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Testing nuclear forces by polarization transfer coefficients in $d(\vec{p},\vec{p})d$ and $d(\vec{p},\vec{d})p$ reactions at $E_{lab}^{p}=22.7$ MeV

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The proton to proton polarization transfer coefficients $K^p_v$, $K^p_r$, and $K^p_s$, and the proton to deuteron polarization transfer coefficients $K^d_v$, $K^d_r$, $K^d_s$, $K^{d--v}$, $K^{d--r}$, $K^{d--s}$, and $K^{d----v}$ were measured in $d(\vec{p},\vec{p})d$ and $d(\vec{p},\vec{d})p$ reactions, respectively, at $E_{lab}^{p}=22.7$ MeV. The data were compared to predictions of modern nuclear forces obtained by solving the three-nucleon Faddeev equations in momentum space. Realistic (semi)phenomenological nucleon-nucleon potentials combined with model three-nucleon forces and modern chiral nuclear forces were used. The AV18, CD Bonn, and Nijm I and II nucleon-nucleon interactions were applied alone or combined with the Tucson-Melbourne 99 three-nucleon force, adjusted separately for each potential to reproduce the triton binding energy. For the AV18 potential, the Urbana IX three-nucleon force was also used. In addition, chiral $NN$ potentials in the next-to-leading order and chiral two- and three-nucleon forces in the next-to-next-to-leading order were applied. Only when three-nucleon forces are included does a satisfactory description of all data result. For the chiral approach, the restriction to the forces in the next-to-leading order is insufficient. Only when going over to the next-to-next-to-leading order does one get a satisfactory description of the data, similar to the one obtained with the (semi)phenomenological forces.

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I. INTRODUCTION

A rich set of observables provided by three-nucleon (3N) reactions can be used to test modern nuclear forces. Presently, there are two theoretical approaches to construct them. In the traditional approach, the nucleon-nucleon ($NN$) potentials are derived in the framework of the meson-exchange picture alone or mixed with phenomenological assumptions. By adjusting parameters, so-called realistic, high-precision interactions such as the phenomenological AV18 [1] potential and the meson-theoretical CD Bonn [2] together with Nijm I and II [3] potentials are obtained. They provide a very good description of $NN$ data below about 350 MeV nucleon laboratory energy. All these potentials have in common, the fact that they fit the large set of $NN$ data with $\chi^2$ per datum close to 1, indicating essentially phase equivalence.

In a more modern framework of chiral effective field theory, nuclear forces are linked to the underlying strong interaction between quarks and gluons. They are derived from the most general effective Lagrangian for pions and nucleons, which is consistent with the spontaneously broken approximate chiral symmetry of QCD, using chiral perturbation theory ($\chi$PT) [4]. The $\chi$PT approach gives a deeper understanding of nuclear forces than the traditional approach and allows us to construct three- and more-nucleon forces consistent with the $NN$ interactions [5–7]. In practice, various contributions to the nuclear force in the $\chi$PT framework are organized in terms of the expansion in $Q/\Lambda$, where $Q$ is the soft scale corresponding to the nucleon external momenta and the pion mass and $\Lambda$ is the hard scale associated with the chiral symmetry breaking scale or an ultraviolet cutoff. At present, the two-nucleon system has been studied up to next-to-next-to-next-to-leading order ($NNNLO$) in the chiral expansion [7,8]. At this order, the two-nucleon force receives the contributions from one-pion exchange, two-pion exchange at the two-loop level, and (numerically irrelevant) three-pion exchange. In addition, one has to take into account all possible short-range contact interactions with up to four derivatives and the appropriate isospin-breaking effects. In the three- and more-nucleon sectors, the calculations have so far only been performed up to next-to-next-to-leading order (NNLO) [7]. In this work, we will show the results corresponding to the latest version of the chiral $NN$ forces introduced in Refs. [8,9] and based on the spectral function regularization scheme [10].

Recent studies of few-nucleon bound states and of 3N reactions provided numerous indications that three-nucleon forces (3NFs) form an important component of the potential energy of three interacting nucleons [11–16]. In the traditional approach, they are accounted for by adding model 3NFs, such as, e.g., the $2\pi$-exchange Tucson-Melbourne (TM) [17] or Urbana IX [18] interactions, with parameters adjusted to reproduce the experimental triton binding energy. Such a simple treatment allows us to cure some of the discrepancies between data and theory [19–29]. In the approach based on

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\( \chi PT \), nonvanishing 3NFs appear in the next-to-next-to-leading order (NNLO) of the chiral expansion. In addition to the \( 2\pi \)-exchange term, two other topologies appear. One of them corresponds to a contact interaction of three nucleons, and the second to a contact interaction of two nucleons exchanging in addition one pion with the third nucleon. The two free parameters of these two terms are adjusted by fitting two independent 3N observables (e.g., the triton binding and the nd doublet scattering length). The quality of the description of the 3N observables is then similar in both approaches [30]. The study of the details of the 3NFs is a lively topic of present-day few-nucleon system studies.

