Effect of n-value on third harmonic voltage method analyzed by finite element method

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Abstract

To investigate the correctness of the estimation of the critical current density J_{c} in high-T_{c} superconductor by third harmonic voltage measurement method, the third harmonic voltage V_3 and the distributions of magnetic flux and current in a superconducting film with various n-values are calculated by FEM using the n-value model. When the AC current $I_0 \cos\omega t$ is applied to a coil, the threshold coil current I_{th}, which is proportional to J_{c} in the film, is determined by so-called off-set method for V_3/I_0-I_0 curve which is frequently used in the four terminal method. The calculated value of $I_{\mathrm{th}}$is slightly smaller than that of theoretical value by Bean's model. When the coil current I_0 is smaller than I_{th}, although the current density distribution is almost uniformly distributed as predicted by Bean's model, the value of current density is smaller than J_{c}. This behavior is different from the prediction of Bean's model. It is found that the current density is uniform and is equal to J_{c} for high n-value when I_0 is closed to I_{th}. Therefore, it seems to be reasonable to estimate J_{c} by I_{th} in the above method for superconducting films with n-value higher than 20.

Keywords: third harmonic voltage, FEM, n-value

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1 Introduction

Recently long YBCO coated-conductors with high critical current density are successfully fabricated [1]. Hence, simple and reliable measurement methods of the critical current density are required for characterization of long coated conductors. The third harmonic voltage measurement method seems to be suitable for the purpose, since the measurement can be done without any contacts with specimens. The critical current density J_c of a superconducting film can be estimated by measuring the third harmonic voltage induced in a pick-up coil put on the film which is exposed to AC magnetic field [2]. Since the third harmonic voltage rapidly grows when the AC magnetic field penetrates through the film, J_c can be directly measured from the threshold value of AC magnetic field at which the third harmonic voltage V_3 appears. Using this method, the current density J_c based on Bean's model is given by [3]

$$J_\mathrm{c} = \frac{2K}{d} I_\mathrm{th}, \tag{1}$$

where d is the film thickness, I_th is the threshold value of the coil current amplitude I_0 at which the third harmonic voltage is generated in the coil, and K is a constant determined by the coil geometry and distance between the coil and the film surface [4].

In the four terminal measurement method, the critical current density is determined with some criteria due to the difficulty in exact determination of the current at which the voltage rises up. This is also for the case of the third harmonic voltage method for the measurement on high-T_c superconductors, in which V_3 appears gradually with increasing I_0 due to wide statistical distribution of the local critical current density [5]. In this paper, so-called off-set method is used for determination of the threshold current, I_th. It is known that the n-value model is useful to describe E-J characteristics instead of Bean's model ($n \to \infty$) for high-T_c superconductors. However, it is difficult to theoretically estimate the magnetic flux density and the shielding current density

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inside the superconducting film with the n-value model. On the other hand, various analyses have been carried out using FEM [6]–[8]. Therefore, it seems to be possible to calculate the magnetic flux density, the current density and the value of V_3 in the third harmonic voltage measurement method by FEM with the n-value model.

In this paper, distributions of the magnetic flux density B and the current density J in the superconducting film with various n-values are numerically calculated by FEM as a function of the coil AC current amplitude I_0. Especially the distribution of the shielding current density is investigated in detail in the vicinity of $I_{\rm th}$ to find out the real critical condition.

2 Analysis

JMAG Studio version 8.4 of Japan Research Institute is used for the calculation of FEM. The following circumstance shown in Fig. 1 is assumed so as to fit the general experimental condition. The specifications of the coil and the superconducting film are given in Table 1 and Table 2, respectively. The coil constant K given in Eq. (1) is estimated as 47000 m^{-1} for the coil configuration illustrated in Fig .1 [4]. The region of $r > 0$ in Fig. 1 was modeled from symmetry. The superconducting layer was divided into ten elements from the surface to the other side. The magnetic flux density B and the current density J were calculated for three cycles of AC current of frequency of 1 kHz. The magnetic field and the current density in the last cycle are approximately regarded as the results in the steady state.

