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A Method to Obtain the Parameters of the lightning Wave Form

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Abstract

This paper provides a method to obtain the parameters of the lightning wave form function which is a linear combination of two exponential functions. Due to heavy use of this function in transient analysis simulation programs such as the EMTP (Electro-Magnetic Transients Program), determining the wave form parameters is important. The novel aspect of this paper is to use the homotopy method in obtaining the parameters of the wave form function.

1. Introduction

In recent insulation coordination design of electric power system, it is very important to accurately grasp the effect of switching surge or lightning surge on the system in view of the economical efficiency as well as from the equipment protection. For such a purpose, transient analysis simulations using the EMTP for electric power system are frequently used in insulation design to examine the effect of the surge. Thus, the expression of the shape of the surge wave form in simulations is extremely important in wave propagation analysis, and therefore the wave form should exactly be determined, especially in lightning wave propagation analysis. Fitting the wave form expressed by the function which is a linear combination of two exponential functions to a digitally recorded surge impulse is also important and the matter is discussed in the literature [7-9].

The standard wave form for lightning surge current is defined by the wave-front T_f and the wave-length T_t in IEC60060-2 [6] such as $T_f = 2\mu s$ and $T_t = 70\mu s$ as shown in Figure 1. The wave form is mathematically expressed by the formula

$$f(t) = A(\exp(-\alpha t) - \exp(-\beta t)). \tag{1}$$

In a simplest form in which T_f is defined by the time to the crest and T_t by the time to the half value of the crest, parameters A, α, and β must inversely be determined by solving the equations,

$$\begin{aligned} F(t_1) &= E, \\ F(t_2) &= 0.5E, \\ F'(t_1) &= 0, \end{aligned} \tag{2}$$

where E denotes the crest value of the wave form. Bewley [2] introduced an ingenious method to eliminate the two unknown parameters, and consequently we only have to search for the remaining one parameter. He provided a graph to determine these parameters.

However, if we want to define the lightning wave form by effective front and length shown as in Figure 1, it becomes a little bit tedious to find the three parameters. The equations are

$$\begin{aligned} &F(t_1) = k_1 E, \\ &F(t_2) = k_2 E, \\ &F(t_3) = E, \\ &F(t_4) = 0.5E, \\ &F'(t_3) = 0, \\ &T_f = \frac{t_2 - t_1}{k_2 - k_1}, \\ &T_t = t_4 + \frac{k_1}{k_2 - k_1}(t_2 - t_1) - t_1, \end{aligned} \tag{3}$$

because t_1, t_2, t_3, and t_4 must simultaneously be obtained with parameters A, α, and β. Typically, the values of k_1 and k_2 are $k_1 = 0.9$ and $k_2 = 0.3$,0.1. The standard Newton-Raphson method requires a carefully selected initial guess in finding the solution [3]. Some methods to reduce the unknown parameters do exist, but we do not go deeply into such methods. Rather, we investigate how to solve equations (3) in stable.

2. Homotopy Method

2.1 Equation formulation

First, we transform the parameters,

$$
\begin{aligned}
&\exp(-\alpha \times 10^{-6}) = x,\\
&\exp(-\beta \times 10^{-6}) = y,\\
&A/E = z,\\
&t_1 \times 10^6 = u,\\
&t_2 \times 10^6 = v,\\
&t_3 \times 10^6 = w,\\
&t_4 \times 10^6 = r,\\
&T_f \times 10^6 = s_1,\\
&T_t \times 10^6 = s_2,
\end{aligned}
\tag{4}
$$

in order to deal with numerical computation easily. Then, equations (3) become

$$
\begin{aligned}
&f_1(\theta) = x^u - y^u - k_1 z = 0,\\
&f_2(\theta) = x^v - y^v - k_2 z = 0,\\
&f_3(\theta) = x^w - y^w - z = 0,\\
&f_4(\theta) = x^r - y^r - 0.5z = 0,\\
&f_5(\theta) = x^w \log x - y^w \log y = 0,\\
&f_6(\theta) = v - u - (k_2 - k_1)s_1 = 0,\\
&f_7(\theta) = s_2 - r - 0.5v + 1.5u = 0,\\
&\quad (\theta = (x, y, z, u, v, w, r)^T).
\end{aligned}
\tag{5}
$$

The solution of equations (5) can be obtained by using the Newton-Raphson method if an initial guess is appropriately selected, but finding the initial guess is difficult in general. The homotopy method can make good this deficiency by expanding the domain of attraction of the Newton-Raphson method.

