PAPER *Special Section on Discrete Mathematics and Its Applications*

A Minimum Feedback Vertex Set in the Trivalent Cayley Graph

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SUMMARY In this paper, we study the feedback vertex set problem for trivalent Cayley graphs, and construct a minimum feedback vertex set in trivalent Cayley graphs using the result on cube-connected cycles and the Cayley graph representation of trivalent Cayley graphs.
key words: *minimum feedback vertex set, trivalent Cayley graphs, cube-connected cycles, Cayley graph representation*

1. Introduction

Let $G = (V(G), E(G))$ be a finite and undirected simple graph. A *feedback vertex set* F of G is a subset of $V(G)$ such that the induced subgraph $\langle V(G) - F\rangle$ has no cycle. The minimum feedback vertex set problem is to find a feedback vertex set of minimum cardinality in G.

The feedback vertex set problem has been extensively studied in [2], [4], [5], [7]. It is NP-hard for general graphs [6]. The best known approximation algorithm for this problem has approximation ratio 2 [1]. This problem is fundamental in combinatorial optimization and find applications in many different settings: graph layout, deadlock prevention, combinatorial circuit design, the problem of converters placement in optical networks.

The following inequality shows a lower bound to the size of a minimum feedback vertex set in any graphs.

Proposition 1: [2] For feedback vertex set F of a graph $G = (V(G), E(G))$ with maximum degree $\Delta(G)$, it holds that

$$|F| \geq \left\lceil \frac{|E(G)| - |V(G)| + 1}{\Delta(G) - 1} \right\rceil.$$

Trivalent Cayley graphs were introduced in [12] to design interconnection networks. The graph has a logarithmic diameter in the number of nodes, is regular of degree 3, and some properties are shown in [8], [13], [14].

In this paper, we propose a minimum feedback vertex set in trivalent Cayley graphs with the size equal to the lower bound. It depends on the results on cube-connected cycles obtained in [11] and Cayley graph representation of trivalent Cayley graphs developed in [10].

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2. Preliminaries

A *subdivision* of G is a graph obtained from G by removing some edge $e = (u, v)$ and adding a new vertex w and edges (u, w) and (w, v).

Let H be a subset of $V(G)$. Then the *induced subgraph* $\langle H\rangle$ is a subgraph of G having vertex set H and whose edge set consists of those edges of G joining two vertices in H.

A graph H is said to be a *spanning subgraph* of G if H is a subgraph of G such that $V(G) = V(H)$. On other terminology and notation, we refer to [3].

We give some propositions with respect to the feedback vertex set.

Proposition 2: Let F be a feedback vertex set of G. For any spanning subgraph G' of G, F is a feedback vertex set of G'.

Proposition 3: Let F be a feedback vertex set of G. For any subdivision G' of G, F is a feedback vertex set of G'.

In [12], Vadapalli and Srimani defined the trivalent Cayley graph $TC(n)$ with n symbols and their forms. In this paper, we use another definition of the trivalent Caylay graph. Equivalency of the following definition and the original definition in [12] was proved in [9] and [10].

Definition 1: Let $n \geq 2$ be a positive integer. *The trivalent Cayley graph* $TC(n)$ has the vertex set $V(TC(n)) = \{(k, v_0v_1 \cdots v_{n-1}) | 0 \leq k \leq n-1, v_i \in \{0, 1\}\}$ and each vertex $v = (k, v_0v_1 \cdots v_{n-1})$ in $V(TC(n))$ is adjacent to vertices $u = (k + 1 \bmod n, v_0v_1 \cdots v_{k-1}\overline{v_k}v_{k+1} \cdots v_{n-1})$, $x = (k - 1 \bmod n, v_0v_1 \cdots v_{k-2}\overline{v_{k-1}}v_k \cdots v_{n-1})$ and $y = (k, v_0v_1 \cdots v_{k-1}\overline{v_k}v_{k+1} \cdots v_{n-1})$ where $\overline{v_k}$ means the complement of v_k.

Figure 1 and Fig. 2 shows $TC(3)$ and $TC(4)$, respectively.