Numerical analysis by FEM is conducted based on the A-ϕ method using the vector potential $\vec{A}$ and the scalar potential ϕ. The difference from usual cases is the assumption of the E-J characteristics of the superconductor:

$$E = E_{\rm c}\left(\frac{J}{J_{\rm c}}\right)^{n}, \tag{2}$$

where n is a parameter showing the sharpness of superconducting-normal transition. E_c is the electric field criterion to determine the critical current density J_c. Here, $E_c = 1.0\times10^{-4}$ V/m and $J_c = 2.6\times10^{10}$ A/m^2 are assumed as used in the previous work[9]. Equation (2) assumes the effective electric conductivity σ_s:

$$\sigma_s = \frac{J}{E} = \frac{J_c}{E_c}\left(\frac{J_c}{J}\right)^{n-1}. \tag{3}$$

In FEM, Maxwell equations with Eq. (2) are applied to all the elements of the analysis region. Since σ_s has nonlinearity, it is necessary to determine the convergence condition:

$$\Sigma\Delta\sigma_s/\Sigma\sigma_s < \varepsilon \tag{4}$$

for all analysis regions, where ε is the error span and $\Delta\sigma_s$ is the difference from the last value of σ_s. The initial value of σ_s is assumed as

$$\sigma_{\rm init} = \frac{J_c}{E_c} = 2.6\times10^{14}\ [\rm S/m]. \tag{5}$$

The maximum electric conductivity $\sigma_{\max} = 2.6\times10^{17}$ S/m is introduced as an upper limit of σ_s [10].

3 Results and Discussion

Fig. 2 shows the numerically analyzed results of V_3-I_0 characteristics for the various n-values. V_3 is not zero even for the small current and increases almost linearly with I_0. Appearance of V_3 in this range is considered to be caused by the change in the distribution of the shielding current inside the superconducting film even below the critical state [8]. The threshold current $I_{\rm th}$ is determined by the current at which a tangential line from part of the V_3/I_0-I_0 curve at 50 $\mu\Omega$ crosses to an initial straight line at low I_0 value as shown in inset figure of Fig. 2. This is so-called the off-set method which is frequently used in the four terminal method for V-I curve. Since the slope of V_3/I_0-I_0 changes slowly for the case of low n value, the determination of the tangential line is difficult, resulting in large error of estimation of $I_{\rm th}$. $I_{\rm th}$

for each n-value is estimated as 0.153 ± 0.016 A ($n = 10$), 0.137 ± 0.006 A ($n = 20$), 0.136 ± 0.002 A ($n = 30$) and 0.135 ± 0.001 A ($n = 40$). Since $I_{\rm th}$ calculated by Eq. (1) is 0.138 A with $K = 47000$ m^{-1} and $d = 0.5\mu$m, the result for the higher n-value is slightly smaller than the theoretical prediction of Bean's model or agrees well, while the deviation is large in the case of $n = 10$. The value of V_3 grows rapidly when I_0 exceeds $I_{\rm th}$ and is larger for the higher n-value.

Fig. 3 (a)–(e) shows the distributions of the magnetic flux density and the shielding current density in the superconducting film just under the winding section of the coil for $n = 30$ at $I_0 = 0.090$ A during the change in the phase of the AC magnetic field $H_0\cos\omega t$ from $\omega t = \pi$ to 2π. Here, the abscissa shows the distance from the surface of the film, and the distributions of the magnetic field density and the current density predicted by the critical state model are also shown together in (b) at $\omega t = 5\pi/4$. It is found that the obtained distributions are slightly different but approximately the same as the predictions of Bean's model up to 0.36 μm. Since $I_0 = 0.090$ A for Fig. 3 is smaller than $I_{\rm th} = 0.136$ A ($n = 30$), the shielding current density J is smaller than $J_{\rm c} = 2.6 \times 10^{10}$ A/m^2, and the distribution is not uniform inside the superconducting film (see Fig. 3(a) and (e) for $\omega t = \pi$ and 2π). The magnetic flux does not completely reach the other side of the film. Hence, it is understood that the critical state is not reached.

