Reconstructing a Polyhedron from the Apparent Shape of its Edges

by

Seiichiro KAMATA, Seiji ISHIKAWA and Kiyoshi KATO

Synopsis

This paper describes a technique involving reconstructing a polyhedron from its projected images. The proposed technique is based on binocular vision, and it makes use of the gradients and the lengths of the projected edges of a polyhedron. As the gradient of a projected edge changes greatly under small movement of an observer, depending on the location of the edge, one may expect a binocular vision system with small parallax. This possibility distinguishes the present technique from other standard stereo vision techniques. An experiment employing the proposed technique was carried out for the reconstruction, and satisfactory results were obtained under small parallax.

1 Introduction

Polyhedral scene analysis is a basic research field among many computer vision problems. However, it is of great importance, since it offers a general idea for analyzing a scene with various objects [1], [2], [3].

Among several useful techniques [5], [6], [7] for extracting the three-dimensional information from a polyhedral scene, the binocular vision technique [4] is a natural idea derived from the human mechanism for detecting distance between an observer and an object. Binocular vision requires correspondence between two images, that suggests the need for small parallax with the vision system, since the image differences are only slight under the small parallax, which implies high correspondence. In general, however, the small parallax is incompatible with obtaining effective depth information. An effective technique involving the small parallax is provided by using the gradients of projected edges. The gradient of a projected edge changes greatly under small movements by an observer depending on its location. Therefore, by making use of the change in the gradient between the left and the right view, it is possible to obtain practical depth information regarding several bodies, even if the parallax is small. This possibility distinguishes the present technique from other standard stereo vision techniques.

Ishikawa [8] shows a technique for reconstructing faces on a polyhedron from the change in the gradients of its projected edges. There he refers only to the theoretical aspect of the face reconstruction problem. It is in fact necessary to show how it can be applied to practical polyhedral reconstruction problems by performing some experiments. Moreover, sensitivity analysis has also to be performed for the purpose of estimating the influence of the errors contained in the measurement of a polyhedral scene. In the present paper, a technique is described for locating the absolute positions of the edges on the polyhedron in the three-dimensional space from the orientation of these faces obtained from the reconstruction technique [8], and the experiment for the reconstruction is also presented with some results. The errors contained in the experimental result are also discussed, and satisfactory results were
obtained under small parallax.

2 Reconstructing a Polyhedron

The followings are assumed in the present paper: (a) projection is orthographic; (b) bodies are polyhedra; (c) the vision system is binocular; (d) complete line drawings are obtained after preprocessing from the images acquired by the vision system.

In the following argument, two coordinate systems are considered. In Fig. 1, the $xyz$ rectangular coordinate system is used, while the $uvw$ rectangular coordinate system represents an observer-oriented coordinate system. There, the $u$ axis is superimposed on the $y$ axis. Since orthographic projection is assumed, an observer looks at a scene from a point at infinity on the $w$ axis, which makes the angle $-\psi$ with the $x$ axis. The two coordinate systems are related with each other according to the equation

$$
\begin{pmatrix}
u \\
x \\
w
\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 \\
-\sin \phi & 0 & \cos \phi \\
\cos \phi & 0 & \sin \phi
\end{pmatrix}
\begin{pmatrix}
y \\
z
\end{pmatrix}
$$

It is shown in [8] that, since an orthographic projection is adopted, the orientation of a face on a polyhedron is recovered from the gradients of its projected edges. The origin is set at an arbitrary point to obtain a three-dimensional model of the polyhedron. Once the origin is given, the location of the faces which pass the origin is specified. A vertex on the intersection between each adjacent pair of these faces is then fixed in the three-dimensional space. In Fig. 2 (a), $V_s$ and $V_t$ are the vertices on a polyhedron, and $F_i$ and $F_j$ are the faces which make the intersection $V_sV_t$. Assume that the three-dimensional coordinates of $V_s$ are already known. Then the three-dimensional coordinates for $V_t$ are obtained, since the absolute locations of $F_i$ and $F_j$ are determined. Let faces $F_i$ and $F_j$ be expressed by

