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journal or publication title	Physics Procedia
volume	65
page range	241-244
year	2015-06-20
URL	<a href="http://hdl.handle.net/10228/5814">http://hdl.handle.net/10228/5814</a>

doi: info:doi/10.1016/j.phpro.2015.05.134

27th International Symposium on Superconductivity, ISS 2014

# Evaluation of trapped magnetic field properties in superconducting MgB<sub>2</sub> bulk magnets by finite element method

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## Abstract

The trapped magnetic field properties in superconducting MgB<sub>2</sub> bulk magnets with various kinds of shape such as a disk, a ring and a pair of disks were calculated by the finite element method (FEM). For simplicity, field cool magnetization was replaced by a simple magnetization process at constant temperature to obtain equivalent distribution of magnetic field, and the thermal equation in FEM was omitted. It was confirmed that the result of FEM agreed well with the result by analytical method in infinite long cylinder. We compared the trapped magnetic field property between FEM result and experimental result in reference in order to research the simple evaluation method of the trapped magnetic field of MgB<sub>2</sub> bulk magnet. It was found that the result of FEM agreed with the experimental result and it can explain the distribution of trapped magnetic field of superconducting MgB<sub>2</sub> bulk magnet. From these results, it was found that it was possible to be calculated in various kinds of shape with using simple evaluation by FEM. Therefore, the optimization of the maximum trapped magnetic field in superconducting MgB<sub>2</sub> bulk magnet can be discussed.

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Peer-review under responsibility of the ISS 2014 Program Committee.

*Keywords:* MgB<sub>2</sub>; superconducting bulk magnet; finite element method; trapped magnetic field; simulation

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## 1. Introduction

Superconducting MgB<sub>2</sub> bulk magnet has been investigated, since the critical temperature is rather high among metallic superconductors and the manufacturing process is easy compared with oxide superconductors due to the simple intermetallic compound of two elements. The maximum trapped magnetic field in MgB<sub>2</sub> was reported 5.4 T at 12 K [1] and 4.02 T at 11 K in a pair of bulks [2]. MgB<sub>2</sub> bulk magnets have potential as quasi-permanent magnets that take place of oxide superconducting bulk magnets. It is expected to be applied for magnetic resonance imaging (MRI), motors and magnetically levitated trains [3], generator systems [4] in the future. Therefore, the trapped magnetic field of MgB<sub>2</sub> bulk magnet has been analyzed by the numerical calculation using finite element method (FEM). And it has been shown that the calculated result by FEM agrees well with experimental data [5]. In this calculation by FEM, they took into account of both electromagnetic and thermal equations for field-cooled magnetization (FCM) [5]. Hence, the calculation by FEM is relatively complicate. They reported the temperature dependence of FCM process of trapped magnetic field, and the temperature raise within 1 K was depended on the rate of the external magnetic field decreasing in the final process of FCM. On the other hand, the maximum trapped magnetic field was almost constant for the rate of the external magnetic field decreasing [5]. Therefore, the magnetic field distribution in the MgB<sub>2</sub> bulk magnet in FCM could be calculated only the consideration of the electromagnetic equations at the constant temperature.

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In this paper, the trapped magnetic field properties in superconducting  $\text{MgB}_2$  bulk magnets are calculated by FEM and analytical method. For simplicity, only the electromagnetic equations are used and thermal equation is omitted in the FEM with using a simple magnetization process in the constant temperature to obtain the equivalent distribution of magnetic field instead of FCM. Then, the trapped magnetic field properties calculated by FEM are compared with analytical result and experimental result in reference in order to research the simple evaluation method of trapped magnetic field of  $\text{MgB}_2$  bulk magnets.