In the present paper, we analyze the proton to proton and the proton to deuteron spin transfer coefficients measured in \( d(p, d)p \) and \( d(p, d')p \) reactions, respectively, at \( E_{\text{lab}}^p = 22.7 \text{MeV} \) [31–33]. The existing data for polarization transfer coefficients in elastic nucleon-deuteron (Nd) scattering are restricted to few experiments in the \( np \) restricted to few experiments in the \( np \) system (\( E_{\text{lab}}^p = 19 \text{MeV} \) [37]). After a short description of the theoretical calculations in Sec. II, we show in Sec. III the data and compare them to different theoretical predictions. The summary and conclusions follow in Sec. IV.

II. CALCULATIONS

In this work, we will employ two different methods to solve the nucleon-deuteron scattering problem. Our first scheme is based on Faddeev equations. The nucleon-deuteron elastic scattering with neutron and protons interacting through a \( NN \) potential \( V \) and through a 3NF \( V_4 \) is described in terms of a breakup operator \( T \) satisfying the Faddeev-type integral equation [15,38,39]

\[
T = tP + (1 + tG_0)V_4^{(1)}(1 + P) + tPG_0T \\
+ (1 + tG_0)V_4^{(1)}(1 + P)G_0T.
\]

The two-nucleon (2N) \( t \)-matrix \( t \) results from the interaction \( V \) through the Lippmann-Schwinger equation. The permutation operator \( P = P_{12}P_{23} + P_{13}P_{23} \) is given in terms of the transposition \( P_{ij} \) which interchanges nucleons \( i \) and \( j \), and \( G_0 \) is the free 3N propagator. Finally, the operator \( V_4^{(1)} \) appearing in Eq. (1) is part of the full 3NF \( V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)} \) and is symmetric under exchange of nucleons 2 and 3. For instance, in the case of the \( 2\pi \)-exchange 3NF, such a decomposition corresponds to the three possible choices of the nucleon which undergoes off-shell \( \pi-N \) scattering. It is understood that the operator \( T \) acts on the incoming state \( |\phi_i\rangle = |\tilde{q}_0\rangle|\phi_2\rangle \) which describes the free nucleon-deuteron motion with relative momentum \( \tilde{q}_0 \) and the deuteron wave function \( |\phi_2\rangle \). The physical picture underlying Eq. (1) is revealed after iteration which leads to a multiple scattering series for \( T \).

The elastic Nd scattering transition operator \( U \) is given in terms of \( T \) by [15,38,39]

\[
U = PG_0^{-1} + PT + V_4^{(1)}(1 + P) + V_4^{(1)}(1 + P)G_0T.
\]

We solve Eq. (1) in momentum space using a partial wave decomposition for each total angular momentum \( J \) and parity of the 3N system. To achieve converged results, a sufficiently high number of partial waves have been used. Calculations with and without 3NF were performed including all 3N partial wave states with total two-body angular momenta up to \( j = 5 \). In the case when the 3NF is switched off, Eq. (1) is solved for \( J \) up to 25/2. When the shorter ranged 3NF is also active, it is sufficient to go only up to \( J \leq 13/2 \). In all calculations, we neglect the total isospin \( T = 3/2 \) contribution in the \( 1S_0 \) state and use in this state a \( np \) form of the \( NN \) interaction. Such a restriction to the \( np \) force for the \( 1S_0 \) state does not have a significant effect on the polarization transfer coefficients.

The second scheme is based on the Kohn variational principle, the \( S \)-matrix elements corresponding to a 3N scattering state with total angular momentum \( J \) can be obtained as the stationary point of the functional

\[
\left[ J S_{LL}^{\text{SS}} \right] = J S_{LL}^{\text{SS}} + i (\Psi_{LSJ}^T | H - E | \Psi_{LSJ}^T).
\]

The wave function \( \Psi_{LSJ}^T \) describes a 3N scattering state in which asymptotically an ingoing nucleon is approaching the deuteron in a relative angular momentum \( L \) and total spin \( S \). The parity of the state is given by \((-1)^L\). The wave function is expanded, using a partial wave decomposition, in terms of the pair correlated hyperspherical harmonic (PHH) basis as described in Ref. [29]. As in the Faddeev scheme, states up to \( J = 25/2 \) have been considered.