Fig. 4(a),(b) shows the distribution of the magnetic flux density and the current density in the superconducting film at $\omega t = 2\pi$ for $n = 30$ in $I_0 = 0.090$ and 0.120 A. When I_0 is smaller than $I_{\rm th}$, the current distribution changes and its value is smaller than $J_{\rm c}$, although it is almost uniform at $I_0 = 0.120$ A. This behavior is different from the prediction of Bean's model.

Fig. 5(a)–(c) shows the distribution of the magnetic flux density and the current density at $\omega t = 2\pi$ for $n = 30$ in the region of $I_0 = 0.128$–0.132 A. It is found that the current density J is equal to $J_{\rm c}$ at $I_0 = 0.130$ A. This value is slightly smaller than 0.136 A by the off-set method and 0.138 A by Bean's model. However, $J_{\rm c}$-value estimated by the off-set method is 2.56×10^{10} A/m^2

and is only 1.7% smaller than 2.6×10^{10} A/m^2. Therefore, the estimation of J_c from I_{th} by the off-set method seems to be reasonable.

The distributions of the magnetic flux density and the current density in the superconducting film at $\omega t = 2\pi$ where the current density is equal to J_c for various n-values are shown in Fig. 6. It is found that $I_0 = 0.124$ A for the case of $n = 10$ is far smaller than 0.153 A estimated by the off-set method. Therefore, the estimation of J_c is only reasonable for the n-value higher than 20.

4 Summary

The distributions of the magnetic flux density and the current density in superconducting films with various n-values are calculated by using FEM with the n-value model. The threshold current I_{th} is determined by the off-set method. It is found that the value of V_3 grows rapidly when I_0 exceeds I_{th}, which is larger for the higher n-value. When I_0 is smaller than I_{th}, although the current distribution is almost uniform, the value of the current density is smaller than J_c and there is a room of additional transport current. This behavior is different from the prediction by Bean's model. The current density J is equal to J_c at I_0 slightly smaller than I_{th} for high n-value. Therefore, it seems to be reasonable to estimate J_c by I_{th} defined by the off-set method from V_3/I_0-I_0 characteristics for the n-value higher than 20.

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Table 1: Specification of coil.

inner diameter (mm)	outer diameter (mm)	height (mm)	number of turns (turn)
0.80	4.2	1.0	400

Table 2: Specification of specimens.

thickness d (nm)	J_{c} (A/m^2)	n value
500	2.6×10^{10}	10, 20, 30, 40

Figure captions

Fig. 1 Schematic illustration of arrangement of coil and superconducting film for calculation by FEM.

Fig. 2 V_3-I_0 characteristics by FEM for $n = 10, 20, 30, 40$.

Fig. 3 Distributions of the magnetic flux density and the current density in the superconducting film for $n = 30$ at $I_0 = 0.090$ A during the change in the phase of the AC magnetic field from $\omega t = \pi$ to 2π.

Fig. 4 Distributions of the magnetic flux density and the current density in the superconducting film at $\omega t = 2\pi$ for $n = 30$ in (a) $I_0 = 0.090$ A and (b) 0.120 A.

Fig. 5 Distributions of the magnetic flux density and the current density in the superconducting film at $\omega t = 2\pi$ for $n = 30$ in (a) $I_0 = 0.128$ A, (b) 0.130 A and (c) 0.132 A.

Fig. 6 Distributions of the magnetic flux density and the current density in the superconducting film at $\omega t = 2\pi$ in (a) $I_0 = 0.124$ A ($n = 10$), (b) 0.128 A ($n = 20$) and (c) 0.130 A ($n = 40$).

