ANOMALY IN TOTAL INTERNAL REFLECTION OF OPTICAL PLANE WAVES AT THE BOUNDARY OF ISOTROPIC AND ANISOTROPIC DIELECTRIC MEDIA

by

Kazunori HANO

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SYNOPSIS

In this paper we report the anomalous phenomena of total internal reflection of optical plane waves at the boundary between lossless isotropic and lossless uniaxial anisotropic dielectric media. When TE or TM plane waves impinge upon the interface from a high-index isotropic medium region at a certain angle so as to be totally reflected, each amplitude coefficient of ordinary and extraordinary evanescent waves in anisotropic media diverges, but they cancel out due to the phase difference of π. The total electromagnetic fields in anisotropic media are, therefore, not divergent, but become indefinite at this incidence angle. Moreover, the reflection coefficients also become indefinite at this angle because of the boundary conditions.

This anomalous phenomenon occurs when the decay constants become equal of ordinary and extraordinary evanescent waves in uniaxial anisotropic media whose optic axis lies in the plane of boundary.

1. INTRODUCTION

Propagation of plane electromagnetic waves in lossless or lossy anisotropic media, and reflection and transmission at the boundary between lossless isotropic and anisotropic media have well been studied.[1,2] And the phenomena of total internal reflection at the boundary between two dielectric media have attracted much attention with respect to ray shifts (Goos-Haenchen and Imbert shifts) upon reflection,[3,4] waveguiding in thin-film optical waveguides,[5,6] FTR spectroscopy,[7] and total reflection holography,[8] etc.

There are in general two characteristic (ordinary and extraordinary) waves in (uniaxial) anisotropic media, to which two different indices of refraction belong. When a plane electromagnetic wave is incident from an isotropic medium region, two characteristic waves are, therefore, excited in an anisotropic medium, and they propagate with different phase velocities. Except the cases where the direction of the wavenormal of incident waves coincides with optic axis in lossless anisotropic media or with singular axis[2] in lossy anisotropic media, to the author's knowlegde, two refractive indices can not become identical.

As will be shown in this paper, however, these two indices of refraction are able to agree if the incidence angles are greater than two critical angles for total reflection of ordinary and extraordinary waves in lossless uniaxial anisotropic media and the optic axis lies in the boundary plane. The result is that the decay constants of ordinary and extraordinary evanescent waves coincide, the amplitudes of each wave diverge, but two

Dep. of Electronics, Kyushu Institute of Technology.

(ordinary and extraordinary) waves oscillate with out of phase. This causes the total electric and magnetic fields in anisotropic media to be indefinite because the amplitudes of the fields are expressed as 0/0. At the same time the reflection coefficients of the reflected fields in an isotropic medium region become also indefinite due to the boundary conditions.

In 2 of this paper we give the reflection and transmission coefficients at the boundary when TE or TM plane waves are incident upon a uniaxial anisotropic medium whose optic axis lies in the boundary plane. In 3, the anomalies in the total internal reflection are shown along with numerical examples, and the conditions for which the anomalies should occur are derived in 4.

2. REFLECTION AND TRANSMISSION AT THE BOUNDARY

In Fig. 1 the upper half region ($z<0$) is occupied by a lossless isotropic dielectric with refractive index n_p, and the lower half region ($z>0$) filled with a lossless uniaxial anisotropic medium whose indices of refraction are n_o and n_e for ordinary and extraordinary waves, respectively. Permeability of each medium is assumed to be μ_0 (permeability in vacuum). For a monochromatic plane wave $\exp[i(\omega t-\vec{k}\cdot\vec{r})]$ (ω: radian frequency, $\vec{k}$: wave vector, $\vec{r}$: position vector), Maxwell's eqs. in anisotropic media are reduced to