## 2. Theoretical calculation

We define three types of superconducting  $\text{MgB}_2$  bulk magnet models in order to calculate the magnetic field distributions of magnets using the finite element method (FEM). First type is a disk magnet which thickness is 10 mm with various diameters from 10 mm to 100 mm. Second type is a disk magnet which diameter is 60 mm with various thicknesses from 10 to 100 mm. Third type is a ring-shaped magnet with diameter of 60 mm, and thickness of 10 mm with various inside radiuses  $R_1$  from 5 to 25 mm. In practical case, bulk magnet is magnetized by field-cool magnetization (FCM) with using superconducting magnet. That is, temperature is raised above the critical temperature and then external magnetic field is applied. Then the temperature is cooled down to measurement temperature, and then the external magnetic field is reduced to zero. In some case, pulse magnet is used for magnetization of bulk magnet. However, the calculation of the temperature dependence of the trapped magnetic field is complicate, since the electromagnetic and thermal equations should be simultaneously solved in FEM calculation. In the present work, therefore, the external magnetic field is applied enough larger than 2 times of the penetration field  $B_p = \mu_0 J_c d/2$  in the constant temperature in the calculation, where  $J_c$  is the critical current density and  $d$  is the diameter of the disk. Then external magnetic field is reduced to zero. The maximum magnetic field applied to the  $\text{MgB}_2$  bulk magnet is 9 T. In this manner, almost an equivalent distribution of the trapped magnetic field by FCM can be realized. And the thermal equation in FEM can be omitted. It is to be noted that the temperature is slightly increases within 1 K during decreasing magnetic field. However, the maximum trapped magnetic field is reported to be less influenced by increasing temperature within 1 K [5]. The magnetic field dependence of the critical current density of the bulk  $\text{MgB}_2$  at 5—37.5 K was measured by SQUID magnetometer [6], and it is used in the calculation by FEM.

Here we should show the analytical calculation of the distribution of magnetic field in the bulk superconductor. Suppose an infinite cylindrical superconducting bulk magnet with external magnetic field applied along the long axis. Cylindrical coordinates are introduced with the  $z$ -axis along the external magnetic field and  $r$  denoting the distance from this axis. The force balance equation of the Lorentz force and the pinning force is given by

$$-\frac{B_z}{\mu_0} \frac{dB_z}{dr} = \delta F_p(B_z), \quad (1)$$

where  $B_z$  is  $z$ -axis component of magnetic field,  $\delta$  is a sign factor indicating the direction of the flux line motion, e.g.,  $\delta = 1$  indicates that flux line moves to the radial direction [7], and  $F_p$  is the global pinning force density given by  $J_c B_z$ . We approximate the experimental data of  $J_c - B$  property which was measured by SQUID magnetometer as polynomial function, and obtain the fitting parameters. The distribution of the trapped magnetic field property of the superconducting  $\text{MgB}_2$  infinite cylindrical bulk magnet is calculated by Eq. (1) and approximation of  $J_c - B$  property.

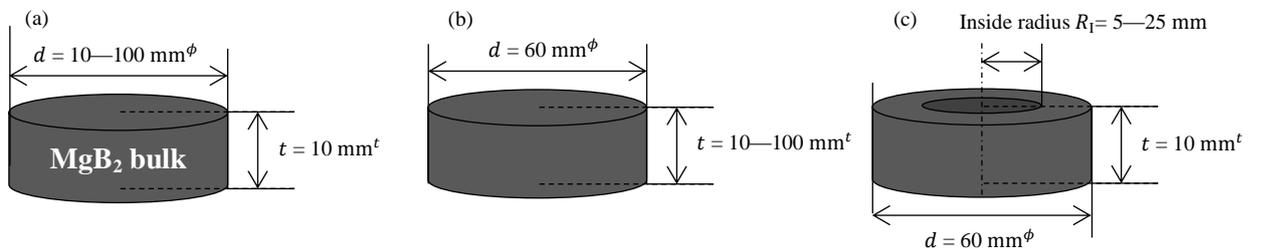


Fig. 1.  $\text{MgB}_2$  bulk magnet models for FEM. (a) disk type with various diameters; (b) disk type with various thicknesses; (c) ring-shape type with various inside radiuses.

### 3. Results and discussion

Fig. 2 shows the result of the distribution of the trapped magnetic field at 5 mm above the top surface of the superconducting  $\text{MgB}_2$  bulk magnet at 20 K which thickness is 10 mm with various diameters from 10 to 100 mm as shown in Fig. 1(a). Maximum trapped magnetic field at the center of the bulk superconductor is monotonically increasing and saturated as increasing diameter of the bulk superconductor. Therefore, it is effective to increase the diameter of the bulk to obtain high trapped magnetic field. On the other hand, it is not effective from the view point of cost to increase the volume of the bulk magnet. The negative magnetic trapped field at the edge of the bulk superconductor is due to so-called the shape effect. It is found that the shape effect becomes significant as increasing radius of the bulk superconductor, since the ratio of thickness to radius becomes small and the shape of bulk deviates from the infinite long cylindrical shape which has no shape effect.