In our Faddeev calculations, the Coulomb interaction between two protons is totally neglected. A measurement

\[
\begin{array}{c}
\text{FIG. 1. Nucleon to nucleon spin transfer coefficients in Nd elastic scattering at } E_{\text{lab}}^{pp} = 22.7 \text{MeV. Crosses are } pp \text{ data from } [31–33]. \text{ Dashed line is the result of the hyperspherical harmonic expansion method with AV18 potential. Solid line is the corresponding result when the } pp \text{ Coulomb force is included.}
\end{array}
\]
effects have been studied on a few polarization transfer coefficients [40] as well as on other polarization observables [29] using the Kohn variational principle in conjunction with the PHH basis. In these calculations, the Coulomb force between the two protons has been considered without approximations, and the results confirm sizable Coulomb force effects in the energy range considered here. Therefore, to remove possible Coulomb force effects from the studied polarization transfer coefficients, we proceed in the following manner. Using the PHH expansion, we evaluate the studied polarization transfer coefficients with and without Coulomb force and employ the AV18 $NN$ interaction. This is displayed in Figs. 1–3. In this manner, we read off the shifts caused by the $pp$ Coulomb force.

Then we generate $nd$ data by applying those shifts to our $pd$ data. For the studied polarization transfers, the Coulomb force effects are restricted mostly to forward angles and to the region around $\theta_{c.m.} \approx 120^\circ$. For the proton to proton spin transfer coefficient $K_x^{(d)}$, they are of minor importance, whereas for $K_y^{(d)}$ and $K_x^{(d)}$, the Coulomb force effects significantly change the magnitude of these coefficients around $\theta_{c.m.} \approx 120^\circ$ (see Fig. 1). For the proton to vector-deuteron spin transfers,
Coulomb force effects are rather small with the exception of very forward angles, where they significantly decrease the magnitude (see Fig. 2). For $K_x'$ also some effects are seen around $\theta_{\text{c.m.}} \approx 120^\circ$. In the case of the proton to tensor-deuteron transfers shown in Fig. 3, only $K_y'$ (in the steep slope), $K_y'z'$, and $K_z'y'$ exhibit large Coulomb force effects in the region of c.m. angles around $\theta_{\text{c.m.}} \approx 120^\circ$. In subsequent figures, we include both the $pd$ and $nd$ data.

III. RESULTS

In Figs. 4–6, we show our data and compare them to theoretical predictions based on (semi)phenomenological $NN$ potentials alone or combined with the TM99 [41] or Urbana IX [18] 3NFs. The theoretical results were obtained by using the Faddeev approach, where we were able to also employ nonlocal interactions. The corresponding comparison for chiral forces is presented in Figs. 7–9. Comparison with experiment always means the Coulomb-corrected $nd$ data.

For the traditional approach based on high-quality (semi)phenomenological interactions, we took the $NN$ potentials AV18, CDBonn, Nijm I, and Nijm II and combined each of them with the $2\pi$-exchange TM99 3NF, adjusting the cutoff parameter of TM99 individually to get the experimental triton binding energy. The resulting cutoffs for these potentials are, respectively, 5.215, 4.856, 5.120, and 5.072 (in units of the pion mass $m_\pi$). From the predictions of these potentials alone...
or combined with the TM99 3NF, two bands, light and dark, respectively, were formed and are shown in Figs. 4–6.

For the proton to proton spin transfer coefficients (see Fig. 4), the effects of the Coulomb interaction are located in the region of c.m. angles around $\theta_{c.m.} \approx 120^\circ$. In that region, the differential cross section has its minimum. They are significant only for $K_y^{\gamma}$ and $K_z^{\gamma}$, and are practically negligible for $K_x^{\gamma}$. The realistic potentials alone provide a good description only of $K_y^{\gamma}$ and fail to reproduce the data for $K_x^{\gamma}$ and $K_z^{\gamma}$, especially around $\theta_{c.m.} \approx 120^\circ$. Including the TM99 3NF and, in case of AV18, also Urbana IX changes only slightly the predictions for $K_x^{\gamma}$. For $K_y^{\gamma}$ and $K_z^{\gamma}$, the effects of these 3NFs are significant in the region of c.m. angles around $\theta_{c.m.} \approx 120^\circ$, and their inclusion leads to a good description of the data.

For the corresponding proton to deuteron spin transfer coefficients ($K_x^{\gamma}$, $K_y^{\gamma}$, and $K_z^{\gamma}$, see Fig. 5), effects of the Coulomb force are practically negligible at angles where data exist and they are seen only for $K_y^{\gamma}$. For these observables also, the effects of the TM99 and Urbana IX 3NFs are small and the realistic $NN$ potentials alone or combined with these 3NFs provide quite a good description of the data.