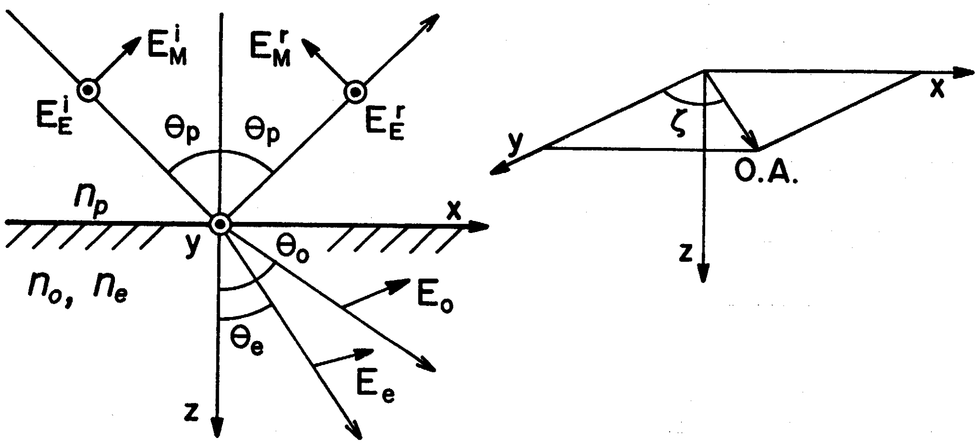

Fig. 1 Geometry of the problem and the orientation ζ of optic axis. θ_p indicates the incidence (and reflection) angle, and θ_o and θ_e are the refraction angles of ordinary (E_o) and extraordinary (E_e) waves, respectively. O.A. denotes the optic axis.

$$\vec{k}\times\vec{E}=\mu_0\vec{H}, \tag{1a}$$

$$\vec{k}\times\vec{H}=-\varepsilon_0[\varepsilon]\vec{E}, \tag{1b}$$

where $\vec{E}$ and $\vec{H}$ are electric and magnetic field vectors, respectively. $\varepsilon_0[\varepsilon]$ in (1b) (ε_0 is a permittivity in vacuum) denotes a dielectric tensor of uniaxial anisotropic media whose optic axis lies in the boundary plane, and $[\varepsilon]$ is given as

$$\left.\begin{aligned}
&\varepsilon_{xx}=n_o^2\cos^2\zeta+n_e^2\sin^2\zeta\\
&\varepsilon_{yy}=n_o^2\sin^2\zeta+n_e^2\cos^2\zeta\\
&\varepsilon_{zz}=n_o^2\\
&\varepsilon_{xy}=\varepsilon_{yx}=(n_e^2-n_o^2)\sin\zeta\cos\zeta\\
&\varepsilon_{yz}=\varepsilon_{zy}=\varepsilon_{zx}=\varepsilon_{xz}=0
\end{aligned}\right\} \tag{2}$$

where ζ is the angle between optic axis and positive y axis.

Suppose that a TE (E^i_E) or TM (E^i_M) wave

$$\vec{\mathrm{E}}^i=(\mathrm{E}^i_\mathrm{M}\cos\theta_p,\ \mathrm{E}^i_\mathrm{E},\ -\mathrm{E}^i_\mathrm{M}\sin\theta_p)\exp[-ik\kappa_p z], \tag{3a}$$

$$\vec{\mathrm{H}}^i=\mathrm{Y}(-\kappa_p\mathrm{E}^i_\mathrm{E},\ n_p\mathrm{E}^i_\mathrm{M},\ N\mathrm{E}^i_\mathrm{E})\exp[-ik\kappa_p z], \tag{3b}$$

is obliquily incident from an isotropic medium region, there appear the reflected waves

$$\vec{\mathrm{E}}^r=(-\mathrm{E}^r_\mathrm{M}\cos\theta_p,\ \mathrm{E}^r_\mathrm{E},\ -\mathrm{E}^r_\mathrm{M}\sin\theta_p)\exp[ik\kappa_p z], \tag{4a}$$

$$\vec{\mathrm{H}}^r=\mathrm{Y}(\kappa_p\mathrm{E}^r_\mathrm{E},\ n_p\mathrm{E}^r_\mathrm{M},\ N\mathrm{E}^r_\mathrm{E})\exp[ik\kappa_p z], \tag{4b}$$

in the isotropic medium region, and the transmitted (ordinary and extraordinary) waves

