Numerical calculation of third harmonic voltage induced by a shielding current in a superconducting thick film

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Abstract

When an ac magnetic field of a suitable magnitude is applied locally normal to a wide surface of a superconducting thick film, a third harmonic voltage V_3 is induced in a pick-up coil. It is predicted by the critical state model that V_3 is proportional to the driving current amplitude I_0^2 when I_0 is smaller than the threshold current $I_{\rm th}$ at which the shielding current in the film reaches the opposite side of the film. A numerical analysis using Finite Element Method (FEM) is carried out for the V_3-I_0 characteristics for the film. It is found that V_3 is proportional to I_0^2 only when I_0 is smaller enough than $I_{\rm th}$, and V_3 takes a smaller value than predicted by the theory near $I_0 = I_{\rm th}$. The deviation between the theoretical prediction and the result of FEM is found to be attributed to the different distribution of the shielding current in the film.

Key words: critical current density, third harmonic voltage, FEM
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1 Introduction

An inductive technique to measure nondestructively the critical current density J_c of a superconducting thin film was proposed by Claassen *et al.*, in which a third harmonic voltage induced by an applied ac magnetic field is detected [1]. In this method, an ac magnetic field is applied to a superconducting film by a small flat coil carrying an ac drive current $I_0 \cos\omega t$. A third harmonic voltage $V_3 \cos(3\omega t - \theta_3)$ is generated in the small coil due to a nonlinear magnetic response of the superconductor which shields the ac magnetic field. The V_3-I_0 relationship is derived by using the critical state model with a simple assumption that the magnetic field is locally parallel to the flat surface of the film [2]. The critical current density J_c is proportional to I_{th}/d, where I_{th} is threshold current at which the ac magnetic field reaches the opposite surface of the film and d is the film thickness [2]. Therefore, J_c can be estimated from I_{th}. Our previous studies with Finite Element Method (FEM) clarified a validity of the theoretical prediction for a thin film [3,4].

On the other hand, in the case of thick film, Mawatari *et al.* theoretically predicted that V_3 is proportional to $\omega I_0^2/J_c$ for $I_0 < I_{th}$ based on the same simple assumption [5]. Therefore, J_c in the surface area of the thick film is determined from this relationship, even in the case where J_c is distributed in the direction of film depth. In addition, the in-depth average critical current density J_{ct} is measured as well as the case of thin film [2]. J_c and J_{ct} of a Bi-2223 thick film were measured with this method by Yamasaki *et al.* [6]. The experimental value of V_3 was proportional to $\omega I_0^2/J_c$, when I_0 was smaller enough than I_{th}, and V_3 was smaller than the predicted value near I_{th}. This deviation seems to be attributed to the simple assumption in the theory that the magnetic field in the thick film is assumed to be parallel to the surface of the thick film. Therefore, it is desired to clarify the mechanism of this deviation. However, an analytical investigation seems to be difficult.

In this paper, the magnetic field distribution in a superconducting thick film is numerically calculated by FEM to estimate the third harmonic voltage V_3 as a function of I_0. This result is compared with the result of theoretical analysis by Mawatari *et al.* It is confirmed that the experimental result can be explained by the analytic result of FEM. Then, the reason for the disagreement between the calculated result and the theoretical result is discussed from the viewpoint of the distribution of the shielding current in the thick film. In addition, the effect of distance between the coil and the thick film on the penetration depth of the magnetic field in the superconductor is also investigated.

2 Simulation

JMAG studio version 8 of Japan Research Institute is used for the calculation of FEM. The following circumstance shown in Fig. 1 is assumed so as to fit the experimental condition by Yamasaki *et al.* [6]: a single coil with inner and outer diameters of 1.0 mm and 3.6 mm and a height of 1.0 mm is mounted at a position of 0.25 mm apart from the surface of infinitely long superconducting film of thickness 0.4 mm. The value of J_c of the thick film is 1.0×10^7 A/m^2. The number of turns of winding in the coil is 400. An ac magnetic field is applied normal to a wide surface of the thick film by a sinusoidal driving current $I_0 \cos \omega t$. The magnetic flux density distribution in the space including the thick film and coil system is calculated by FEM in each phase of ac field assuming the critical state model. Because of the azimuthal symmetry of the system, the field and current distributions in 1/360 of the whole system is calculated under a suitable boundary condition. Fig. 2 shows an example of result of FEM calculation for $I_0 = I_{\rm th}$. It is found that the magnetic flux density is concentrated at the bottom of the coil.