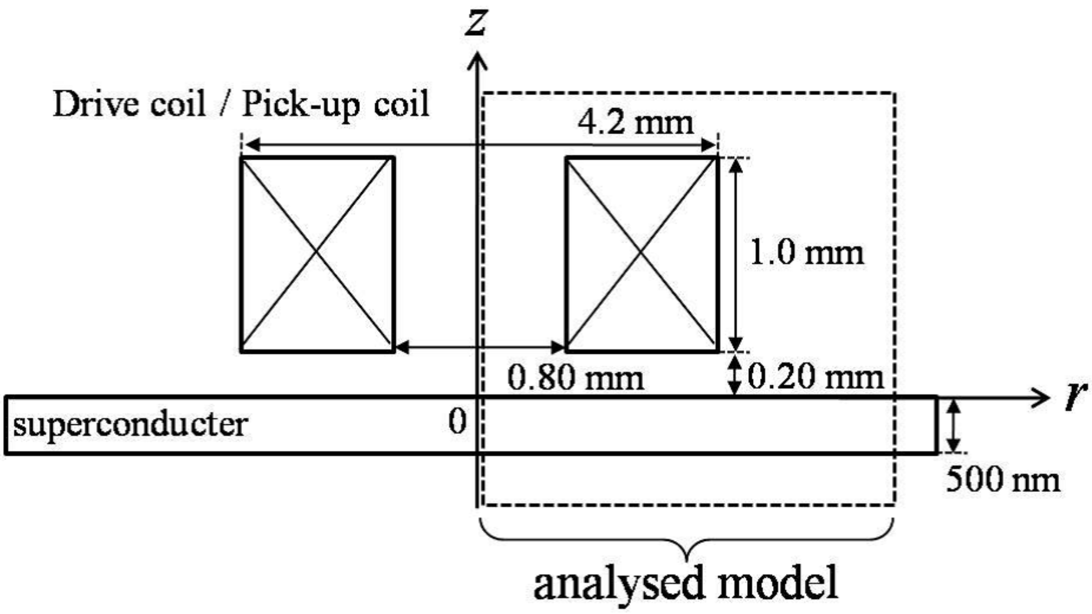

Fig. 1: T. Yoshida *et al.* WTP–114/ISS2007

