Numerical Calculation of Critical Current Density and Third Harmonic Voltage in Superconducting Films

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\textbf{Abstract}

When an AC magnetic field of a suitable magnitude is applied locally normal to a wide surface of a superconducting film, a third harmonic voltage is induced in a pick-up coil. This measurement is used for an evaluation of the critical current density of the film. Mawatari \textit{et al}. analyzed the relationship between the third harmonic voltage amplitude ($V_3$) and the driving current amplitude for the field ($I_0$) by using the critical state model and a simple assumption that the magnetic field is locally parallel to a flat surface. In this study, this relationship is numerically analyzed by Finite Element Method (FEM) for a condition similar to the experiment by Mawatari \textit{et al}. This result is compared with the above theoretical and experimental results. It is found that the scaling of $V_3$-$I_0$ curves holds and $I_0$ at which $V_3$ starts to appear is proportional to the critical current density, as prediction by Mawatari \textit{et al}. The effect of imperfections in the film on the $V_3$-$I_0$ curve is also investigated.

Keywords: Critical current, Superconducting films, FEM

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1 Introduction

A simple contactless measuring method of critical current density $J_c$ in superconducting thin films or coated tapes is required. Claassen et al. [1] proposed an inductive method to measure a third harmonic voltage of amplitude $V_3$ generated by a current induced in a superconducting thin film due to an external AC magnetic field, which was generated by a sinusoidal driving current $I_0 \cos \omega t$ flowing in a small coil placed close to the surface of the film. It is empirically known that the local $J_c$ is proportional to a threshold value of $I_0$ ($I_{c0}$) at which $V_3$ starts to appear.

The relationship between $I_0$ and $V_3$ was theoretically given by Mawatari et al. [2] by using the critical state model with an assumption that the magnetic field is parallel to the surface due to a shielding effect where magnetic flux penetrates the superconducting thin film. According to their theoretical analysis, $V_3$ is almost zero when $I_0$ is smaller than $I_{c0}$, which is proportional to $J_c d$, with $d$ denoting the film thickness. When $I_0 > I_{c0}$, on the other hand, the third harmonic voltage is expressed as $V_3 \exp(-i \theta_3) = \omega I_{c0} G(I_0/I_{c0})$, where $G(x)$ is a scaling function determined by a configuration of the coil and $\theta_3$ is a phase of the third harmonic voltage. However, the magnetic field is not exactly parallel to the surface as assumed in [2], especially around the central area of the coil where the magnetic field is strong. Hence, the validity of the above approximation should be clarified.

In this paper, the magnetic field distribution in a superconducting film was numerically calculated by Finite Element Method (FEM) to estimate the third harmonic voltage and the scaling of $V_3 I_0$ curves was examined. This result is compared with the above theoretical and experimental results by Mawatari.
et al. In addition, the effect of locally deteriorated region in a film on the signal was also investigated to examine the applicability of this measuring method for characterization of superconducting thin films.

2 Simulation

J MAG studio version 7 of Japan Research Institute was used for the calculation. For comparison with experimental results, a similar circumstances to experiment was assumed: A single coil of internal and external diameters of 2 mm and 5 mm and a height of 1 mm is mounted at a position of 0.2 mm apart from the surface of a large-area superconducting film of thickness 0.6 μm. The number of windings of the coil is 400 turns. The coil axis is directed normal to a wide surface of a superconducting film. The magnetic flux density distribution is calculated assuming the critical state model inside the superconducting film.

Because of the azimuthal symmetry of the system, we calculated the field and current distributions inside the region of 1 degree, 1/360, of the whole system. This region was divided into 60 parts in radial direction and 56 layers in axial direction: 12 layers in the film and 34 and 10 layers in the space above and below the film.

The numerical analysis was carried out using the Bean model in the absence of external magnetic field. On the other hand, the value of $J_c$ was changed in the range of $1.0 \times 10^9$ to $1.0 \times 10^{10}$ A/m². This was equivalent to the application of external magnetic field as in experiment, since the superposition of uniform DC magnetic field does not influence the current distribution. The distributions of magnetic field and current inside the superconductor were determined, and
the voltage induced in the driving coil was derived from the magnetic flux which interlinked the driving coil. The amplitude $V_3$ and phase $\theta_3$ of third harmonic voltage were derived by Fourier analysis.