Definition 2: Let $n \geq 3$ be a positive integer. *The cube-connected cycles* $CCC(n)$ has the vertex set $V(CCC(n)) = \{(k, v_0v_1 \cdots v_{n-1}) | 0 \leq k \leq n-1, v_i \in \{0, 1\}\}$ and each vertex $v = (k, v_0v_1 \cdots v_{n-1})$ in $V(CCC(n))$ is adjacent to vertices $u = (k + 1 \bmod n, v_0v_1 \cdots v_{n-1})$, $x = (k - 1 \bmod n, v_0v_1 \cdots v_{n-1})$ and $y = (k, v_0v_1 \cdots v_{k-1}\overline{v_k}v_{k+1} \cdots v_{n-1})$.

For a vertex $v = (k, v_0v_1 \cdots v_{n-1})$ in $TC(n)$ or $CCC(n)$, the value k is called the *level* of v. The sequence $v_0v_1 \cdots v_{n-1}$ is called the *sequence* of v. If the parity of the sequence $v_0v_1 \cdots v_{n-1}$ is odd, that is, a cardinality of the set $\{v_i | v_i =$

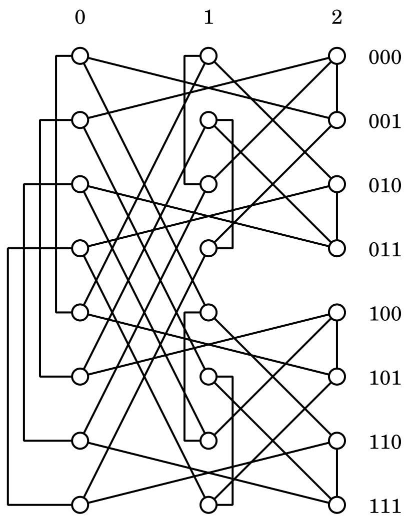

Fig. 1 Trivalent Cayley graph $TC(3)$.

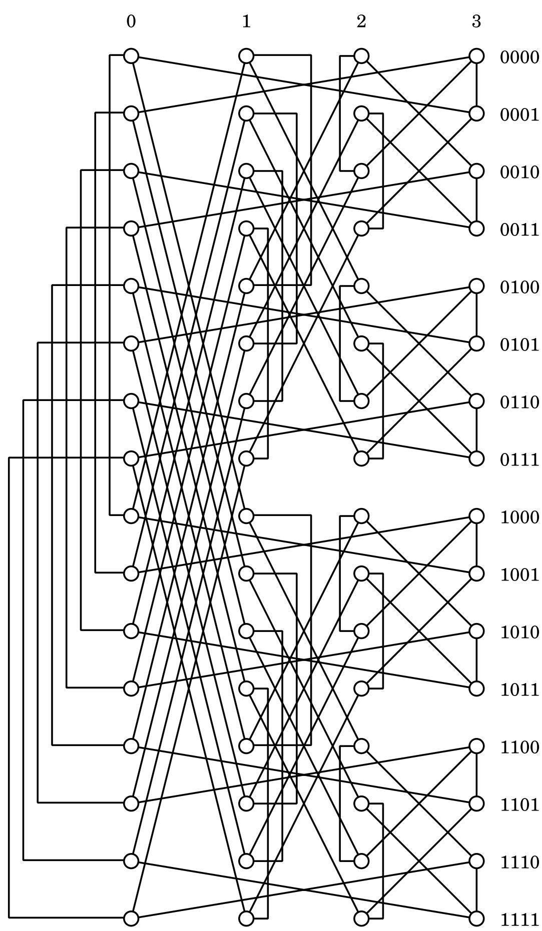

Fig. 2 Trivalent Cayley graph $TC(4)$.

$1, 0 \leq i \leq n-1\}$ is odd, then the sequence of v is called *odd*, otherwise the sequence of v is called *even*.

3. Constructing a Minimum Feedback Vertex Set

In this section, we construct minimum feedback vertex sets in trivalent Cayley graphs. We first show the lower bound of the size of the minimum feedback vertex set in trivalent Cayley graphs.

From Proposition 1, the following proposition is obtained.