The voltage induced in the pick-up coil is derived from the magnetic flux which interlinked the pick-up coil. The amplitude, V_3, and the phase, θ_3, of the third harmonic voltage as a function of I_0 are derived by Fourier analysis. The threshold current, $I_{\rm th}$, is determined by the I_0 value at which the shielding current reaches the bottom of the thick film.

3 Results and Discussion

Fig. 3 shows the V_3-I_0 calculated by FEM. The calculated value of V_3 is proportional to I_0^2 as predicted by the theory for $I_0 < 20$ mA, where the maximum penetration depth of the magnetic field in the superconductor is 0.13 mm. Therefore, it is found that $J_{\rm c}$ in the surface area of the superconductor can be estimated by using the theory. On the other hand, V_3 deviates from the I_0^2 behavior and becomes smaller than this prediction for $I_0 > 20$ mA, as was observed by Yamasaki *et al.* [6]. Therefore, the experimental result of the driving current amplitude dependence of V_3 can be qualitatively explained by the result of FEM.

The difference between the theoretical prediction and the FEM result is now discussed in detail. The distribution of the shielding current in the thick film is examined to clarify the difference. Fig. 4 shows the distribution of the shielding current in the thick film at $\omega t = \pi$ at each amplitude of I_0, where the dot-dashed line represents the center of the coil. When the driving current $I_0\cos\omega t$ flowing in the coil produces ac magnetic field, the theory predicts that the radial component of magnetic field at the film surface is expressed as

$$H_0(\rho) = -I_0 F_1(\rho)\cos\omega t, \tag{1}$$

where ρ is a radius from the center and F_1 is the coil-factor function and is determined by the configuration of the coil and the superconducting thick film [2]. According to Bean's model, the penetration depth $\lambda(\rho)$ is given by

$$\lambda(\rho) = H_0/J_c. \tag{2}$$

The pattern of the shielding current distribution is shown in Fig. 4. The dotted line shows the penetration depth given by Eq. (2) and the shaded region shows the distributed region calculated by FEM.

The calculated shielding current distribution agrees well with the theoretical prediction at $I_0 = 10$ and 20 mA. However, the distribution of the shielding current deviates from the theory with a further penetration of the shielding current in the central region when $I_0 = 40$ and 56 mA as shown in regions (a) of the figure. At the same time, the shielding current disappears in region (b). The difference of the shielding current distribution is considered to be caused by the fact that the normal component of the magnetic field which is found by FEM as shown in Fig. 5 is not assumed in the theoretical analysis. Furthermore, the shielding current in the region (a) in Fig. 4 weakly contributes to the increase of V_3, since the diameter of the shielding current is smaller than the diameter of the coil. On the other hand, the shielding current in the region (b) in Fig. 4 contributes to the decrease of V_3, since the diameter of the shielding current is similar to the coil size. Therefore, it is found that the proportionality of V_3 to I_0^2 occurs only for $I_0 < 20$ mA. In addition, it is clarified that V_3 does not agree with the theoretical value for $I_0 > 20$ mA because of the difference of the distribution in the shielding current.

In the present calculation by FEM, $I_{\rm th}$ is estimated as 56 mA and $J_{\rm ct}$ is estimated as $1.08\times10^7\,{\rm A/m^2}$. On the other hand, $I_{\rm th}$ predicted by the theory is 52 mA for $J_{\rm ct}=1.0\times10^7\,{\rm A/m^2}$. It is almost the same value of the specified $J_{\rm ct}$. Although the theoretical prediction gives a slightly small value, this deviation may be attributed to the limit of resolution of FEM. Therefore, we need a further analysis for the accuracy of the estimation of $J_{\rm ct}$ from $I_{\rm th}$ value.

