Cross Section Minima in Elastic *Nd* Scattering: Possible Evidence for Three-Nucleon Force Effects

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Neutron-deuteron elastic scattering cross sections are calculated at different energies using modern nucleon-nucleon (NN) interactions and the Tucson-Melbourne three-nucleon force adjusted to the triton binding energy. Predictions based on NN forces only underestimate nucleon-deuteron data in the minima at higher energies starting around 60 MeV. Adding the three-nucleon forces fills up those minima and reduces the discrepancies significantly. [S0031-9007(98)06806-9]

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Substantial progress has been made recently in the study of the three-nucleon (3N) system both experimentally and theoretically. The set of data is being significantly enriched for cross sections and spin observables in elastic neutron-deuteron (nd) and proton-deuteron (pd)scattering and in the 3N breakup process. Theoretical formulations and numerical algorithms have been significantly improved, with the result that 3N bound and ndscattering states can be solved exactly. Recently, in the still pending pp Coulomb force problem for the pd system, a step forward has been achieved below the deuteron breakup threshold [1]. In addition, the nucleon-nucleon (NN) system is still very intensively investigated and the increased data set provides a sound foundation for reliable modern phase-shift analysis [2]. Based on these phases, modern NN forces have been constructed by different groups [3-5]. These interactions reproduce the NN data set with unprecedented accuracy as measured by a χ^2 /datum very close to 1. Although those forces are not yet linked to the underlying quantum chromodynamics (QCD) due to well-known reasons, they cover a wide spectrum of expected properties and form an interesting basis to study few-nucleon systems. Thus theoretical tools and data are available to probe the dynamics of three interacting nucleons. In the future, OCD should provide theoretically consistent NN and 3N forces and specifically the relative importance of the latter ones for binding energies and scattering matrices. The first steps on that ground are being done in chiral perturbation theory [6]. Despite the still restricted theoretical insight from QCD, one can go ahead and compare the theoretical predictions obtained with modern NN interactions and model 3N forces to experimental 3N data. There might be a clear-cut signal coming from certain observables which cannot be explained by 3N Hamiltonians based on modern NN interactions only. Such a "smoking gun" observable would then put limits on present day 3N force models and would also be of great importance to test the future QCD-based dynamics.

The three-nucleon binding energy by itself is a first signature. The modern *NN* interactions underbind ³H, but to a different extent [7]. The essentially local ones lack binding energy of about 800 keV out of 8.48 MeV, whereas the nonlocal CD Bonn interaction [5] underbinds only by \approx 500 keV. That information from ³H on insufficient dynamics based on present day *NN* forces only should be enriched by further evidence from the 3*N* continuum.

Such a search for 3N continuum observables, which could serve as possible evidence for 3NF effects, has been pursued since 3N continuum calculations have become feasible [8]. With the advent of the optimally tuned NN forces and the feasibility to also include threenucleon forces (3NF's) into 3N continuum calculations, the conclusive power of such calculations has increased tremendously. It is the aim of this article to point to such a smoking gun in the 3N continuum based on modern 3NFaddeev calculation.

Before coming to that, let us briefly describe the situation in 3N continuum studies. A detailed overview has been given recently [9]. The bulk of 3N scattering observables below about 100 MeV nucleon lab energy can be described quite well in the NN force picture only. A beautiful example is the total nd cross section [10]. This most simple picture is also quite stable in the sense that the most modern phase-equivalent NN force models yield essentially the same predictions. But there are exceptions, "time dependent ones," which were removed by subsequent measurements [11], and more important true ones, where the data are reconfirmed by independent measurements. Such a distinguished case is the low energy vector analyzing power A_v in elastic Nd scattering [12]. A drastic discrepancy between the predictions based on NN forces only, and both nd and pd data, has been found. Present day 3NF models have insignificant effects and do not remove that discrepancy. It is known, that A_{y} depends very sensitively on the ${}^{3}P_{i}$ NN forces. Thus a trivial explanation might be that the ³P NN phase-shift parameters from modern phase-shift analysis have not been settled to the true ones [13]. Presently, it is an unsolved puzzle. If the reason does not lie in the *NN* forces, a 3NF of still unknown properties will be responsible. In Ref. [14] arguments are given for that scenario since the considered changes in the *NN* forces, excluding the well-established propriety of the one pion exchange, were not capable of solving that puzzle. The closely related deuteron vector analyzing power iT_{11} is equally not understood [13].

