matrices between the consistent PPE (2) and the coupled equation system (1). The iteration number required by the iterative solver to solve the linear equation system is dependent on its coefficient matrix condition. Thus we evaluate the matrix condition using the iteration number at the first time step of the analysis. To measure the inhomogeneity of submatrix elements between the fluid and the structure, we use the characteristic ratio between them \( R \), which is defined by

\[
R = (\rho' \Delta t^2 h + \Delta t^2 hE)/(\rho' \Delta x^3),
\]

where the reference element length \( \Delta x \) is defined by the minimum element length \( \Delta x_{\text{min}} = 0.125 \text{ cm} \). Note that this \( R \) is the general expression of that appears in Eq. (5).

a. Influence of Material Property

The problem (a) described in Section 2 is examined here. The Young's modulus which appears in \( R \) (8) is given \( E = 1.0 \times 10^2, 1.0 \times 10^3, ... , 1.0 \times 10^{11} \text{ Pa} \), while the other parameters are given the same values used in Section 4.1.1. As shown in Fig. 7, the condition of the coefficient matrix of Eq. (1) becomes worse rapidly as \( R \) increases, while the coefficient matrix of Eq. (2) keeps its condition. This numerical result is consistent with the theoretical result in Section 3.2.

b. Influence of Time Increment

The problem (a) described in Section 2 is also examined here. The time increment which appears in \( R \) (8) is given \( \Delta t = 5.0 \times 10^{-5}, 1.0 \times 10^{-4}, 5.0 \times 10^{-4}, 1.0 \times 10^{-3} \text{ sec} \), while the other parameters are given the same values used in Section 4.1.1. As shown in Fig. 8, the condition of the coefficient matrix of Eq. (1) becomes worse rapidly as \( R \) increases, while the coefficient matrix of Eq. (2) keeps its condition. This numerical result is consistent with the theoretical result in Section 3.2.
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Fig. 7  The relation between the nondimensional parameter $R$ and the iteration numbers of iterative solvers for the conventional and the proposed monolithic methods, where the Young’s modulus is altered. ○, ×, □ denote the iteration numbers of the BiCGSTAB method with diagonal scaling for the conventional method, the CG and the BiCGSTAB methods for the proposed method, respectively.

Fig. 8  The relation between the nondimensional parameter $R$ and the iteration numbers of iterative solvers for the conventional and the proposed monolithic methods, where time increment is altered. ○, ×, □ denote the iteration numbers of the BiCGSTAB method with diagonal scaling for the conventional method, the CG and the BiCGSTAB methods for the proposed method, respectively.

c. Influence of Plate Thickness

The problem (b) described in Section 2 is examined here. The plate thickness $h$ is changed since it affects the coefficient matrix condition of the shell stiffness matrix, while the fluid mass density $\rho'$ is also changed to keep the same value of $R$ (8). The other parameters are given the same values used in Section 4.1.1. As shown in Fig. 9, the condition of the coefficient matrix of Eq. (1) becomes worse rapidly as $R$ increases, while the coefficient matrix of Eq. (2) keeps its condition. The latter seems to be caused by the formulation for the shell stiffness matrix in the coefficient matrix of Eq. (2), i.e. the shell stiffness matrix is reduced to its Schur complement associated with the translational DOFs, whose condition is improved compared with that of the shell stiffness matrix (8).

4.1.3 Relation between the proposed PPE and Fluid PPE

The relation between the proposed PPE and fluid PPE is examined. The time increment, Young's modulus, the plate thickness and the structural mass density are given $\Delta t = 1.0 \times 10^{-9}$ sec, $E = 1.0 \times 10^2$ Pa, $h = 1.25 \times 10^{-3}$ m and $\rho' = 1.18 \times 10^5$, ..., $1.18 \times 10^5$ kg/m$^3$, respectively. The other parameters are given the same values used in Section 4.1.1. The above setup makes $R$ (8) approximately equivalent to $R$ appears in Eq. (5) since the second term in the numerator of Eq. (8) is far smaller than the other terms. Fig. 10 shows the relation between $R$ and the iteration number required by the CG method to solve the following three PPEs: (a) the proposed PPE, (b) the fluid PPE in the case of no plate, and (c) the fluid PPE in the case that the plate is replaced with the completely fixed wall. As shown in Fig. 10, the iteration number in the case of (a) approaches that in the case of (b) as $R$ approaches 0, and that in the case of (c) as $R$ approaches $\infty$. These numerical results are consistent with the theoretical results in Section 3.2.
4.2 Performance Evaluation of Parallel Analysis

The results and discussions in Section 4.1 suggest that the proposed PPE does not require any preconditioners to use the CG method in spite of the problems pointed out in Section 2. Therefore we select the combination of the proposed PPE and the CG method without the preconditioners to analyze the FSI problems in parallel. It is expected that our method can achieve the efficient parallel analyses of shell-fluid interaction problems in the distributed memory parallel computers with low speed network. In this section, we evaluate the parallelization efficiency of our method, whose parallel solution procedure is described in Section 3.3.