2.2 Homotopy method

The homotopy method can find the solutions of m nonlinear equations in principle whenever trivial solutions are available (see [1,4,5]). Let $f : R^m \to R^m$ be a smooth function. We want to find the solution of $f(\theta) = 0$, where $\theta = (\theta_1, \theta_2, ..., \theta_m)^T$. In general, the homotopy deformation $h : R^m \times [0,1] \to R^m$ can be defined such that

$$
h(\theta, 0) = g(\theta), \quad h(\theta, 1) = f(\theta), \tag{6}
$$

where $g : R^m \to R^m$ is a trivial smooth map having known zero points and h is also smooth. For instance, h is often defined as follows:

$$h(\theta, t) = tf(\theta) + (1-t)\{f(\theta) - f(\theta^{(0)})\}, \tag{7}$$

where $\theta^{(0)}$ is a solution when $t = 0$. We can trace a set $\{\theta(t) : 0 \leq t \leq 1 \mid h(\theta(t), t) = 0\}$ from the zero point of (7) at $t = 0$ to a zero point of (7) at $t = 1$ continuously. This set is a curve in R^m parameterized by t, and the zero point becomes a solution of $f(\theta) = 0$. Thus, the solution of the log-likelihood equations can certainly be obtained as long as the curve is traced until $t \geq 1$.

By differentiating $h = 0$ with respect to t, a system of differential equations

$$\frac{d}{dt}\theta(t) = -\left(h_\theta(\theta(t), t)\right)^{-1} h_t(\theta(t), t) \tag{8}$$

is obtained. By applying an Eulerian method to solve (8), a successive scheme,

$$\theta^{(i+1)} = \theta^{(i)} - \delta \cdot (J^{(i)})^{-1} f(\theta^{(0)}), \quad i = 0, 1, ..., \tag{9}$$

will find a solution, where $J^{(i)}$ denotes a Jacobian of $f(\theta^{(i)})$ and δ is a small number. The sequence of $\theta^{(i)}$ are obtained to be located on the solution curve of $h = 0$. This is called the naive homotopy method.

Although the scheme is very similar to the Newton-Raphson scheme expressed by:

$$\theta^{(i+1)} = \theta^{(i)} - (J^{(i)})^{-1} f(\theta^{(i)}), \quad i = 0, 1, ..., \tag{10}$$

the homotopy method has a global convergence property unlike the Newton-Raphson method, as mentioned above. The Newton-Raphson method traces zero points induced from approximate local linear equations related to the original nonlinear equations, while the homotopy method traces zero points of function $h(\theta, t) = 0$; the solution can be expected as long as the solution $h(\theta, 1) = 0$ is obtained.

The naive homotopy method will fail at possible turning points of t because t cannot decrease. To circumvent this inconvenience, arclength s and curve $c(s)$, which consist of zero points $h^{-1}(0)$, are introduced, since s is monotone increasing. The homotopy deformation is then denoted as $h(c(s))$. By differentiating $h = 0$ with respect to s,

$$h'(c(s)) \cdot \dot{c}(s) = 0 \tag{11}$$

is obtained, where $\dot{c}(s) = dc/ds$. To reduce one free parameter a constraint

$$\|\dot{c}(s)\| = 1, \tag{12}$$

should be imposed, where $\|\cdot\|$ denotes an Euclidian norm (l^2 norm). With an assumption that $\text{rank}(h'(c(s))) = m$, an augmented Jacobian matrix,

$$A(s) = \begin{pmatrix} h'(c(s)) \\ \dot{c}(s)^T \end{pmatrix}, \tag{13}$$

becomes nonsingular, because $h'(c(s))$ is orthogonal to $\dot{c}(s)$ (see (11)). Thus, the direction of traversing the curve $c(s)$ should be determined by a constraint,

$$\det(A(0)) \cdot \det(A(s)) > 0. \tag{14}$$

Using (11)-(14), increments $(dc(s))^{(i)}$ are obtained by solving a system of linear equations (11). Then, a new point $(\check{c}(s))^{(i)}$ is obtained by $((c(s))^{(i)} + (dc(s))^{(i)})$. This step is called the predictor step. However, this point is not necessarily on the curve $c(s)$. A correction process is needed for finding a point $(c(\tilde{s}^{(i)}))$ such that it is on the curve $c(s)$.