Another possible signature for 3NF effects is the space star configuration in the 3N breakup process at 13 MeV [11]. Two *nd* measurements agree essentially with each other but deviate from theory (in the *NN* picture only and including 3NF models). The situation poses even more questions since *pd* data deviate very severely from the *nd* data pointing to unexpectedly large Coulomb force effects [15].

In the present study we investigate the angular distribution in elastic *Nd* scattering. The transition amplitude for this process is composed of the nucleon exchange part (PG_0^{-1}) , the direct action of a 3NF and a part having its origin in the multiple interactions of three nucleons through 2N and 3N forces:

$$\langle \phi' | U | \phi \rangle = \langle \phi' | P G_0^{-1} + V_4^{(1)} (1 + P) + P \tilde{T} + V_4^{(1)} (1 + P) G_0 \tilde{T} | \phi \rangle.$$
(1)

That rescattering part is expressed in terms of a \tilde{T} operator which sums up all multiple scattering contributions through the integral equation [16]

$$\tilde{T}|\phi\rangle = tP|\phi\rangle + (1 + tG_0)V_4^{(1)}(1 + P)|\phi\rangle + tPG_0\tilde{T}|\phi\rangle + (1 + tG_0)V_4^{(1)}(1 + P)G_0\tilde{T}|\phi\rangle.$$
(2)

Here G_0 is the free 3N propagator, t is the NN t matrix, and P is the sum of a cyclical and anticyclical permutation of three objects. The 3NF V₄ is split into three parts

$$V_4 = \sum_{i=1}^3 V_4^{(i)}, \tag{3}$$

where each one is symmetrical under exchange of two particles. For the π - π exchange 3NF, for instance [17], this corresponds to the three possible choices of the nucleon, which undergoes the (off-shell) π -N scattering. The asymptotic state $|\phi\rangle$ ($|\phi'\rangle$) is a product of the deuteron wave function and the momentum eigenstate of the third particle.

The exchange part comprises two processes where the incoming nucleon ends up as a constituent of the final deuteron, and the constituents of the initial deuteron are free in the final state. Because of the nature of this term its contribution to the elastic scattering cross section is peaked at backward angles. The contribution from the driving term $tP|\phi\rangle$ and the rescattering terms in t (NN force contributions only) are peaked at forward angles. Therefore the elastic scattering cross section exhibits a

characteristic minimum in the angular range, where the contributions of the exchange and the rescattering terms are of comparable order and both are small. This angular range around the minimum could thus be a place where the 3NF signal, if sufficiently strong, should appear. It would happen at those energies where the pure 3NF contribution to the elastic scattering amplitude in that minimum is comparable or larger than the contributions of the exchange part and the pure 2N rescattering terms.

The pure 3NF contribution to the transition operator Uresults from Eqs. (1) and (2) when only the 3NF is active: $U^{3NF} = P\tilde{T}^{3NF} + V_4^{(1)}(1+P) + V_4^{(1)}(1+P)G_0\tilde{T}^{3NF}$ (4)

with

$$\tilde{T}^{3NF} |\phi\rangle = V_4^{(1)} (1+P) |\phi\rangle + V_4^{(1)} (1+P) G_0 \tilde{T}^{3NF} |\phi\rangle.$$
(5)

We expect that the contribution of U^{3NF} alone is uniformly distributed over all angles.

In order to check these expectations we solved Eqs. (2) and (5) at the nucleon laboratory energies of 12, 65, 140, and 200 MeV using the modern NN interactions: AV18 [4], CD Bonn [5], Nijm I, and Nijm II [3]. As the 3NF we took the 2π -exchange Tucson-Melbourne (TM) model [17], where the strong cutoff parameter Λ has been adjusted individually together with each NN force to the experimental triton binding [7]. In the calculations including 3NF's, all partial wave states with total angular momenta in the two-nucleon subsystem up to $j_{\text{max}} =$ 3 were taken into account. It is the most extensive calculation with 3NF's in the continuum which we can presently perform. At the higher energies they are not fully converged with respect to j_{max} . The importance of partial waves with higher two-nucleon angular momenta is illustrated in fully converged solutions in the case when only 2N forces are active. Then we included all states up to $j_{\text{max}} = 5$. Our theoretical results are shown in Figs. 1-4 in comparison to data. Our theory does not include the pp Coulomb force. Therefore we should compare to nd data. This is only possible at rather low energies, where *nd* data exist and which agree perfectly with NN force predictions only [9]. The pd data also existing there agree with the *nd* data, except at very forward angles, where Rutherford scattering has to show up. That interference with Rutherford scattering can clearly be seen in Figs. 1 and 2 at forward angles, where the data bend towards smaller values. Aside from that, there is a very good agreement at 12 MeV with theory. This, together with the smallness of the Coulomb force effects on the elastic scattering cross section in the region of its minimum, as shown by exact calculations under the deuteron breakup threshold [1], supports the conjecture that a comparison of *nd* theory with *pd* data at even higher energies makes sense. Figures 1-4 show the expected result, that the pure 3NF contribution is