Fig. 6 shows the result of V_3-I_0 curve, when the coil is just above the surface of the thick film, i.e., the distance between the coil and the thick film is zero. V_3 is found to be proportional to I_0^2 when I_0 is smaller than 24 mA, where the maximum penetration depth of the magnetic field in the superconductor is 0.23 mm. In addition, $I_{\rm th}$ becomes smaller (38 mA) compared with the previous configuration (56 mA). Thus, the prediction of the theory is useful for wider range of I_0 when the distance between the surface of the superconducting thick film and the coil becomes short, the effect of the normal component of magnetic field is less than previous configuration.

4 Summary

The magnetic field distribution in a superconducting thick film was numerically calculated by FEM to estimate the third harmonic voltage V_3 as a function of I_0. This result was compared with the theoretical results by Mawatari *et al.* and the disagreement between the calculated result and theoretical results is discussed. In addition, the effect of distance between the coil and the thick film on the penetration depth of the magnetic field in the superconductor was also investigated.

1. It is shown that V_3 is proportional to I_0^2 and J_c can be estimated as predicted by the theory only when I_0 is sufficiently smaller than I_{th}.
2. The deviation from the theory at $I_0 > 20$ mA is explained by the difference of the distribution of the shielding current in the thick film.
3. The prediction of the theory is useful for a wider range of I_0 when the distance between the surface of the superconducting thick film and the coil becomes short.

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Fig. 1 Schematic illustration of arrangement of coil and superconducting thick film for calculation by FEM.

Fig. 2 Distribution of magnetic flux density calculated by FEM at $I_0 = I_{\rm th}$.

Fig. 3 Third harmonic voltage amplitude V_3 vs driving current amplitude I_0.

Fig. 4 Distribution of shielding current in superconducting thick film at $\omega t = \pi$ for various amplitudes I_0.

Fig. 5 Distribution of magnetic flux in superconducting thick film at $\omega t = \pi$ at $I_0 = I_{\rm th}$.

Fig. 6 Third harmonic voltage amplitude V_3 vs driving current amplitude I_0 in case that the coil is put just above the surface of the film.