Proposition 4: Let $F(n)$ be a feedback vertex set in the trivalent Cayley graph $TC(n)$. Then,

$$|F(n)| \geq n \times 2^{n-2} + 1.$$

In Sect. 3.1, we give minimum feedback vertex sets for $TC(2), TC(3)$ and $TC(4)$, individually. For $n \geq 5$, we prove the existence of the minimum feedback vertex set by induction in Sect. 3.2.

3.1 Minimum Feedback Vertex Sets for Small n

For $n = 2, 3$ and 4, minimum feedback vertex sets for $TC(n)$ are shown in the following theorem.

Theorem 1: For $n = 2, 3$ and 4, the size of the minimum feedback vertex set $F(n)$ in the trivalent Cayley graph $TC(n)$ is $n \times 2^{n-2} + 1$.

Proof: The following sets $F(2), F(3)$ and $F(4)$ are minimum feedback vertex sets with size 3, 7 and 17 of $TC(2)$, $TC(3)$, and $TC(4)$, respectively.

$$F(2) = \{(0,00),(0,01),(0,10)\},$$

$$F(3) = \left\{\begin{matrix}(0,000),(1,101),(1,111),(2,000),\\(2,010),(2,100),(2,110)\end{matrix}\right\},$$

$$F(4) = \left\{\begin{matrix}(0,0101),(1,0011),(1,0100),(1,0110),\\(1,1000),(2,0001),(2,0100),(2,1101),\\(2,1111),(3,0001),(3,0010),(3,0101),\\(3,0111),(3,1000),(3,1010),(3,1100),\\(3,1110)\end{matrix}\right\}.$$

□

Figure 3, Fig. 4 and Fig. 5 show $TC(2)-F(2)$, $TC(3)-F(3)$ and $TC(4)-F(4)$, respectively. These graphs do not include any cycle.

3.2 A Minimum Feedback Vertex Set for Large n

For $n \geq 5$, we show a feedback vertex set whose size equal to the lower bound, by induction on n. First, we focus on two induced subgraphs in $TC(n)$. One is isomorphic to the subdivision of $TC(n-1)$, and the other is isomorphic to a subgraph of cube-connected cycles $CCC(n-1)$. We give feedback vertex sets in these subgraphs and construct a minimum feedback vertex set in $TC(n)$ from those sets.

We decompose $V(TC(n))$ into X, Y such that

$$X = \{(k, 0v_1v_2\cdots v_{n-1})\} \cup \{(0, 1v_1v_2\cdots v_{n-1})\},$$
$$Y = V(TC(n)) - X$$

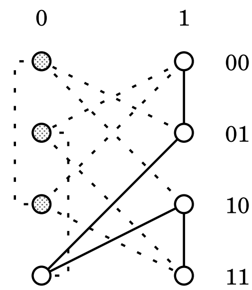

Fig. 3 A minimum feedback vertex set in $TC(2)$.

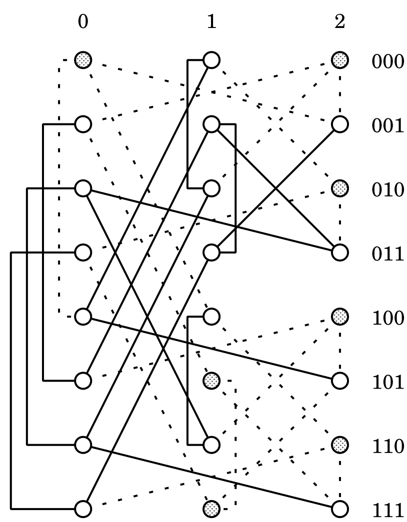

Fig. 4 A minimum feedback vertex set in $TC(3)$.

$$= \{(k, 1v_1v_2\cdots v_{n-1}) | 1 \le k \le n-1\}.$$

Figure 6 illustrates a partition of $TC(n)$ into X and Y.