Fig. 3 shows the result of the distribution of the trapped magnetic field of the bulk superconducting magnet at 20 K with diameter of 60 mm with various thicknesses from 10 to 100 mm as shown in Fig. 1(b). The thick dashed-dotted line represents the calculated result by using Eq. (1). The maximum trapped magnetic field at the center of the bulk is monotonically increasing with increasing the thickness of the bulk and saturated to the value, and saturated value at thickness of 1000 mm agrees well with the analytical calculation of the infinite long cylinder using Eq. (1). Therefore, it is found that the trapped magnetic field calculated by FEM in the present work seems to be correct. The shape effect becomes small as increasing the thickness of bulk.

Fig. 4 shows the result of temperature dependence of the maximum trapped magnetic field at the surface and the center in the pair of disk  $\text{MgB}_2$  bulks, where the symbols show the experimental data of figure 2 in Ref. [2], and lines show results of FEM. The size of two  $\text{MgB}_2$  bulk magnets is equal, i.e., the thickness is 10 mm and the diameter is 30 mm. As shown in Fig. 4, the results of FEM agree well with the experimental data. Therefore, it is found that the trapped magnetic field can be calculated by FEM in which the only electromagnetic equations are used without thermal equation, and the simple magnetized process is also useful in which the temperature of the bulk does not change. The equivalent distribution of the magnetic field by FCM can be realized by simple magnetized process. If the temperature of the bulk magnet is cooled down to 5 K, the maximum trapped field increases as shown in Fig. 4.

Fig. 5 shows the result of distribution of trapped magnetic field of the ring shape bulk superconducting magnet at 20 K with diameter of 60 mm and thickness of 10 mm with various inside radii from 5 to 25 mm as shown in Fig. 1(c). Maximum trapped magnetic field decreases and position of the maximum trapped magnetic moves from the center to the edge as increasing inside radius of the bulk superconductor. The shape effect at the edge is independent with the inside radius in the calculation. The trapped magnetic field profile is concave at the center, since the trapped magnetic field is obtained at 5 mm above the top surface of the ring bulk just as a hole probe measurement. In some applications, the gradient of the magnetic field in the bulk superconducting magnet is important for trapping the magnetized material such as magnetic separation [8], drag delivery system [9] and so on. In these cases, since the maximum gradient of the magnetic field is obtained at the edge of the ring-shape bulk magnet, it is cost effective than disk-shape bulk magnet. It is known that the mechanical processing of  $\text{MgB}_2$  bulk is easy compared to oxide

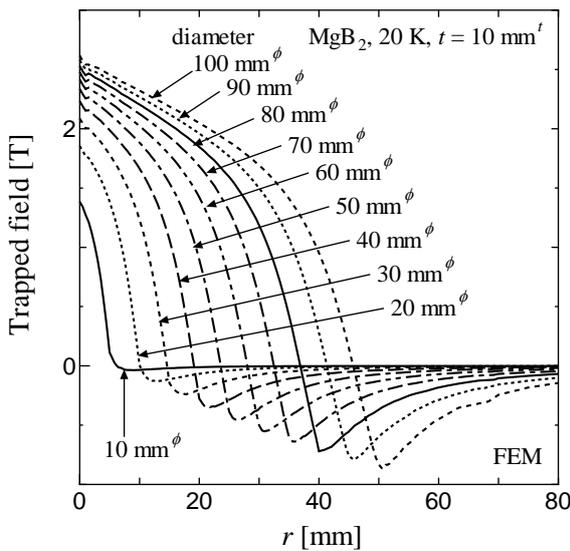


Fig. 2. Distribution of trapped magnetic field in disk bulk superconductor with various diameters.

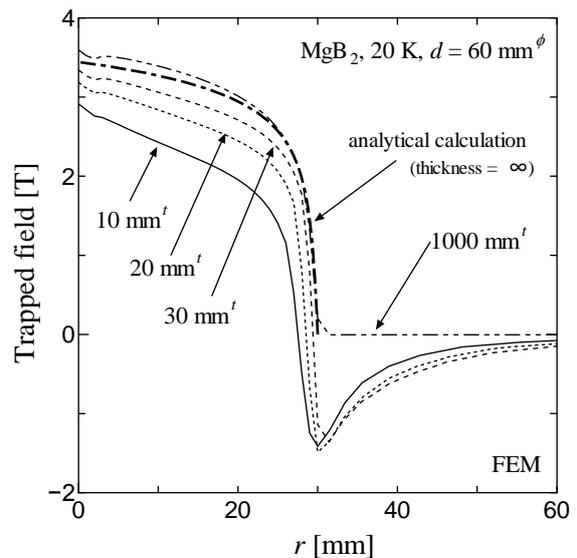


Fig. 3. Distribution of trapped magnetic field of disk bulk superconductor with various thicknesses. The analytical result agrees with the FEM result of infinite long cylinder.