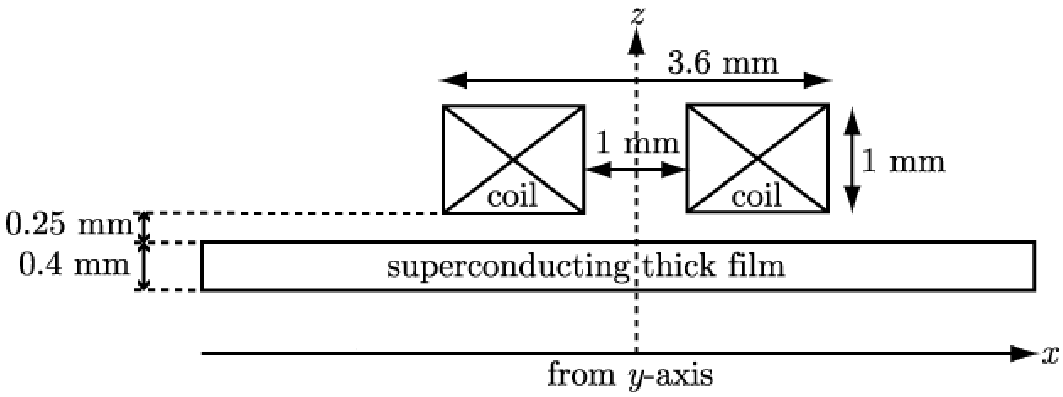


Fig. 1: T. Nadami *et al.* BSP–28/ISS2004

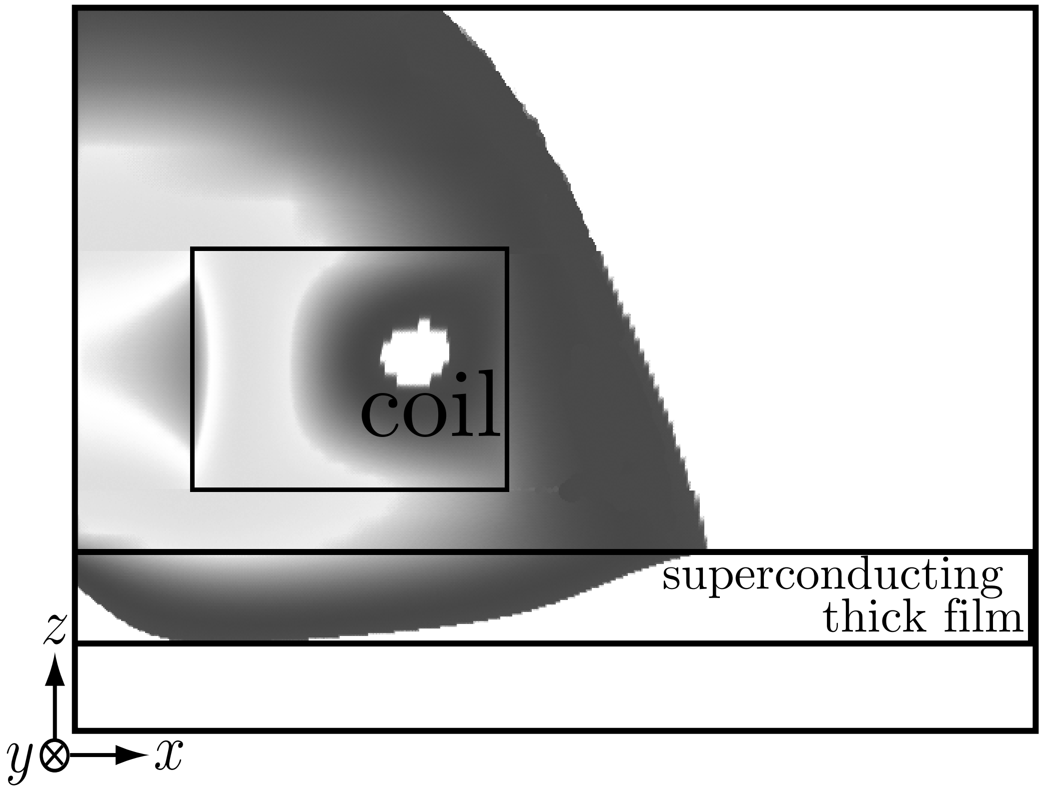