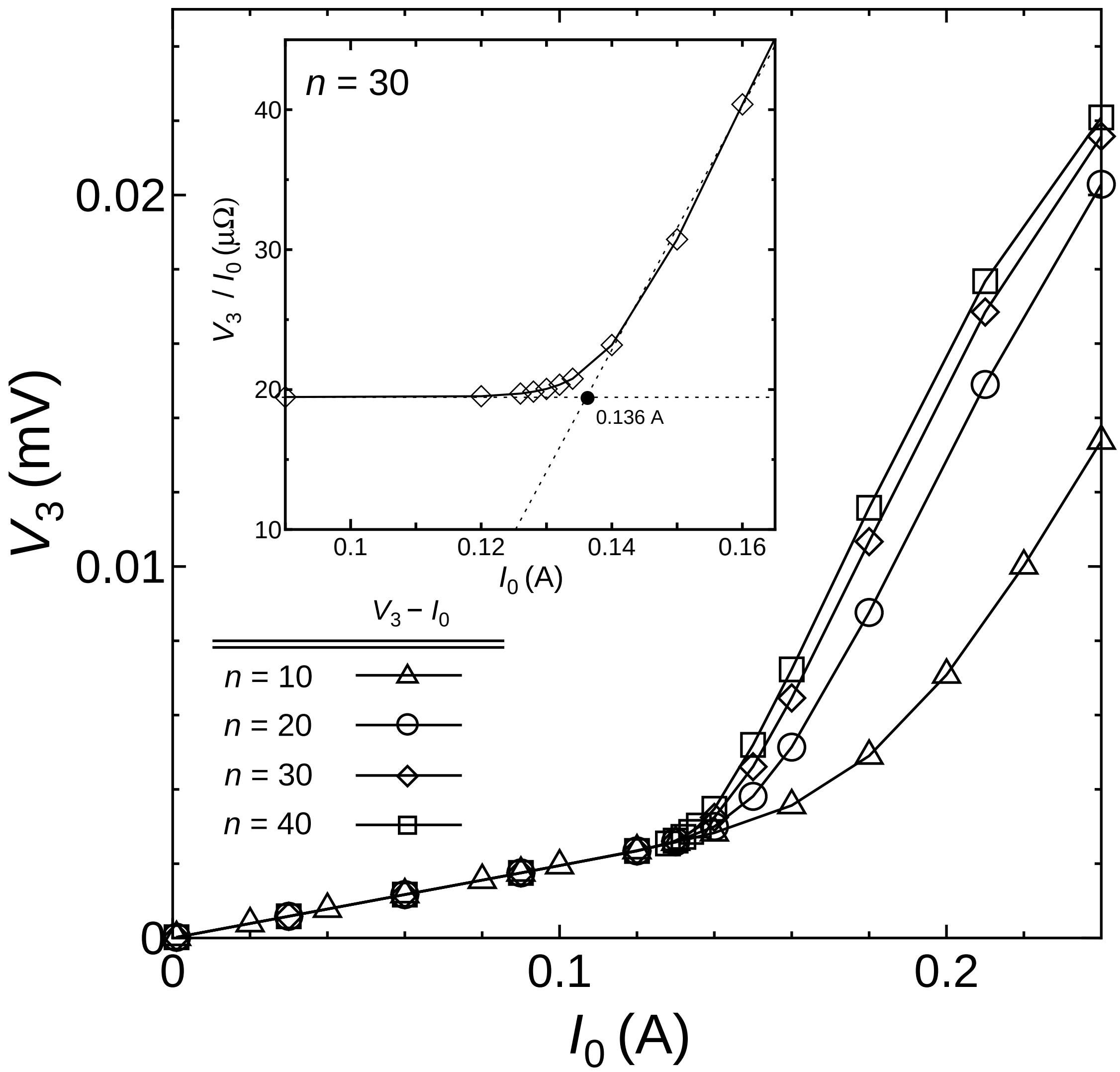

Fig. 2: T. Yoshida *et al.* WTP–114/ISS2007

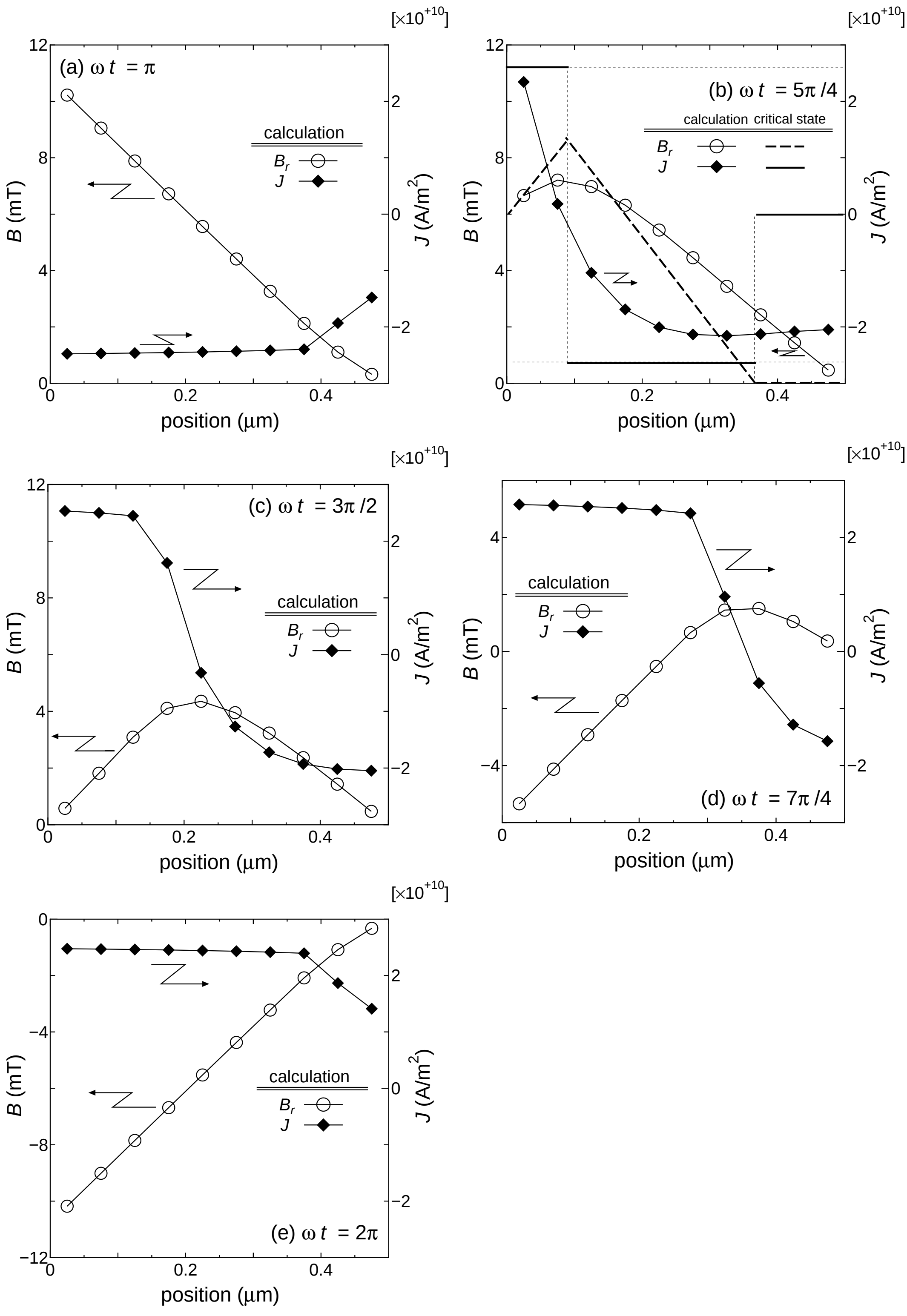