$$a_x + b_y + c_z = d_i,$$

$$a_x + b_y + c_z = d_j,$$

respectively. Assume that $V_sV_t = (x, y, z)$, and the length of the projection of $V_sV_t$ onto the $uv$ plane is equal to $p$. Then the following equations hold:

$$a_x + b_y + c_z = 0$$

$$a_x + b_y + c_z = 0$$

$$(x^2 + z^2) = p^2.$$

The third equality comes from $a^2 + b^2 = p^2$. Hence, by solving Eq. (1), $V_sV_t$ is determined and $V_t$ is obtained from the rotation $\overrightarrow{OV_t} = \overrightarrow{OV_s} + \overrightarrow{V_sV_t}$. Fig. 2 (b) shows another case of determining the coordinates of a vertex. Assume that face $F_i$ and vertex $V_s$ are already determined, and that $V_sV_t = (x, y, z)$. If we denote the gradient and the length of the projection of $V_sV_t$ onto the $uv$ plane by $g$ and $r$, respectively, the following equations hold:

(a) the case that faces $F_i$ and $F_j$ are known

(b) the case that only face $F_i$ is known

Fig. 1 Coordinate systems

Fig. 2 Determining the coordinates of a vertex $V_q$
Reconstructing a Polyhedron from the Apparent Shape of its Edges

pass vertex 1, their absolute locations are determined. Vertex 8 is then determined employing Eq. (1). Face $F_3$ is determined, since it passes vertex 8, and vertices 3 and 9 are determined using Eq. (1). Finally vertex 9 determines the absolute location of face $F_4$, and vertices 4 and 6 are then determined by Eq. (1). Vertices 2 and 5 are determined by Eq. (2).

3 Experiment

An experiment was performed based on the above argument for the reconstruction. Fig. 4 shows a flow chart of the reconstruction system. A binocular vision system is realized by a TV camera and computer system in the present research. Two pictures of a scene which contains a polyhedron are taken from different directions. Each of them is preprocessed to yield a line drawing using the context sensitive line finder [10]. If it is not a complete line drawing, completion is carried out by adding suitable length of lines. Since attention is focused on showing the way the polyhedron is reconstructed, the cases where completion is necessary are limited to those considered in [7]. Now we have a pair of complete line drawings of a polyhedron on this stage. Edges are corresponded between the two line drawings. Then, each face is determined with its relative location by referring to the correspondence. Finally, a polyhedron is reconstructed by calculating the coordinates of each vertex on it. The practical hardware system contains a TV camera and a computer as well as several peripheral devices (See Fig. 5). A polyhedron is placed on a round table which rotates around the vertical axis that passes the center of gravity of the table. A binocular vision system is realized by employing a single TV camera and rotating the table according to the parallax of the system, instead of employing two TV cameras. Some experimental results were obtained by the reconstruction system. In Fig. 6, dashed lines show the projection of a reconstructed polyhedron onto the $uv$ plane, while solid lines show the projection of the original polyhedron. The angle $\phi$ is $40^\circ$. The parameter denoting the parallax $\theta$ ranges from 2" to 10".
4 Discussion

Let us discuss the errors the reconstruction process yields using actual data obtained in the experiment. The error is evaluated by the difference between the length of each edge on the projection of an original polyhedron and the length of the edge on the projection of a reconstructed polyhedron. The error \( f \) is defined by

\[
f = E \left( \frac{\rho_i - \rho_o}{\rho_o} \right) \times 100(\%),
\]

where \( \rho_i \) is the length of edge \( i \) on the projection of the reconstructed polyhedron, and \( \rho_o \) is the length of edge \( i \) on the projection of the original polyhedron. \( E [\ast] \) is an averaging operator which, in this case, calculates the mean value of the ratios \( |\rho_i - \rho_o| / \rho_o \) for all the edges on the polyhedron. The errors on the right im-
Reconstructing a Polyhedron from the Apparent Shape of its Edges

5 Conclusion

A technique was shown for locating the absolute positions of the faces on a polyhedron in the three-dimensional space from the orientation of these faces obtained from the reconstruction technique [8]. Several experimental results were given along with the estimate of reconstruction errors. The present result confirms that the proposed reconstruction technique works well and it is particularly useful when the parallax is small in a binocular vision system.

References