Fig. 2: T. Nadami *et al.* BSP–28/ISS2004

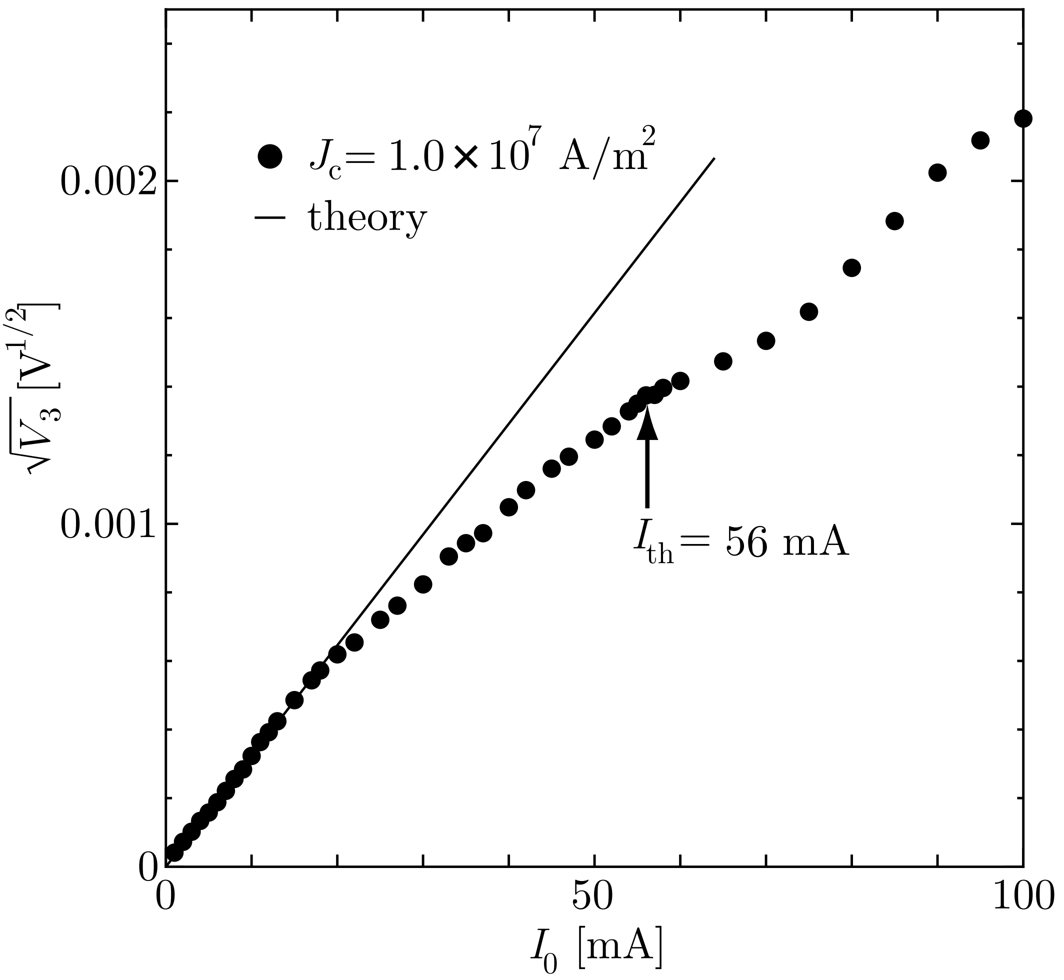