4.2.1 Analysis Setup

We use the same setup in Section 4.1.1. The fluid mesh is decomposed following Fig. 3. Since the computational node P₁ executes both computations concerning submeshes Ωₛ and Ωₓ₁, the uniform mesh decomposition might give the higher computational load on P₁ than that on the other computational nodes. Thus we employ the mesh decomposition taking into account the computational load balancing based on the arithmetic operations number \( N_o(Ω) \), which expresses the arithmetic operations number concerning the domain \( Ω \) to calculate Eq. (6) in the present computer program. First the domain \( Ωₓ₁ \) is defined so that \( N_o(Ωₓ)+N_o(Ωₓ₁) \) are nearly equal to \( (N_o(Ωₓ)+N_o(Ωₓ₁))/(Nₛ-1) \), where \( Nₛ \) denotes the number of the fluid submeshes. Next the fluid domain except \( Ωₓ₁ \) are uniformly decomposed to \( Nₛ-1 \) pieces of...
submeshes \( \Omega_{fi} \) \( (i = 2, 3, \ldots, N_d) \) by using Metis \(^{12}\). As the parallel computation environment, we employ the PCs cluster consists of 16 PCs, whose CPU and memory are Pentium 4 2.8GHz and 1GByte, respectively, which are connected with each other via the network 100Base-TX, and where the MPI library is used for the data communication.

4.2.2 Performance of Parallel Computation

Fig. 11 shows the speed up for the number of CPUs, which is evaluated using the CPU time to solve the proposed PPE in the first time step. The parallelization efficiency is approximately 72 % in the case of \( N=8 \). Taking into account the low data transfer rate, and the averaged nodes number of the fluid submeshes 1,087 in the case of \( N=8 \), we can conclude that the proposed method possesses the fine performance of the parallel computation.

4.3 Application to Large Scale Analysis

5.1 Elastic Wing Flapping in Quiescent Fluid

We consider an elastic wing situated in the large cylinder filled with quiescent fluid as shown in Fig. 12. The narrow strip region of the wing behind the leading edge can easily bend around the span-wise axis due to its low bending rigidity, while the rest is rigid. Following the flapping motion around z-axis with the large stroke amplitude imposed to the leading edge, the flexible region takes a large bending deformation which produces the large pitch angle. Thus the present FSI system takes the large fluid domain changes and structural deformations. The proposed method is suitable for such problems that the strong dependence between the fluid and the structure would occur and the DOFs of fluid analyses tend to become large. The parameters used in the following analysis are chosen as follows. The geometric parameters are the thickness \( h=0.0032 \text{m} \), the span length \( L=0.25 \text{m} \), chord length \( c=0.0938 \text{m} \) of the wing, and the radius \( R=0.80 \text{m} \) and height in the \( z \)-direction \( H=0.36 \text{m} \) of the fluid domain. The material properties are the mass density of the wing \( \rho_s=1000 \text{kg/m}^3 \) and the Young’s modulus of the wing flexible region \( E=0.4 \text{MPa} \), and the fluid mass density \( \rho_f=880 \text{kg/m}^3 \) and the viscosity \( \mu=0.0715 \text{ kg/(m sec)} \) of the fluid, which corresponds to a mineral oil. The wing kinematical parameters are the flapping frequency \( f=145 \text{ mHz} \) and the stroke amplitude \( \Phi=160 \text{ degrees} \). The flapping motion is given so that the acceleration of the stroke angle \( d^2\phi/dt^2 \) follows the eleventh power of the sinusoidal function. We employ the coarser mesh (the numbers of nodes and elements of the fluid are
60,504 and 340,032, respectively, and the number of nodes and elements of the shell structure are 88 and 70, respectively) and the finer mesh (the numbers of nodes and elements of the fluid are 272,205 and 1,574,496, respectively, and the number of nodes and elements of the shell structure are 240 and 210, respectively). Note that the discretization of the finer mesh in all directions in the vicinity of the wing is twice finer than that of the coarser mesh. Since the DOFs of Eq. (1) in the case of the finer mesh is over one million, the analysis can be regarded as a large scale monolithic analysis of FSI problems. Fig. 13 shows the fluid mesh decomposed into 8 pieces of submeshes using the same procedure in Section 4.2.1. We use the same computational environment in Section 4.2.1.

5.2 Results and Discussions

Fig. 14 shows the time history of the lift, which has two peaks in each half stroke. Fig. 15 shows the fluid velocity field around the wing at 0.25 cycle. Two vortices appear at the leading and trailing edges as shown in Fig. 15. As shown in Fig. 14, the lift forces in the cases of the coarser and finer meshes are qualitatively consistent with each other, but their relative errors of the first and second peaks in each half stroke are approximately 10% and 20%, respectively. The present example provides one of the cases where the DOFs of fluid analyses tend to become large, and the parallel analyses are required in such cases. The performances of the proposed method in the case of the finer mesh are summarized in Fig. 16, which shows the speed up for the number of CPUs. We decompose the fluid mesh into $N=2^n$ ($n=2$, 3, 4) pieces of submeshes. In the evaluation of the speed up for the number of CPUs, we start from 4 PCs since the analysis using the finer mesh cannot be executed at our PCs smaller than 4. The parallelization efficiency is approximately 95% in the case of $N=16$, therefore we can conclude that the proposed method possesses the fine performance of parallel computation.
6. Conclusion

In this study, first we presented that the coefficient matrix of the consistent PPE is well-conditioned even though that of the original coupled equation system becomes ill-conditioned due to (a) the inhomogeneity of submatrix elements between the fluid and the structure and (b) the ill-conditioned submatrix of shell structure. Next we combined the monolithic method based on the consistent PPE with the CG method without any preconditioner, and parallelized its solution procedure based on the mesh decomposition. Finally we presented in the numerical examples that the proposed method possesses the fine performance of parallel computations. As a consequence we can conclude that our parallel monolithic method using the consistent PPE is efficient for the parallel analyses of the shell-fluid interaction problems.

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