Two methods for finding the point $(c(\tilde{s}^{(i)}))$ can be considered for the correction; one is to minimize the norm $\|(c(\tilde{s}))^{(i)} - (\check{c}(s))^{(i)}\|$ (correction method A), and the other is to make the vector $((c(\tilde{s}))^{(i)} - (\check{c}(s))^{(i)})$ perpendicular to the vector $(dc(s))^{(i)}$ (correction method B). We obtain the point $(c(\tilde{s}))^{(i)}$ by solving $h((c(\tilde{s}))^{(i)}) = 0$ with correction method B. This is called the corrector step. The process which consists of these two steps is called the predictor-corrector homotopy method.

3. Applying the Homotopy Method to Find the Wave Form Parameters

An example of $T_f = 2\mu s$ and $T_t = 70\mu s$ in which the homotopy method works well is introduced here. The initial guess is $\theta = (.92, .2, 1., 1., 2., 2., 70.)^T$, and the traces of parameters in the homotopy procedure are illustrated in Figure 2. The most sensitive parameter is x in this case, and the Newton-Raphson method fails to find the solution when such an initial guess is used. The domain of attraction for x in the Newton-Raphson method is $(.97, .99)$, while that in the homotopy method is $(.92, .99)$. Why the domain of attraction in the homotopy method cannot be expanded dramatically is due to the existence of the multiple solutions. However, the this approach did help us to find the solution.

Using the method proposed here, some typical parameters of the lightning wave form function often used in transient analysis of lightning wave propagation are computed. Table 1 provides the results of the computation.

4. Conclusion

A tedious procedure in finding the parameters of the lightning wave form function in the traditional Newton-Raphson method can be lightened by using the homotopy method which has the property of global convergence in finding the zero point of the non-linear equations. Although the expansion of the domain of attraction is not so large, the homotopy method can reduce to find the initial guess in the Newton-Raphson method. Some typical parameters of the lightning wave form function are presented in this paper.

Table 1. Typical Parameters of the Lightning Wave Form

equation	k_1/k_2	T_f/T_t	A	α	β
		1/70	1.0117	1.0068×10^4	6.4767×10^6
		2/70	1.0244	1.0247×10^4	2.8188×10^6
(2)		4/70	1.0529	1.0638×10^4	1.1899×10^6
		1/40	1.0212	1.7852×10^4	5.8016×10^6
		1.2/50	1.0202	1.4264×10^4	4.8763×10^6
		1/70	1.0228	1.0251×10^4	3.0561×10^6
		2/70	1.0441	1.0574×10^4	1.4612×10^6
(3)	0.3/0.9	4/70	1.0900	1.1248×10^4	6.7496×10^5
		1/40	1.0388	1.8363×10^4	2.9537×10^6
		1.2/50	1.0373	1.4659×10^4	2.4689×10^6
		1/70	1.0265	1.0284×10^4	2.5792×10^6
		2/70	1.0514	1.0636×10^4	1.2303×10^6
(3)	0.1/0.9	4/70	1.1057	1.1374×10^4	5.6594×10^5
		1/40	1.0451	1.8458×10^4	2.4884×10^6
		1.2/50	1.0433	1.4732×10^4	2.0803×10^6