Fig. 3: T. Nadami *et al.* BSP–28/ISS2004

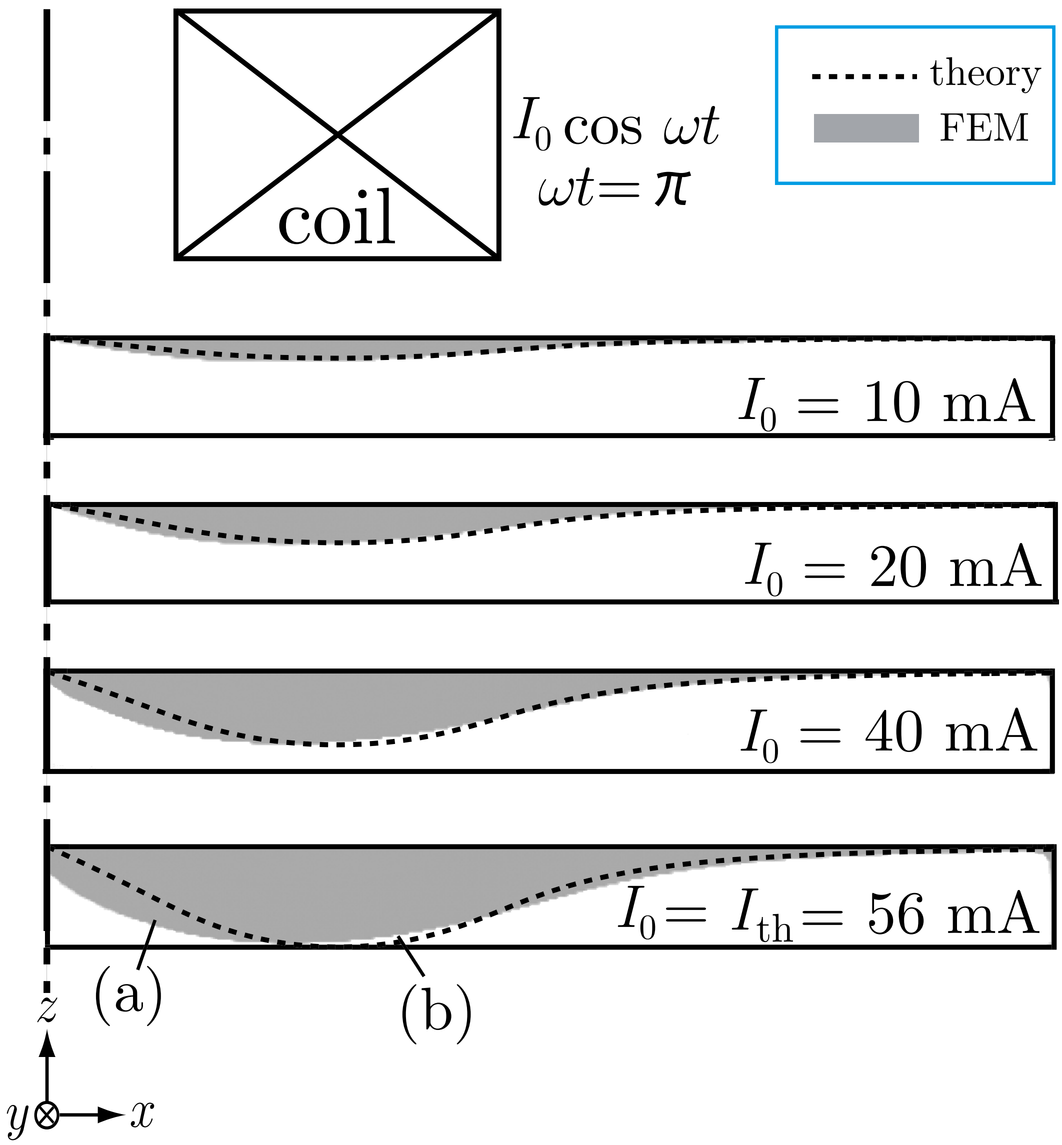
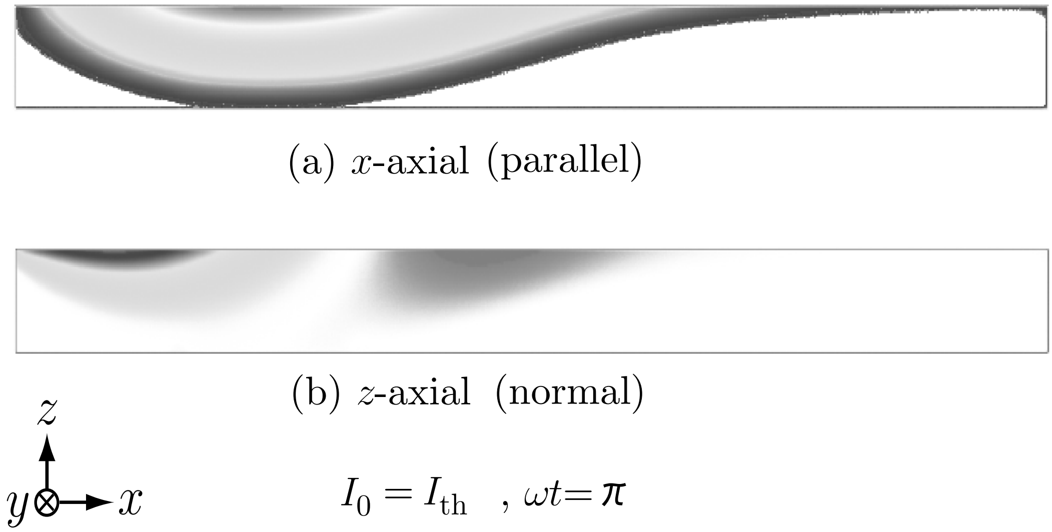