$$\vec{\mathrm{E}}^t=\textstyle\sum_{m=o,e}(a_m,\ b_m,\ c_m)\mathrm{E}_m\exp[-ik\kappa_m z], \tag{5a}$$

$$\vec{\mathrm{H}}^t=\mathrm{Y}\textstyle\sum_{m=o,e}(-b_m\kappa_m,\ a_m\kappa_m-c_mN,\ b_mN)\mathrm{E}_m\exp[-ik\kappa_m z], \tag{5b}$$

in the anisotropic medium region, where

$$\mathrm{Y}=\sqrt{\varepsilon_0/\mu_0},$$

$$k=2\pi/\lambda\ (\lambda \text{ is a wavelength in vacuum}),$$

$$N=n_p\sin\theta_p, \tag{6}$$

$$\kappa_p=\sqrt{n_p^2-N^2}, \tag{7}$$

$$\kappa_o=\begin{cases}\sqrt{n_o^2-N^2} & (N<n_o)\\ -iq_o & (N\geqq n_o),\end{cases} \tag{8}$$

$$\kappa_e=\begin{cases}\sqrt{n_e^2-\{\cos^2\zeta+(n_e^2/n_o^2)\sin^2\zeta\}N^2} & \\ \qquad (N<n_on_e/\sqrt{n_o^2\cos^2\zeta+n_e^2\sin^2\zeta}) & \\ -iq_e \qquad (N\geqq n_on_e/\sqrt{n_o^2\cos^2\zeta+n_e^2\sin^2\zeta}), & \end{cases} \tag{9}$$

$$q_0=\sqrt{N^2-n_o^2}, \tag{10}$$

$$q_e=\sqrt{\{\cos^2\zeta+(n_e^2/n_o^2)\sin^2\zeta\}N^2-n_e^2}. \tag{11}$$

In eqs. (3) through (5) we choose the field representation so as to satisfy the continuity of x component kN of wave vectors, and the common factor $\exp[i(\omega t-kNx)]$ is suppressed. Relations (6) to (11) are derived from the dispersion relation in the medium. Imaginary κ_o and κ_e give rise to the ordinary and extraordinary evanescent waves, respectively. In eq. (5) Σ designates the sum of ordinary (o) and extraordinary (e) waves, and the direction cosines a, b, c are given as follows.

$$\left.\begin{aligned}a_o&=-\kappa_o^2\cos\zeta/\sqrt{|\kappa_o|^2+N^2\cos^2\zeta}\\ b_o&=\kappa_o^2\sin\zeta/\sqrt{|\kappa_o|^2+N^2\cos^2\zeta}\\ c_o&=\kappa_o\,N\cos\zeta/\sqrt{|\kappa_o|^2+N^2\cos^2\zeta}\end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned}a_e&=-\kappa_o^2\sin\zeta/\sqrt{(|\kappa_o|^4+N^2|\kappa_e|^2)\sin^2\zeta+n_o^4\cos^2\zeta}\\ b_e&=-n_o^2\cos\zeta/\sqrt{(|\kappa_o|^4+N^2|\kappa_e|^2)\sin^2\zeta+n_o^4\cos^2\zeta}\\ c_e&=N\kappa_e\sin\zeta/\sqrt{(|\kappa_o|^4+N^2|\kappa_e|^2)\sin^2\zeta+n_o^4\cos^2\zeta}\end{aligned}\right\} \tag{13}$$

Applying the boundary condition that the tangential components of electric and magnetic fields be continuous at the interface, we can obtain the reflection and transmission coefficients. These coefficients are defined as follows.

$$\begin{bmatrix} E_M^r \\ E_E^r \\ E_o \\ E_e \end{bmatrix} = \begin{bmatrix} r_{MM} & r_{ME} \\ r_{EM} & r_{EE} \\ t_{oM} & t_{oE} \\ t_{eM} & t_{eE} \end{bmatrix} \begin{bmatrix} E_M^i \\ E_E^i \end{bmatrix}$$