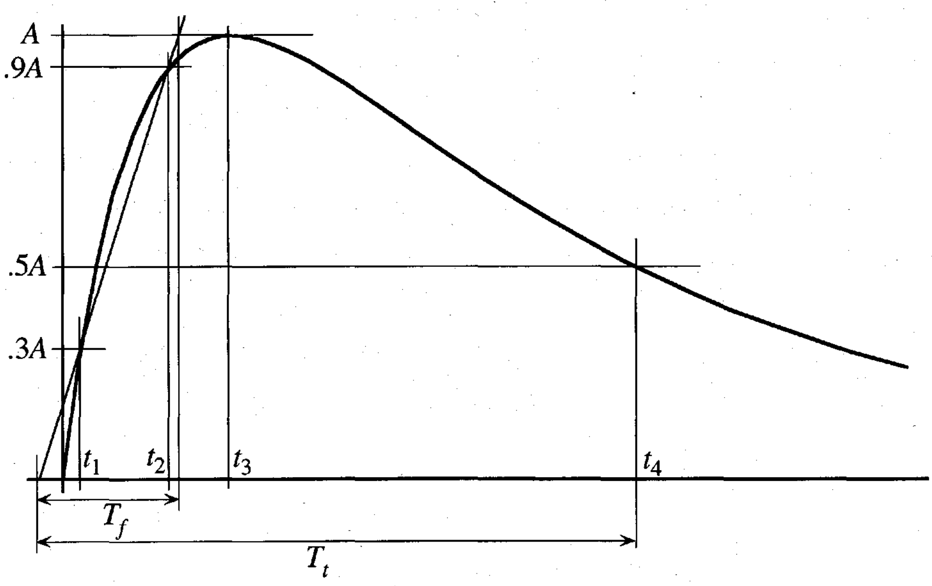

Figure 1. Typical Lightning Surge Wave Form

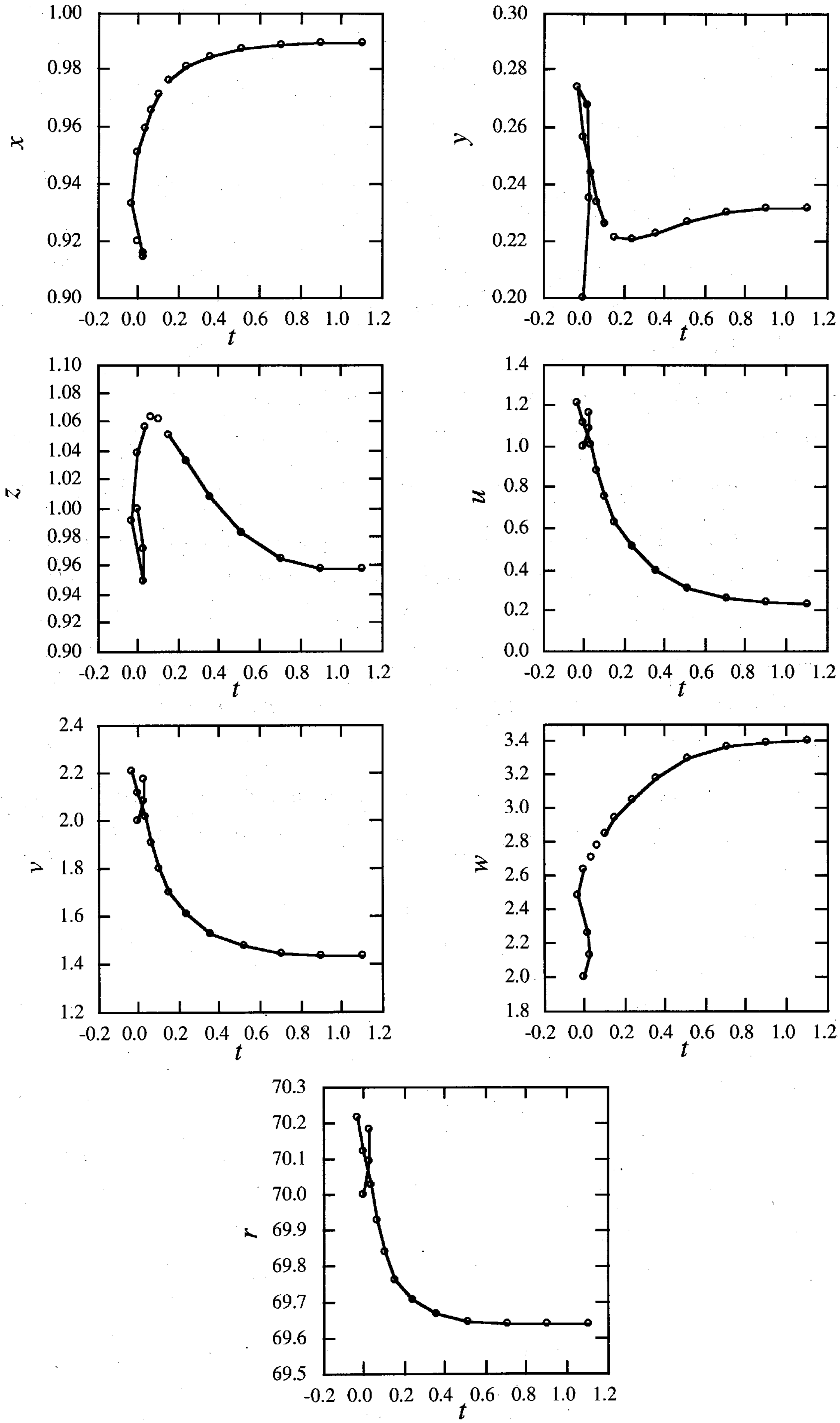

Figure 2.   Traces of Parameters in the Homotopy Procedure

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