Fig. 3: T. Yoshida *et al.* WTP–114/ISS2007

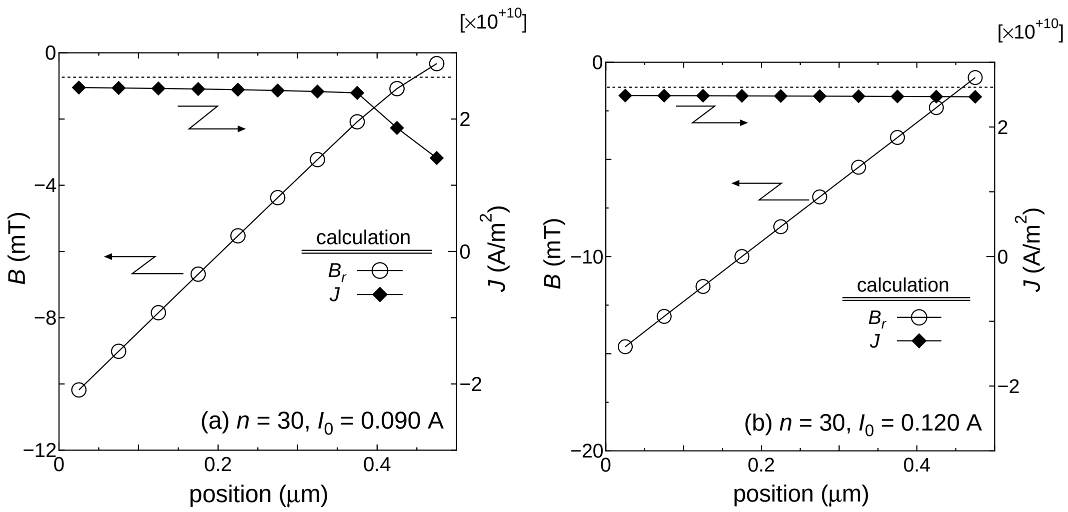