The matrix elements are

$$r_{MM} = [b_e(\kappa_p+\kappa_e)\{a_o(\kappa_p\kappa_o-n_p^2)-N\kappa_p c_o\} - b_o(\kappa_p+\kappa_o)\{a_e(\kappa_p\kappa_e-n_p^2)-N\kappa_p c_e\}]/R \tag{14}$$

$$r_{EE} = [b_e(\kappa_p-\kappa_e)\{a_o(\kappa_p\kappa_o+n_p^2)-N\kappa_p c_o\} - b_o(\kappa_p-\kappa_o)\{a_e(\kappa_p\kappa_e+n_p^2)-N\kappa_p c_e\}]/R \tag{15}$$

$$r_{EM} = 2n_p\kappa_p b_o b_e(\kappa_e-\kappa_o)/R \tag{16}$$

$$r_{ME} = 2n_p\kappa_p\{a_o a_e(\kappa_e-\kappa_o)-N(a_o c_e-a_e c_o)\}/R \tag{17}$$

$$t_{oM} = 2n_p\kappa_p b_e(\kappa_p+\kappa_e)/R \tag{18}$$

$$t_{eM} = -2n_p\kappa_p b_o(\kappa_p+\kappa_o)/R \tag{19}$$

$$t_{oE} = -2\kappa_p\{a_e(\kappa_p\kappa_e+n_p^2)-N\kappa_p c_e\}/R \tag{20}$$

$$t_{eE} = 2\kappa_p\{a_o(\kappa_p\kappa_o+n_p^2)-N\kappa_p c_o\}/R \tag{21}$$

$$R = b_e(\kappa_p+\kappa_e)\{a_o(\kappa_p\kappa_o+n_p^2)-N\kappa_p c_o\} - b_o(\kappa_p+\kappa_o)\{a_e(\kappa_p\kappa_e+n_p^2)-N\kappa_p c_e\} \tag{22}$$

In case of total reflection, we have the following relations[6]

$$\left.\begin{aligned} &|r_{EE}| = |r_{MM}| = u\,, \\ &|r_{EM}| = |r_{ME}| = v\,, \\ &u^2+v^2=1\,, \\ &\phi_{MM}+\phi_{EE} = \phi_{ME}+\phi_{EM}\pm\pi\,, \end{aligned}\right\} \tag{23}$$

where ϕ_{MM}, ϕ_{EE}, ϕ_{ME}, and ϕ_{EM} stand for the phases of r_{MM}, r_{EE}, r_{ME}, and r_{EM}, respectively.

3. ANOMALY IN TOTAL REFLECTION

To investigate the reflection and transmission coefficients given in 2 in the case of total internal reflection, we introduce the relation between q_o and q_e. Eqs. (10) and (11) give rise to the following

$$q_o^2-q_e^2=(n_o^2-n_e^2)(N^2\sin^2\zeta-n_o^2)/n_o^2\,. \tag{24}$$

Because of $n_o \neq n_e$, we have from eq. (24)

$$q_o=q_e \tag{25}$$

when

$$N\sin\zeta=n_o, \qquad i.e., \qquad n_p\sin\theta_p\sin\zeta=n_o\,. \tag{26}$$

Now take notice of R, the denominator of all amplitude coefficients, in eq. (22). As seen, R is antisymmetric with respect to the subscripts o and e. Therefore, we obtain

$$R=0 \tag{27}$$

when $q_o = q_e$ (Note that we have $a_o/a_e = b_o/b_e = c_o/c_e$ if $q_o = q_e$; see eq. (28)). In this case the amplitudes of the transmission coefficients t_{oM} to t_{eE} of eqs. (18) through (21) are seen to diverge, for the numerators don't vanish. On the other it is easy to see that the reflection coefficients r_{MM} to r_{ME} of eqs. (14) through (17) become indefinite (0/0) because the numerators of these coefficients also become null when $q_o = q_e$. From now on we shall call the incidence angles θ_p which satisfy eq. (26) the anomalous incidence angles.