FIG. 1. The differential *Nd* cross section at $E_N^{\text{lab}} = 12$ MeV. The prediction of the CD Bonn *NN* interaction without (shortdashed curve) and with 3NF (solid curve) is compared to *pd* data (circles (\bigcirc) from [21] and crosses (+) from [22]). The long-dashed curve is the pure 3NF prediction. All of the calculations are truncated at $j_{\text{max}} = 3$.

essentially uniform in its angular dependence, and we see that at 12 MeV it is totally negligible. At 65 MeV there are also a few nd data [18] and, as shown in Fig. 2, they come close to NN force predictions only, whereas the pd data [19] deviate strongly in the minimum. Without a rigorous calculation, including the pp Coulomb force, it has to remain an open question whether the deviation



FIG. 2. The differential *Nd* cross section at $E_{n}^{lab} = 65$ MeV. The prediction of the CD Bonn *NN* interaction for $j_{max} = 3$ (short-dashed curve) and $j_{max} = 5$ (long-dashed curve) is compared to 64.5 MeV *pd* data [(\bigcirc) from [19]] and *nd* data [(+) from [18]]. The CD Bonn calculation including the 3*NF* for $j_{max} = 3$ fills the minimum (solid curve). The pure 3*NF* prediction is shown as intermediately long-dashed curve.



FIG. 3. The differential *Nd* cross section at $E_N^{\text{lab}} = 140 \text{ MeV}$. Curve descriptions are the same as in Fig. 2. The *pd* data are 145.5 MeV (\bigcirc) from [23] and 146 MeV (+) from [24]. The triangles (\triangle) are 152 MeV *nd* data from [25].

between the pd data and the NN force predictions is due only to our neglect of Coulomb forces in the theoretical calculations. On the other hand, the nd data of Fig. 3 are compatible with pd data in this energy range and indicate only small Coulomb force effects corresponding to our conjecture. Apparently, precise nd data in the angular range of the minima for 65 MeV and higher would be highly desirable. Independent of that important issue, we can go ahead and display possible 3NF effects in these minima. The discrepancy of the theory based on NNforces only to the pd data increases with energy, as seen in



FIG. 4. The differential *Nd* cross section at $E_N^{\text{lab}} = 200 \text{ MeV}$. Curve descriptions are the same as in Fig. 2. The *pd* data are 198 MeV (\bigcirc) from [26], 200 MeV (+) from [20], 181 MeV (\triangle) from [23], and 216.5 MeV (\times) from [23].

Figs. 2-4. Higher angular momentum states do not cure that discrepancy. They are a significant contribution, however, to the cross section at the higher energies at forward angles [20], as seen especially in Figs. 3 and 4. As expected, the pure 3NF contribution remains essentially uniform also at the higher energies. With increasing energy, however, this contribution becomes significant in relation to the minimum value of the cross section. Being totally negligible at 12 MeV, it overshoots the minimum value by a factor of ≈ 6 at 200 MeV. At 65 MeV, the 3NF signal becomes sufficiently large to be seen in the minimum region. Indeed, as shown in Figs. 2-4, including the 3NF in addition to the 2N interactions in the 3N Hamiltonian removes a large part of the discrepancy in the cross section minimum at the higher energies. We consider that filling of the minima as a smoking gun for 3NF effects. Very precise data, in both the nd and the pd systems, would therefore be highly valuable.

We have to expect additional modifications, especially at the highest energies, due to relativistic effects, which have not been taken into account in our calculation. First estimates just based on kinematical factors indicate indeed a small shift of all angular distribution at higher energies toward higher values.

Finally, we want to emphasize that our conclusions do not depend on the particular NN interaction used. Taking different modern NN interactions and the corresponding TM 3NF leads to practically the same results.

In summary, we have shown that the minima of the elastic Nd scattering cross sections are probably a smoking gun for 3NF effects. A large part of the discrepancy between modern NN potential predictions and data in this angular range can be removed when the TM 3NF, properly adjusted to the triton binding, is included in the 3N Hamiltonian. In order to check more accurately this conclusion, precise Nd elastic scattering data at different energies in the region of the cross section minima are required. The optimal data would be in the nd system to avoid the theoretical uncertainty of ppCoulomb force effects.

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