Induced subgraphs $\langle X\rangle$ and $\langle Y\rangle$ have the following properties:

Lemma 1: Let $TC'(n-1)$ be a subdivision of $TC(n-1)$ such that every edge $((n-2, v_0v_1\cdots v_{n-2}), (0, v_0v_1\cdots v_{n-3}\overline{v_{n-2}}))$ is replaced into a 3-path $(n-2, v_0v_1\cdots v_{n-2}), (n-1, v_0v_1\cdots v_{n-2}), (n, v_0v_1\cdots v_{n-2}), (0, v_0v_1\cdots v_{n-3}\overline{v_{n-2}})$. Then $TC'(n-1)$ is isomorphic to $\langle X\rangle$.

Proof: The mapping $g : V(TC'(n-1)) \to X$ is defined as follows:

$$g((k, v_0v_1\cdots v_{n-2})) = \begin{cases} (k+1, 0v_0v_1\cdots v_{n-2}) & \text{if } 0 \le k \le n-2, \\ (0, 0v_0v_1\cdots v_{n-3}\overline{v_{n-2}}) & \text{if } k = n-1, \\ (0, 1v_0v_1\cdots v_{n-3}\overline{v_{n-2}}) & \text{if } k = n. \end{cases}$$

The mapping g is a bijection. Next, we prove that g is an isomorphism. Each vertex of level $n-1$ and n in $TC'(n-1)$ is of degree 2 and others are of degree 3. Then, $|E(TC'(n-1))| = |E(\langle X\rangle)| = (3n+1)2^{n-2}$. To prove that g is an isomorphism, it is sufficient to prove that g preserves adjacency. For an edge $(u, v) = ((k, v_0v_1\cdots v_{n-2}), (k, v_0v_1\cdots \overline{v_k}\cdots v_{n-2}))$ in $E(TC'(n-1))$, $0 \le k \le n-2$,

$$g(u) = (k+1, 0v_0v_1\cdots v_{n-2}),$$

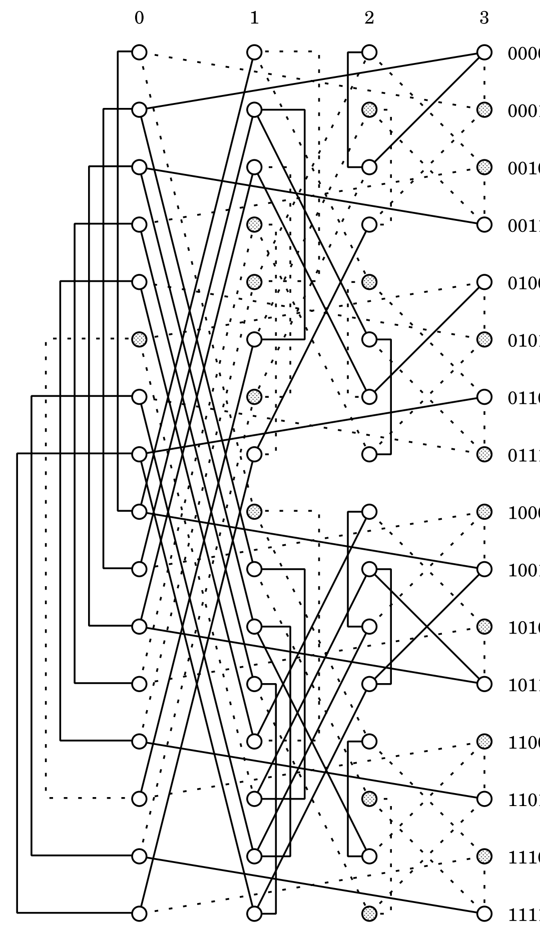

Fig. 5 A minimum feedback vertex set in $TC(4)$.