Fig. 4: T. Yoshida *et al.* WTP–114/ISS2007

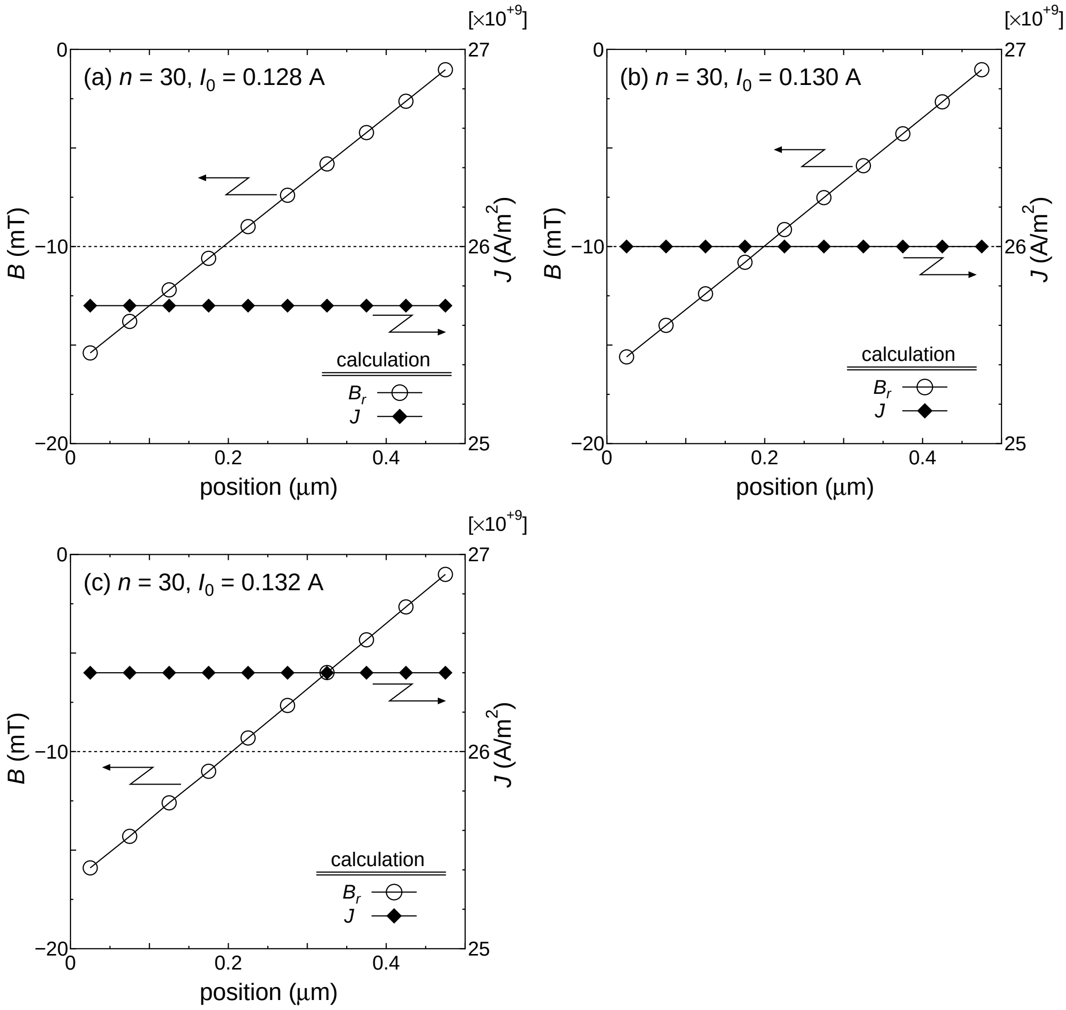

Fig. 5: T. Yoshida *et al.