For the proton to tensor-deuteron spin transfer coefficients, the Coulomb forces are significant for $K_y^{\gamma^{\epsilon}}$, $K_x^{\gamma^{\epsilon}}$, and $K_z^{\gamma^{\epsilon}}$ (see Fig. 6). In the case of $K_x^{\gamma^{\epsilon}}$ and $K_y^{\gamma^{\epsilon}}$, see Fig. 6) they are negligible at angles for which data exist. For these two coefficients also, the effects of the TM99 and Urbana IX 3NFs are small and the $NN$ potentials alone provide a good description of the data. This is similar for $K_y^{\gamma^{\epsilon}}$ and $K_z^{\gamma^{\epsilon}}$, the effects of these 3NFs are nonnegligible, especially in the region of angles around $\theta_{c.m.} \approx 120^\circ$. While for $K_y^{\gamma^{\epsilon}}$ the inclusion of the TM99 or Urbana IX 3NFs improves the description of the data; in the case of $K_z^{\gamma^{\epsilon}}$, it shifts the theory away from the data points.

Based on the chiral interactions, we show in Figs. 7–9 two bands of predictions based on forces derived in NLO and NNLO. Each band is based on five predictions obtained with different cutoff combinations: (450, 500), (600, 500), (550, 600), (450, 700), and (600, 700) (see [7] for more details). The results for the chiral $NN$ potentials in NLO are shown by the light band. In NNLO, nonzero contributions from chiral three-nucleon interactions arise for the first time, and the predictions based on the full chiral Hamiltonian in NNLO are shown by the dark band.

It is clearly seen that the restriction to NLO only is quite insufficient even at low energy of our experiment. The predictions based on the chiral $NN$ potential obtained in this low order are far away from the data. At NNLO, one could show the $NN$ force predictions alone. We refrain from doing that since this is ambiguous. As is well known, unitarily transformed $2N$ forces, which do not affect two-nucleon observables, lead to different results in the $3N$ system [42]. It is only the complete $3N$ Hamiltonian that provides unambiguous results for the $3N$ observables. Since the chiral approach systematically improves the nuclear force description with increasing order and provides strong internal links between $2N$ forces and forces beyond, we deviate here from the usual presentation of exhibiting $3N$ force effects separately. This was done above in the standard approach because there is no internal consistency between $NN$ and $3N$ forces. Now Figs. 7–9 show that the full NNLO predictions lead to quite as good a...
description as the traditional approach including 3N forces. The exceptions are $K^x_z$ for the proton to proton transfer, where the chiral approach differs from the data, and $K^{y'z'}_z$ for the proton to deuteron transfer, where it leads to an agreement with the data. In the standard approach, it is the opposite.

It will be of great interest to see the outcome for NNNLO, for which the $NN$ forces have already been worked out [8]. At that order, a whole host of parameter-free 3NFs contribute, including first relativistic effects.

IV. SUMMARY AND CONCLUSIONS

We presented new data for spin transfer coefficients in elastic $pd$ scattering, both for the proton to proton and for the proton to deuteron transfers. They were measured using a polarized proton beam with energy $E^p_{lab} = 22.7$ MeV. The data were corrected for Coulomb force effects using our theoretical framework of a hyperspherical expansion. This led to $nd$ data to which we compared our theory. In some cases, the Coulomb force effects are quite significant, especially at $\theta_{c.m.} \approx 120^\circ$, where the differential cross section has its minimum.

The theoretical predictions are obtained by solving the 3N Faddeev equations with two different dynamical inputs. One is the standard approach of the so-called high-precision $NN$ forces supplemented by the TM99 and Urbana IX 3NFs. The other is an effective field theory approach constrained by chiral symmetry. In the first case, where the forces have mostly phenomenological character, we show $NN$ force prediction separately in addition to the results obtained by adding the 3NFs. The inclusion of 3NFs clearly improves the description, and the comparison with the data is quite successful. However, it should be pointed out that the higher energy $Nd$ scattering data seem to be more favorable to the study of 3NFs. Generally, the statistical discrepancies between a theory based on $NN$ potentials and experiment become larger with increasing energy of the 3N system [19–28]. In the case of the approach based on chiral nuclear forces, NLO is quite insufficient; but at NNLO, the combined dynamics of $NN$ and 3N forces does essentially as well as the standard approach. Exceptions are the spin transfer coefficients $K^x_z$ from proton to proton and $K^{y'z'}_z$ from proton to deuteron. The first is well described in the standard approach but not the second one. Just the opposite is true in the chiral approach.

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[32] W. Kretschmer (private communication).