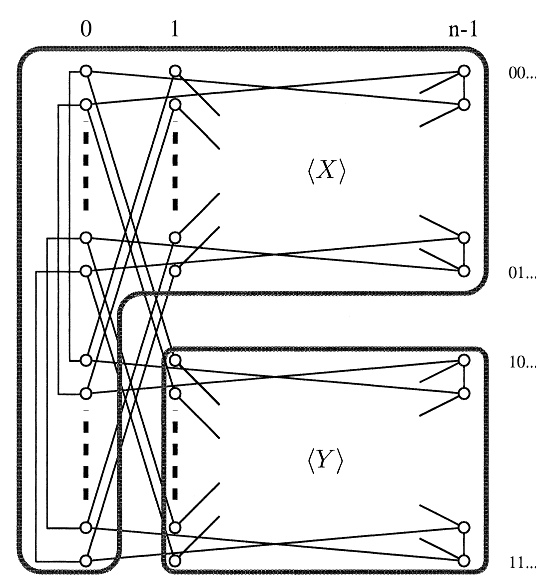

Fig. 6 Partition of $TC(n)$ into X and Y.

$$g(v) = (k+1, 0v_0v_1\cdots \overline{v_k}\cdots v_{n-2}).$$

Thus $(g(u), g(v)) \in E(\langle X\rangle)$. For an edge $(u, v) = ((k, v_0v_1\cdots v_{n-2}), (k+1, v_0v_1\cdots \overline{v_k}\cdots v_{n-2}))$ in $E(TC'(n-1))$,

$0 \le k \le n-3$,

$$g(u) = (k+1, 0v_0v_1\cdots v_{n-2}),$$
$$g(v) = (k+2, 0v_0v_1\cdots \overline{v_k}\cdots v_{n-2}).$$

Thus $(g(u), g(v)) \in E(\langle X\rangle)$. For an edge $(u,v) = ((n-2, v_0v_1\cdots v_{n-2}), (n-1, v_0v_1\cdots v_{n-2}))$ in $E(TC'(n-1))$,

$$g(u) = (n-1, 0v_0v_1\cdots v_{n-2}),$$
$$g(v) = (0, 0v_0v_1\cdots \overline{v_{n-2}}).$$

Thus $(g(u), g(v)) \in E(\langle X\rangle)$. For an edge $(u,v) = ((n-1, v_0v_1\cdots v_{n-2}), (n, v_0v_1\cdots v_{n-2}))$ in $E(TC'(n-1))$,

$$g(u) = (0, 0v_0v_1\cdots v_{n-3}\overline{v_{n-2}}),$$
$$g(v) = (0, 1v_0v_1\cdots v_{n-3}\overline{v_{n-2}}).$$

Thus $(g(u), g(v)) \in E(\langle X\rangle)$. For an edge $(u,v) = ((n, v_0v_1\cdots v_{n-2}), (0, v_0v_1\cdots \overline{v_{n-2}}))$ in $E(TC'(n-1))$,

$$g(u) = (0, 1v_0v_1\cdots v_{n-3}\overline{v_{n-2}}),$$
$$g(v) = (1, 0v_0v_1\cdots v_{n-3}\overline{v_{n-2}}).$$

Thus $(g(u), g(v)) \in E(\langle X\rangle)$. The mapping g preserves adjacency. Since $|E(TC'(n-1))| = |E(\langle X\rangle)|$, g is an isomorphism. □

Let us leave the investigation of $\langle X\rangle$, and turn to $\langle Y\rangle$. Before we treat the subgraph $\langle Y\rangle$, we notice on a specific subgraph of the cube-connected cycles.

Definition 3: Let $n \ge 3$ be an integer. The n-dimensional *unwrapped cube-connected cycles* $UC(n)$ is a spanning subgraph of cube-connected cycles $CCC(n)$ obtained by removing all edges joining level 0 vertex and level $n-1$ vertex.

Lemma 2: $UC(n-1)$ is isomorphic to $\langle Y\rangle$.

Proof: A mapping $f : V(UC(n-1)) \to Y$ is defined as follows:

$$f((k, v_0v_1\cdots v_{n-2}))$$
$$= (k+1, 1v_0v_1\cdots v_{k-1}\overline{v_k}\,\overline{v_{k+1}}\cdots\overline{v_{n-2}}).$$

The mapping f is a bijection. We prove that f preserves adjacency. For an edge $(u,v) = ((k, v_0v_1\cdots v_{n-2}), (k+1, v_0v_1\cdots v_{n-2}))$ in $E(UC(n-1))$, $0 \le k \le n-3$,

$$f(u) = (k+1, 1v_0v_1\cdots v_{k-1}\overline{v_k}\,\overline{v_{k+1}}\cdots\overline{v_{n-2}}),$$
$$f(v) = (k+2, 1v_0v_1\cdots v_{k-1}v_k\overline{v_{k+1}}\cdots\overline{v_{n-2}}).$$