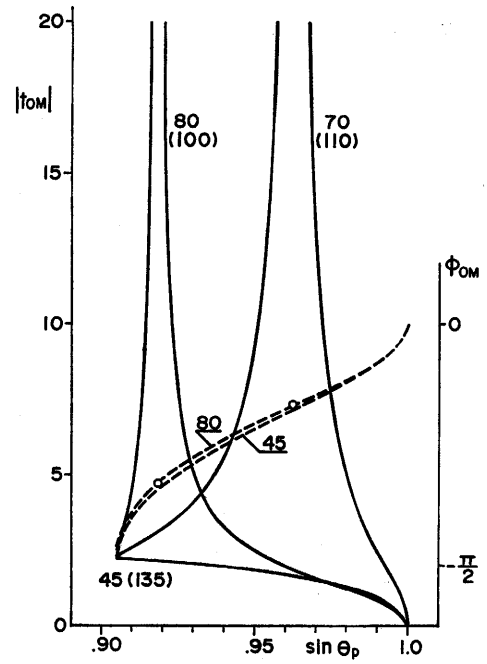

Fig. 2 Transmission coefficient t_{oM} of ordinary evanescent wave for TM wave incidence. The solid and broken lines denote the magnitude $|t_{oM}|$ and the phase ϕ_{oM}, respectively. The phase variation is so small that the plots of ϕ_{oM} are shown for $\zeta = 80°$ and 45°. Though $|t_{oM}|$ becomes infinite and ϕ_{oM} is indefinite at $\sin\theta_p = 0.9183$ and 0.9624 for $\zeta = 80°$ (or 100°) and 70° (or 110°), respectively, there is no such anomalous behavior for $\zeta = 45°$ (or135°).

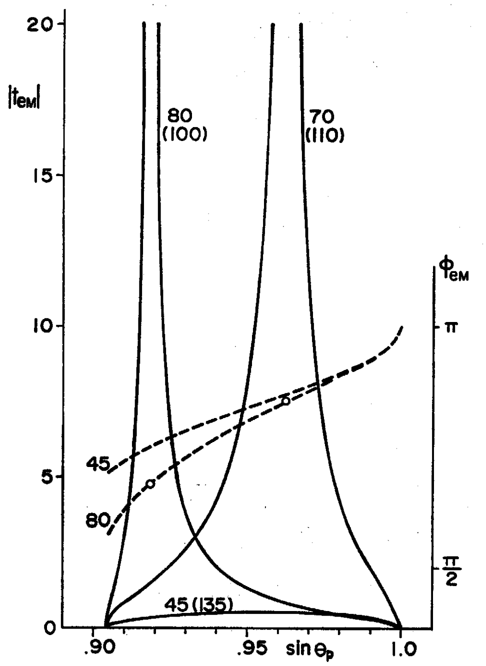

Fig. 3 Transmission coefficient t_{eM} of extraordinary evanescent wave for TM wave incidence. Refer the caption of Fig. 2 as to the lines and the parameter. Note that at the marked points (∘) the phase differences between t_{oM} and t_{eM} are precisely 180° (see Fig. 2).

Now under eq. (25) we check how the fields are in uniaxial anisotropic media. From eq. (5) it is seen that the wave-vectors of ordinary and extraordinary waves are precisely the same. Substiution of eq. (25) into eqs. (12) and (13) results in the folowing

$$a_o : b_o : c_o = a_e : b_e : c_e , \tag{28}$$

which means the same direction of vibration of ordinary and extraordinary evanescent waves. Next look into the amplitudes of ordinary and extraordinary evanescent waves. To do this, the transmission coefficients t_{oM} and t_{eM} for the case of TM wave incidence are plotted against $\sin\theta_p$, the sine of incidence angle in Figs. 2 and 3 with the rotation angle ζ of optic axis as a parameter. These figures show that the magnitudes of t_{oM} and

t_{eM} diverge at the anomalous incidence angles corresponding to the paticular values of ζ. Note that for $\zeta=45°$ or $135°$ there is no incidence angle at which the magnitudes of t_{oM} and t_{eM} are infinite. The reason will be made clear in next 4. Moreover, as evident from eqs. (18) and (19), the magnitudes of the numerators of t_{oM} and t_{eM} coincide perfectly when $q_o=q_e$, but the differences of their phases are just π rads.[9] The total fields which are the sum of ordinary and extraordinary waves turn, therefore, indefinite in uniaxial anisotropic media because the amplitudes have the form of 0/0.