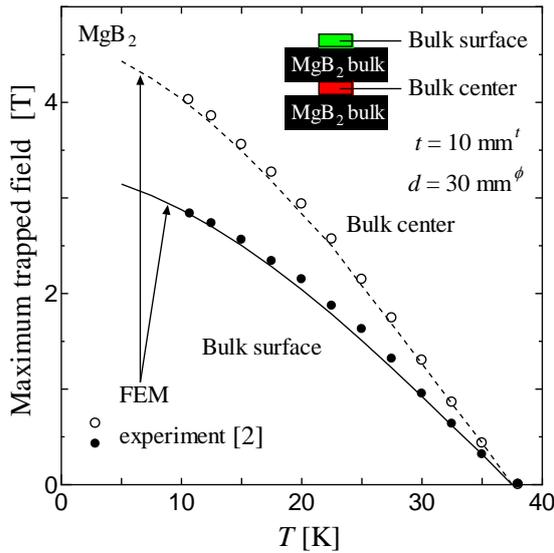


Fig. 4. Temperature dependence of maximum trapped magnetic field at center and surface of superconducting disk pair.

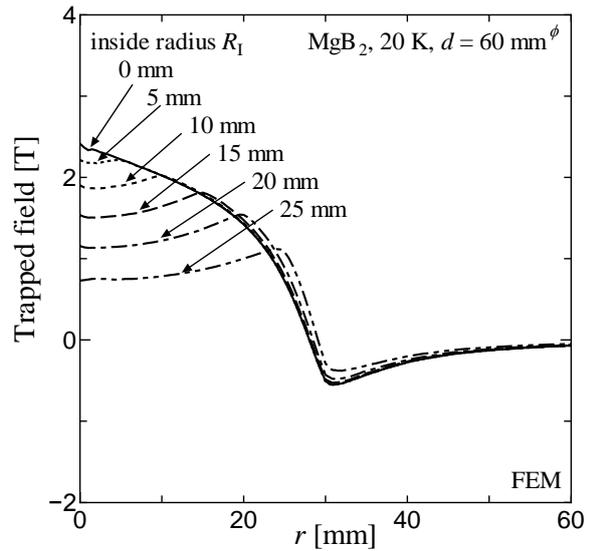


Fig. 5. Distribution of trapped field in the ring shape superconductor with various inside radii.

superconductors. Hence, it is expected to obtain variety of shapes of bulk superconducting magnet made from  $\text{MgB}_2$ . And the distribution of magnetic field in the variety of shapes of bulk magnets can be calculated by FEM as shown in the present study.

#### 4. Conclusion

The trapped magnetic field property of superconducting  $\text{MgB}_2$  bulk magnet was calculated by finite element method (FEM) and analytical method. For simplicity, the only electromagnetic equations were considered in the calculation of FEM and the thermal equation was not involved, since the temperature change during the change of magnetic field was not large during simple magnetization process. The simple magnetized process was used instead of field-cool magnetization (FCM). That is, the magnetic field was applied enough larger than 2 times of the penetration field, then it was reduced to zero. It is confirmed that the result of FEM agrees well with the prediction by the analytical calculation in the case of the infinite long cylinder. It is found that the result of temperature dependence of the maximum trapped magnetic field of the pair of disk  $\text{MgB}_2$  bulks can be explained by the calculation of FEM without the thermal equation. It is also confirmed that the distribution of the magnetic field in FCM is equivalent to that in simple magnetization process. It is possible to calculate various kinds of shape such as ring shape, stacked shape with using FEM. Therefore, the optimization of maximum trapped magnetic field can be discussed.

#### Acknowledgements

We wish to thank Dr. Kohji Kishio and Dr. Akiyasu Yamamoto of University of Tokyo for their kind provision of experimental data and their helpful discussion.

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