Fig. 4: T. Nadami *et al.* BSP–28/ISS2004



(a) x-axial (parallel)

(b) z-axial (normal)

$I_0 = I_{\rm th}$, $\omega t = \pi$

Fig. 5: T. Nadami *et al.* BSP–28/ISS2004

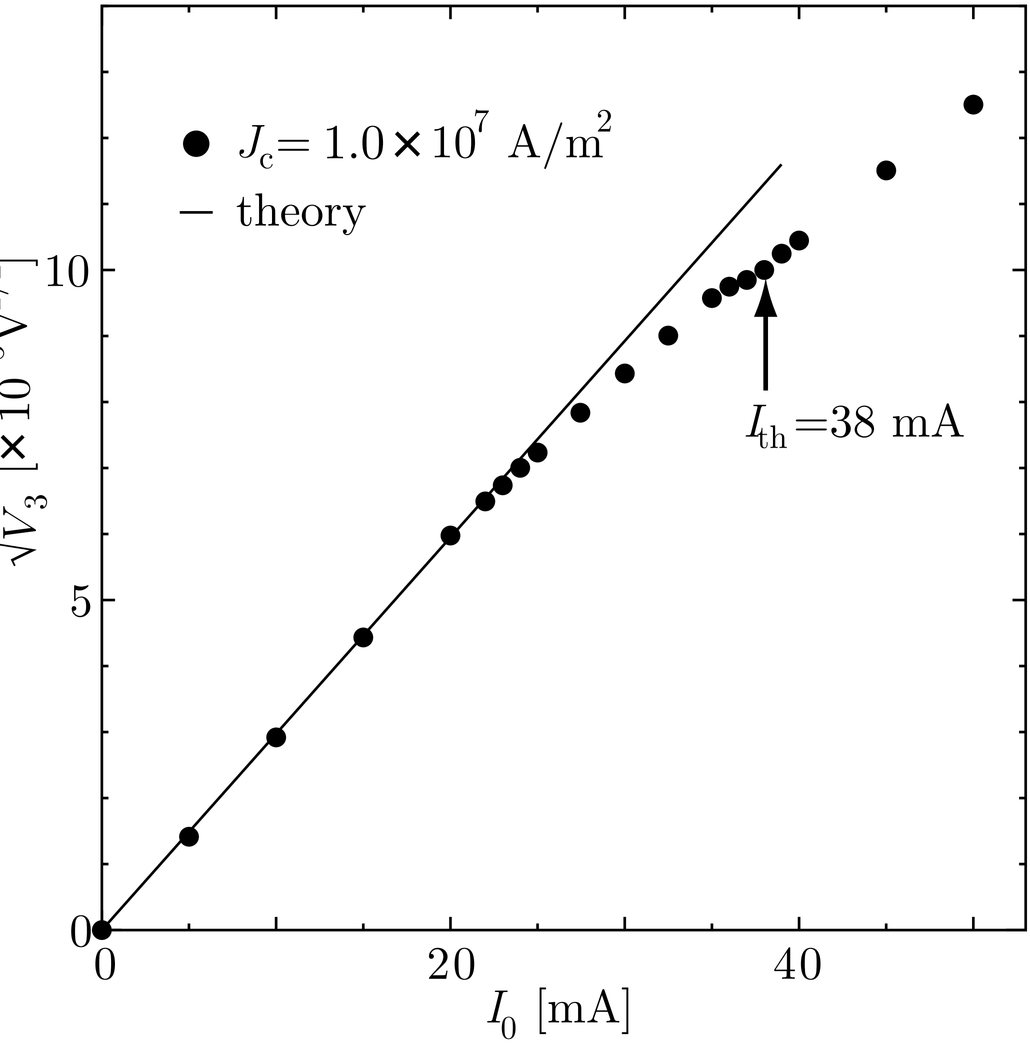


Fig. 6: T. Nadami *et al.* BSP–28/ISS2004