But the total fields are not divergent. This point is confirmed from the boundary conditions. For example, in case of TM wave incidence, the following

$$r_{EM}=b_o t_{oM}+b_e t_{eM} \tag{29}$$

must always be satisfied for the y component of electric fields. r_{EM}, however, is finite as in Fig. 4.

The situations mentioned above are the very same for the case of TE wave incidence, as seen from eqs. (20) and (21), although the plots of t_{oE} and t_{oE} are not given here.

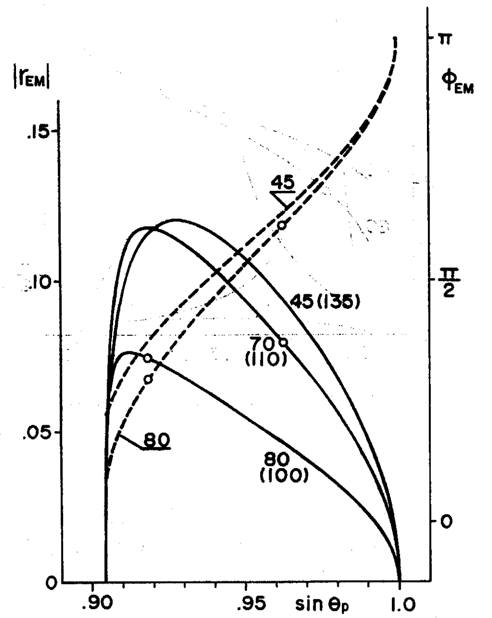

Fig. 4 Conversion coefficient r_{EM} for TM wave incidence. There is no apparent feature of anomaly, but at the marked points (∘) the values are not determined.

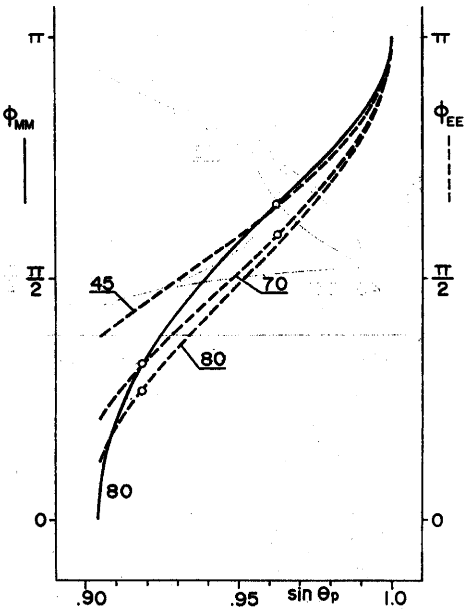

Fig. 5 Phase of reflection coefficients r_{MM} and r_{EE}. The solid line indicates ϕ_{MM} which is almost independent of ζ, and the other broken curves denote ϕ_{EE}. At the marked points (∘) the values are indefinite.

Now examine the reflected waves in an isotropic medium region. As mentioned before, at the anomalous incidence angles, not only R of eq. (22) but also the numerators of r_{MM}, r_{EE}, r_{EM}, and r_{ME} become null. This causes the amplitudes of reflected waves to be indefinite. We show the plots of $r_{EM}(=-r_{ME})$ and the phases ϕ_{MM} and ϕ_{EE} versus $\sin\theta_p$ in Figs. 4 and 5, respectively. As inferred easily from Fig. 4 and the relation (23), the magnitudes of r_{MM} and r_{EE} are nearly equal to unity, and hence omitted in Fig. 5. Although there is no apparent anomalous features in these figures, there exist the anomalous incidence angles as mentioned just above. In these figures, the dependence of r_{MM}, r_{EE}, and r_{EM} on ζ is based upon the fact that for large incidence angles the main component of electric fields of TM waves is the z component E_z which does not "see" the effects of uniaxial anisotropy due to ε_{zz} of eq. (2), while the E_y of TE waves does "see" the optic axis through ε_{yy}.