Thus $(f(u), f(v)) \in E(\langle Y\rangle)$. For an edge $(u,v) = ((k, v_0v_1\cdots v_{n-2}), (k, v_0v_1\cdots v_{k-1}\overline{v_k}v_{k+1}\cdots v_{n-2}))$ in $E(UC(n-1))$, $0 \le k \le n-2$,

$$f(u) = (k+1, 1v_0v_1\cdots v_{k-1}\overline{v_k}\,\overline{v_{k+1}}\cdots\overline{v_{n-2}}),$$
$$f(v) = (k+1, 1v_0v_1\cdots v_{k-1}\overline{\overline{v_k}}\,\overline{v_{k+1}}\cdots\overline{v_{n-2}})$$
$$= (k+1, 1v_0v_1\cdots v_{k-1}v_k\overline{v_{k+1}}\cdots\overline{v_{n-2}}),$$

where $\overline{\overline{v_k}}$ is the complement of complement, that is, equal to v_k. Thus $(f(u), f(v)) \in E(\langle Y\rangle)$ and therefore f perserves adjacency. Since $|E(UC(n-1))| = |E(\langle Y\rangle)| = (3n-5)2^{n-2}$, f is an isomorphism. □

We now prepare two feedback vertex sets in $UC(n)$ corresponding to the cases n is odd or even. If n is odd, we use a feedback vertex set introduced in [11].

Theorem 2: For even $n \ge 4$, let $F_1(n)$ be a subset of the vertex set of $UC(n)$ such that

$$F_1(n) = \left\{(k, v_0v_1\cdots v_{n-1}) \,\middle|\, \begin{array}{l}\text{odd } k,\\ \text{the sequence is odd}\end{array}\right\}$$
$$\cup\{(0, v_0v_1\cdots v_{n-1}) | \text{the sequence is even}\}.$$

Then $F_1(n)$ is a feedback vertex set in $UC(n)$ with the size $(n+2)\times 2^{n-2}$. Moreover, each component in $UC(n) - F_1(n)$ contains exactly one vertex that has either the level 0 or level $n-1$.

Proof: From the definition of $F_1(n)$, all vertices that have the level 0 and the odd sequence are isolated vertices in $UC(n) - F_1(n)$. For a vertex $v = (n-1, v_0v_1\cdots v_{n-1})$ such that the sequence is even and the level is $n-1$, we consider the component C_v that contains the vertex v. The component C_v is illustrated in Fig. 7. Since vertices that have the even sequences exist in $UC(n) - F_1(n)$ except the level 0, C_v contains a path P such that $(n-1, v_0v_1\cdots v_{n-1}), (n-2, v_0v_1\cdots v_{n-1}), \ldots, (1, v_0v_1\cdots v_{n-1})$. Vertices that are of the odd sequence and even level in $UC(n) - F_1(n)$ have the degree one. Thus, each vertex that has the odd sequence is connected to exactly one component in $UC(n) - F_1(n)$. Since there is no edge in $UC(n) - F_1(n)$ joining two vertices that have different even sequences, the component C_v has exactly one vertex that have the level $n-1$, namely, v. Therefore, $UC(n) - F_1(n)$ has at least $2 \times 2^{n-1}$ components.

Finally, we show that there exists no component that contains neither level 0 vertex nor level $n-1$ vertex. The component that contains a level 0 vertex is an isolated vertex. The component that contains a level $n-1$ vertex has $(3n/2) - 2$ vertices. Number of vertices contained in those component is

$$2^{n-1} + 2^{n-1} \times \left(\frac{3n}{2} - 2\right) = (3n-2) \times 2^{n-2}.$$