* WTP–114/ISS2007

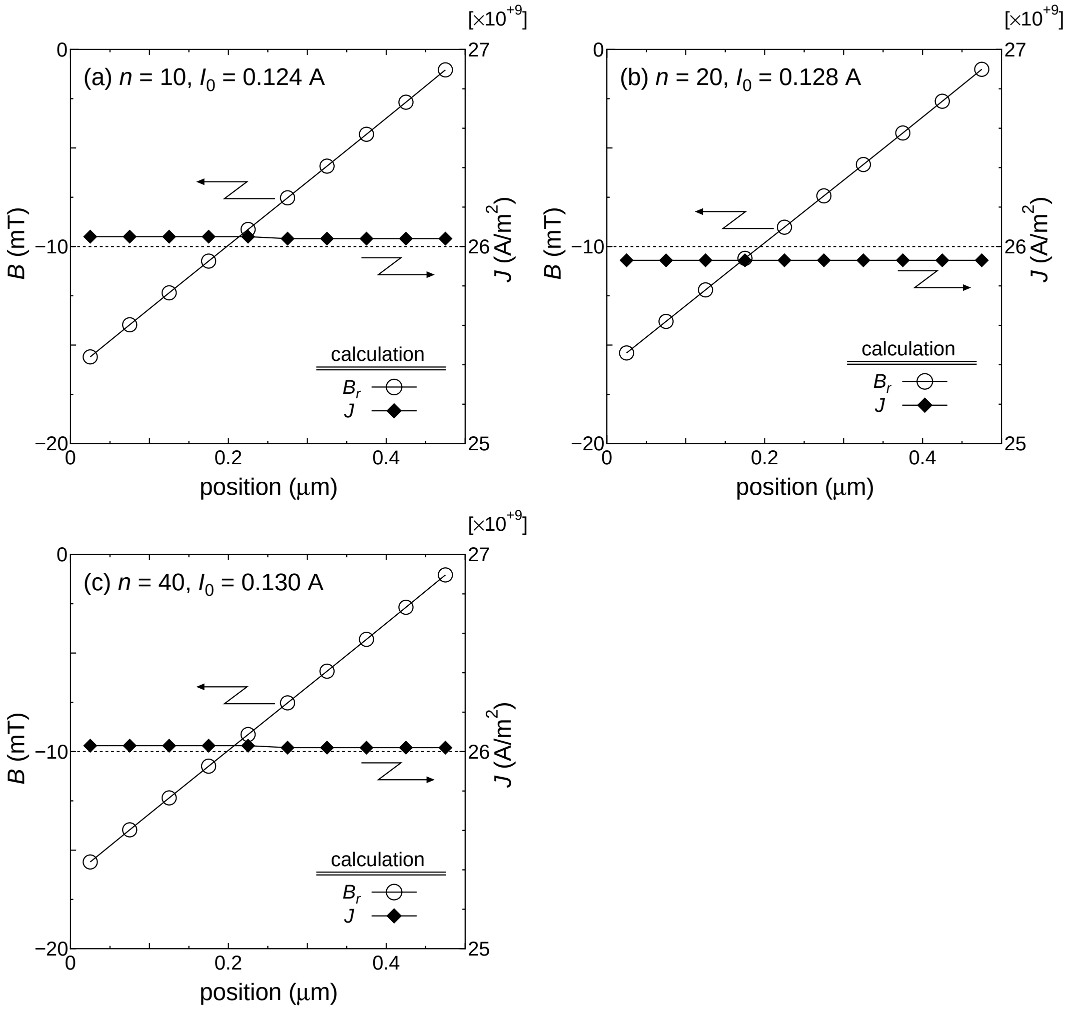

Fig. 6: T. Yoshida *et al.* WTP–114/ISS2007