4. CONDITIONS FOR ANOMALOUS TOTAL REFLECTION

In preceding 3 we have implicitly assumed that eqs. (25) or (26) holds good. Now consider the conditions for which eq. (26) be actually realized. Following inversely the process of derivation of eq. (26), we notice that there is no necessity for κ_o and κ_e to be imaginary in order to get eq. (26). As shown right below, however, eq. (26) is not satisfied in practice for the real κ_o and κ_e; this implies that the anomaly of total reflection does take place only when both ordinary and extraordinary waves are evanescent.

From the second form of eq. (26) we obtain, using the relation $\sin\zeta \leqq 1$,

$$1 \geqq \sin\theta_p > n_o/n_p = \sin\theta_{oc}, \tag{30}$$

where θ_{oc} is the critical angle for total reflection of an ordinary wave. The relation of inequality (30) requires that n_p should be larger than n_o and the incidence angle θ_p be greater than θ_{oc}; the ordinary waves must be evanescent. This leads the extraordinary waves to be also evanescent, for eqs. (25) or (26) cannot be obtained if the ordinary wave is evanescent and the extraordinary wave is not, or vice versa. The second of eq. (26) gives also the following relation

$$1 > \sin\zeta > n_o/n_p, \tag{31}$$

which provides the ranges of ζ in which the anomalies occur for $0° < \theta_p < 90°$. In case of $\sin\zeta = 1$ we have a null ε_{xy}, and then the anomalies will not result because only an ordinary wave is excited in uniaxial anisotropic media by the impingement of TE waves, and only an extraordinary wave by the incidence of TM waves. Because we choose the following materials

$$n_p = 1.830 \text{ (SF optical glass)},$$

$$n_o = 1.655,\ n_e = 1.485 \text{ (calcite)},$$

at $\lambda = 0.6328\ \mu$m in the example, the anomalies take place in the range

$$64.74° < \zeta < 115.26° \quad (\zeta \neq 90°). \tag{32}$$

Dispersion curves of ordinary and extraordinary waves are depicted in Fig. 6 with ζ as a parameter, in which the ordinate denotes κ_o and κ_e (the quarter- circle and ellipses stand for the real κ_o and κ_e, respectively, and the parabolas for the magnitudes q_o and q_e of imaginary κ_o and κ_e, respectively), and the abscissa indicates the x component N of the normalized wavenumber. Solid curves belong to the ordinary waves, and the other to the extraordinary waves. For $\zeta = 45°$ or $135°$, the parabolas of the ordinary and extraordinary evanescent waves are crossed at the point $N = 2.3405$ where eq. (26) should be satisfied. But as seen from Figs. 2 to 5, the anomaly does not occur in this case because ζ does not satisfy the relation (32). On the contrary for $\zeta = 70°$ (or $110°$) and $\zeta = 80°$ (or $100°$) which satisfy the relation (32), two parabolas are crossed (see the inset in the Fig. 6), and then the anomalies should really take place at the anomalous incidence angles. Although there are also crossing points between the parabolas of the extraordinary evanescent waves and the circle of the ordinary waves, it has no meaning because at these points κ_o is real while κ_e is imaginary. It is inferred from Fig. 6 that two parabolas of ordinary and extraordinary evanescent waves agree at $N = n_o$ for $\zeta = 90°$, and at infinite N for $\zeta = 0$. Consequently if we are permitted to obtain the substances with very high-index of refraction n_p with respect