From $|V(UC(n))| - |F_1(n)| = n \times 2^n - (n+2) \times 2^{n-2} = (3n-2) \times 2^{n-2}$, there is no component that contains neither level 0 vertex nor level $n-1$ vertex. Therefore, $F_1(n)$ is a feedback vertex set in $UC(n)$ with cardinality $(n+2)\times 2^{n-2}$. Moreover, each component in $UC(n) - F_1(n)$ contains exactly one vertex that has either level 0 or level $n-1$. □

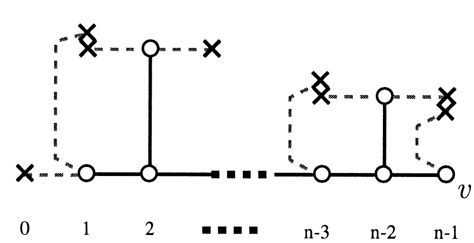

Fig. 7 A component C_v in $UC(n) - F_1(n)$.

Theorem 3: [11] For odd $n \geq 5$, let $F_2(n)$ be a subset of the vertex set of $CCC(n)$ such that

$$
\begin{aligned}
&F_2(n)\\
&= \{(0, v_0 v_1 \cdots v_{n-1}) | \text{the sequence is even}\}\\
&\quad \cup \{(n-1, v_0 v_1 \cdots v_{n-1}) | \text{the sequence is odd}\}\\
&\quad \cup \left\{ (s, v_0 v_1 \cdots v_{n-1}) \,\middle|\, \begin{array}{l} \text{the sequence is odd,} \\ v_0 \neq v_1 \text{ and} \\ \text{odd } s \leq n-4 \end{array} \right\}\\
&\quad \cup \left\{ (t, v_0 v_1 \cdots v_{n-1}) \,\middle|\, \begin{array}{l} \text{the sequence is even,} \\ v_0 = v_1 \text{ and} \\ \text{odd } t \geq 3 \end{array} \right\}\\
&\quad \cup \left\{ (n-3, v_0 v_1 \cdots v_{n-1}) \,\middle|\, \begin{array}{l} \text{the sequence is even} \\ \text{and } v_0 \neq v_1. \end{array} \right\}.
\end{aligned}
$$

Then $F_2(n)$ is a feedback vertex set in $CCC(n)$ with the size $(n+2) \times 2^{n-2}$. Moreover, each component in $CCC(n) - F_2(n)$ contains exactly one vertex either of level 0 or level $n-1$.

From Theorem 3 and Proposition 2, the following corollary is obtained.

Corollary 1: For odd $n \geq 5$, $F_2(n)$ is a feedback vertex set in $UC(n)$ with the property that each component in $UC(n) - F_2(n)$ contains exactly one vertex either of level 0 or level $n-1$.

Lemma 3: For an edge (u, v) in $E(TC(n))$ if $u \in X$ and $v \in Y$, then u has the level 0 and v has the level 1 or $n-1$.

Proof: All vertices in $TC(n)$ have degree 3. Then, if a vertex v in X is adjacent to a vertex in Y, then the degree of v at $\langle X \rangle$ is less than 3. By the definition of $\langle X \rangle$, the level of vertex v must be 0. For the similar reason, if a vertex u in Y has the degree less than 3 at $\langle Y \rangle$, the level of u must be 1 or $n-1$. □

Theorem 4: For the trivalent Cayley graph $TC(n)$, the size of minimum feedback vertex set $F(n)$ is $n \times 2^{n-2} + 1$.

Proof: For $n \geq 5$, we prove by the induction on n. Let $F(n-1)$ be a minimum feedback vertex set in $TC(n-1)$ of size $(n-1) \times 2^{n-3} + 1$. Let $F(n)$ be a subset of the vertex set of $TC(n)$ such that

$$
\begin{aligned}
&F(n)\\
&= \{(k+1, 0v_0 v_1 \cdots v_{n-2}) |\\
&\qquad (k, v_0 v_1 \cdots v_{n-2}) \in F(n-1)\}\\
&\quad \cup \begin{cases} \{(k+1, 1v_0 v_1 \cdots v_{k-1} \overline{v_k}\, \overline{v_{k+1}} \cdots \overline{v_{n-2}}) | \\ \quad (k, v_0 v_1 \cdots v_{n-2}) \in F_1(n-1)\} \text{if } n \text{ is odd} \\ \{(k+1, 1v_0 v_1 \cdots v_{k-1} \overline{v_k}\, \overline{v_{k+1}} \cdots \overline{v_{n-2}}) | \\ \quad (k, v_0 v_1 \cdots v_{n-2}) \in F_2(n-1)\} \text{if } n \text{ is even.} \end{cases}
\end{aligned}
$$

We prove that $F(n)$ is a feedback vertex set in $TC(n)$.