$V_3-I_0$ curves were derived for the critical current density in the range of $1.0 \times 10^9 - 1.0 \times 10^{10}$ A/m$^2$. The predicted scaling of $V_3-I_0$ curves is examined and the proportionality between $I_{c0}$ and $J_c$ is checked. In addition, to investigate a modification of the induced third harmonic voltage $V_3$ by an imperfection of the film, a deteriorated region of a ring shape was introduced in the center of the coil. The width of the ring was 0.1 mm and the inner radius was 0.5, 1.1 and 2.0 mm. The critical current density in the ring was assumed to be 10% of the value of uniform region, $1.0 \times 10^{10}$ A/m$^2$.

3 Results and Discussion

Fig. 1 shows the drive-current amplitude $I_0$ dependence of harmonic voltage amplitude $V_3$. As $I_0$ increases beyond some threshold current $I_{c0}$, $V_3$ starts to appear. $I_{c0}$ is the current at which the magnetic field penetrates through the film as indicated by the magnetic field distribution. In fact, Fig. 2(a) and (b) show the magnetic field distributions just below and above $I_{c0}$, respectively. In addition, Fig. 3 shows that $I_{c0}$ is proportional to $J_c$. These results show that the electromagnetic response becomes nonlinear when the magnetic field penetrates through the film. Fig. 4 shows calculated curves of $V_3/I_0$ vs $I_0/I_{c0}$. It is seen that these curves meet on a single curve. Such a scaling behavior agrees with theoretical and experimental results by Mawatari et al. It means that an assumption by Mawatari et al. that the magnetic field is parallel to the surface where the magnetic flux penetrates the film is correct. When $I_0$
is just above $I_{c0}$, the magnetic flux in the region, where the flux is almost parallel to the surface, penetrates through the film and the magnetic flux in other regions does not penetrate through the film, resulting in no contribution to $V_3$. This is considered as the reason for the correct hypothesis by Mawatari et al.

Fig. 5 shows $V_3$-$I_0$ curves when a damaged region of a ring shape exists in the film. It is seen that the curves are appreciably modified by the existence of such defects. This suggests that this measuring method is useful also for a detection of such damage in the film. Hence, a further detailed analysis is needed for finding the correlation of the type of damage and the $V_3$-$I_0$ curves.

4 Summary

The magnetic field distribution in a superconducting film was numerically calculated by Finite Element Method (FEM) to estimate the third harmonic voltage. This result was compared with the theoretical and experimental results by Mawatari et al. In addition, the effect of locally deteriorated region in a film on the third harmonic voltage was also investigated to examine the applicability of this measuring method for characterization of films.

1. It is shown that $V_3$ starts to appear at the threshold current $I_{c0}$ where the AC magnetic field penetrates through the film. In addition, $I_{c0}$ is found to be proportional to $J_c$. Hence, the critical current density of the film can be estimated from $I_{c0}$.

2. It is shown that the calculated curves of $V_3/I_0$ vs $I_0/I_0$ meet on a single curve. Such scaling behavior agrees with the theoretical and experimental results by Mawatari et al. It means that the simple assumption by
Mawatari et al. is suitable.

3. It is found that the $V_3$-$I_0$ curve is appreciably modified when a damaged region of a ring shape exists in the film. This suggests that this measuring method is useful also for detection of such damages in the film.

References


Figure captions

Fig. 1. Driving current amplitude $I_0$ dependence of third harmonic voltage amplitude $V_3$.

Fig. 2. Distribution of magnetic field at (a) $I_0/I_{c0} = 0.85$ (b) $I_0/I_{c0} = 1.125$.

Fig. 3. Threshold current $I_{c0}$ vs critical current density $J_c$ of film.

Fig. 4. Scaled curves of $V_3/I_{c0}$ vs $I_0/I_{c0}$.

Fig. 5. $V_3$-$I_0$ curves when a damaged region of ring shape exists in the film.
Fig. 1: H. Wada et al./FDP – 48/ISS2002
Fig. 2(a): H. Wada et al. / FDP − 48/ ISS2002
Fig. 2(b): H. Wada et al./FDP – 48/ISS2002
Fig. 3: H. Wada et al. *FDP* – 48/ISS2002
Fig. 4: H. Wada et al./FDP – 48/ISS2002
Figure 5: H. Wada et al. / FDP - 48/ISS2002