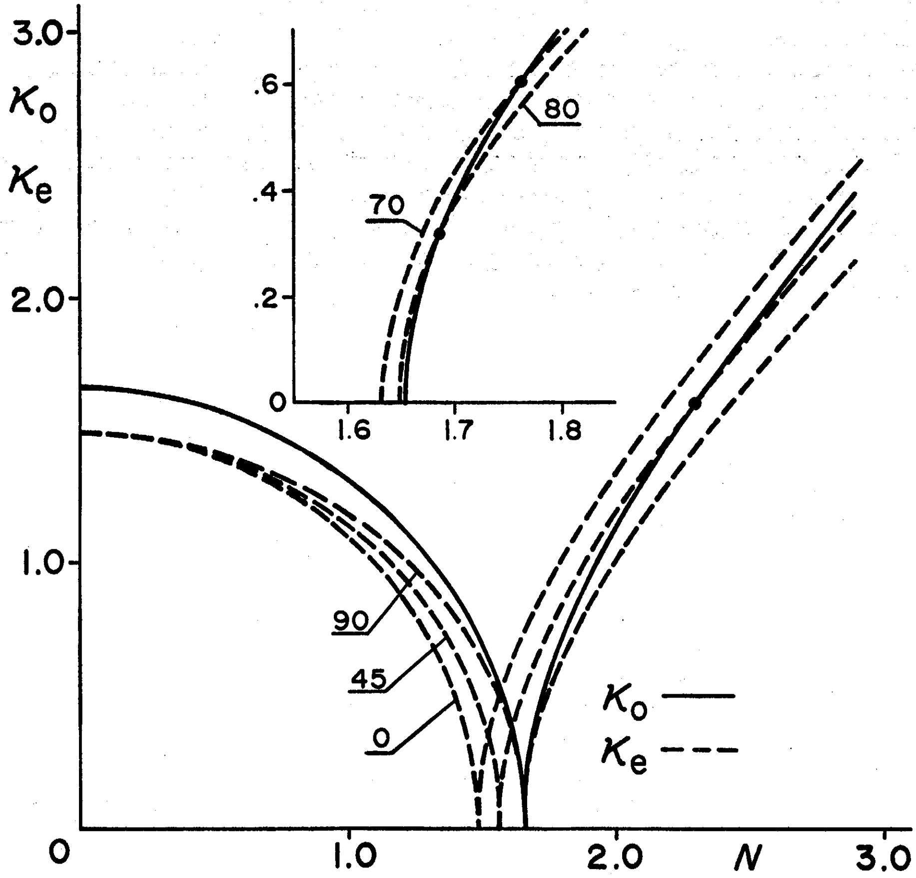

Fig. 6 **Dispersion curves of ordinary (solid lines) and extraordinary (broken curves) waves. Note that for the part of eqauter- circles and ellipses κ_o and κ_e are real, but for the semi-parabolas κ_o and κ_e are imaginary ($\kappa_o=-iq_o$, $\kappa_e=-iq_e$). In the inset q_o and q_e are depicted only near the anomalous incidence angles for $\zeta=70°$ and 80°. At the crossed points between q_o and q_e the anomalies take place.**

to the indices n_o and n_e of a uniaxial anisotropic material, these anomalies would take place for all incidence angles greater than two critical angles of the ordinary and extraordinary waves, corresponding to the direction angle ζ of optic axis.

5. CONCLUDING REMARKS

In this paper we present theoretically the anomaly in total internal reflection along with numerical examples. It takes place when TE or TM plane waves impinge upon the interface from a high-index isotropic medium region at certain angles so that the decay constants q_o and q_e become equal of ordinary and extraordinary evanescent waves in uniaxially anisotropic media whose optic axis lies in the boundary plane. In the case that q_o is nearly equal to q_e, the ordinary and extraordinary evanescent waves become almost identical and the amplitudes of each evanescent wave diverge. But the cancellation between them occur to make the total fields in anisotropic media finite. When q_o equals exactly to q_e, i.e., the ordinary and extraordinary waves are perfectly phase-matched, not only the total electromagnetic fields in uniaxial anisotropic media but the reflected waves in isotropic media become indefinite because the amplitudes have the form of 0/0. We have also given the conditions for which these anomalies should occur.

That the ordinary and extraordinary waves are identical should mean that the anisotropic media behaves as if they were isotropic, for there is no meaning in the distinction between "ordinary" and "extraordinary" waves. Further investigations are needed.

In this paper we have discussed only the case where optic axis is in the boundary $(x-y)$ plane, because the relation (26) essential to the anomaly is not satisfied in the other cases where the optic axis is contained in $y-z$ or $z-x$ plane.

In conclusion the anomaly of this type would also take place in biaxial anisotropic media, and in the stratified structures containing the anisotropic media in which the ordinary and extraordinary waves are evanescent.

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9) Strictly speaking, the phases of two transmitted evanescent waves (or t_{oM} and t_{eM}) are not determined when $q_o=q_e$, but in the *asymototic values* the phase difference is π rads.