The vertex set $V(TC(n)) - F(n)$ is decomposed into X', Y' such that

$$
\begin{aligned}
X' &= (\{(k, 0v_1 v_2 \cdots v_{n-1})\} \cup \{(0, 1v_1 v_2 \cdots v_{n-1})\})\\
&\quad - \{(k+1, 0v_0 v_1 \cdots v_{n-2}) |\\
&\qquad (k, v_0 v_1 \cdots v_{n-2}) \in F(n-1)\},\\
Y' &= (V(TC(n)) - F(n)) - X'.
\end{aligned}
$$

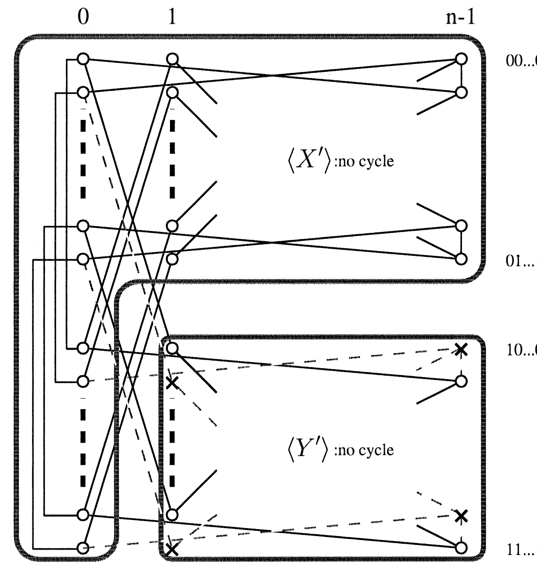

Fig. 8 Partition of $TC(n) - F(n)$ into X' and Y'.

Figure 8 illustrates a partition of $TC(n) - F(n)$ into X' and Y'.

From Lemma 1 and the definition of $F(n)$, $\langle X' \rangle$ has no cycle. Furthermore, from Lemma 2, Proposition 3, Theorem 2 and Corollary 1, $\langle Y' \rangle$ has no cycle without regard to whether n is odd or even. From Lemma 3, each component in $\langle Y' \rangle$ is connected to a component in $\langle X' \rangle$ with exactly one edge. Therefore, they do not produce any cycle. Thus, $F(n)$ is a feedback vertex set in $TC(n)$.

Finally, we consider the cardinality of $F(n)$.

$$
\begin{aligned}
|F(n)| &= |F(n-1)| + \begin{cases} |F_1(n-1)| & \text{if } n \text{ is odd} \\ |F_2(n-1)| & \text{if } n \text{ is even} \end{cases}\\
&= (n-1) \times 2^{n-3} + 1 + (n+1) \times 2^{n-3}\\
&= n \times 2^{n-2} + 1.
\end{aligned}
$$

We can conclude that $F(n)$ is a minimum feedback vertex set in $TC(n)$. □

4. Conclusion

In this paper, we have investigated the feedback vertex set problem for the trivalent Cayley graph $TC(n)$ and have shown a minimum feedback vertex set of it. This result is based on the Cayley graph representation of trivalent Cayley graphs developed in [10] and the result of the MFVS in cube-connected cycles obatined in [11].

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Appendix

Recently we have learned that Y. Suzuki and K. Kaneko have published the following article:
Y. Suzuki and K. Kaneko, "Minimum feedback node sets in trivalent Cayley graphs," IEICE Trans. Inf. & Syst., Vol.E86-D, No.9, pp.1634–1636, Sept. 2003.

In the article, they have shown the minimum feedback vertex sets in trivalent Cayley graphs by constructing examples and that the size of MFVS is equal to the lower bound.

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