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STRESS CONCENTRATION DUE TO SHEAR LAG IN BOX GIRDER UNDER DISTRIBUTED LOAD

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Many studies on shear lag have been conducted. In recent years, the finite element method tends to be used for the study of shear lag. The finite element method is very powerful for engineering solutions. Nevertheless, a variety of stress concentration factors are found in the literature. Which is attributable to the fact that not many researches have carefully dealt with the finite element mesh and the modeling of the load while it is known that stress concentration is sensitive to the modelings. The shear-lag effect in a stiffened box girder under uniformly distributed load is studied by the there-dimensional finite element method in the present study. To be specific, parametric study regarding the geometry of a box girder is conducted numerically. The dependency of the shear-lag effect on the mesh and the modeling of the load are treated with much care. From the numerical results, the empirical formulas for evaluating the stress concentration under uniformly distributed load in the stiffened box girder are proposed.

Keywords: Longitudinal stiffener, Three-dimensional finite element analysis, Parametric study, Multimesh extrapolation method, Empirical formula.

1 INTRODUCTION

The beam theory assumes that the longitudinal normal-stress produced by bending is proportional to the distance from the neutral axis. Consequently, the normal stress is uniform across the flange width. However, for a beam with a wide flange, the uniform stress distribution in the flange becomes unacceptable: the normal stress varies in the flange with the maximum value acting at the flange-web intersection. Which is the so-called shear lag.

Timoshenko and Goodier (1970) have mentioned the work by von Karman, one of the earliest researches on the shear lag. The best-known research in this filed is the work by Reissner (1941) and Reissner (1946). The early researches were conducted analytically, while most of the recent studies have resorted to the finite element method.

The shear lag study is not easy even by the finite element method since the stress concentration is quite sensitive to the way the analysis is carried out. In fact, considerable discrepancy in the shear lag effect can be found in the literature, as pointed out by Lertsima *et al*. (2004) and Yamaguchi *et al*. (2008).

Yamaguchi and his colleagues have conducted a series of shear-lag studies (Lertsima *et al*. 2004, Lertsima *et al*. 2005, Sa-nguanmanasak *et al*. 2007, Yamaguchi *et al*. 2008). They have employed the finite element method and the discretization error has been reduced by the multimesh extrapolation method (Cook *et al*. 1989). Yet those researches have been limited to the unstiffened box girder, while a steel box girder usually has longitudinal stiffeners on the flange.

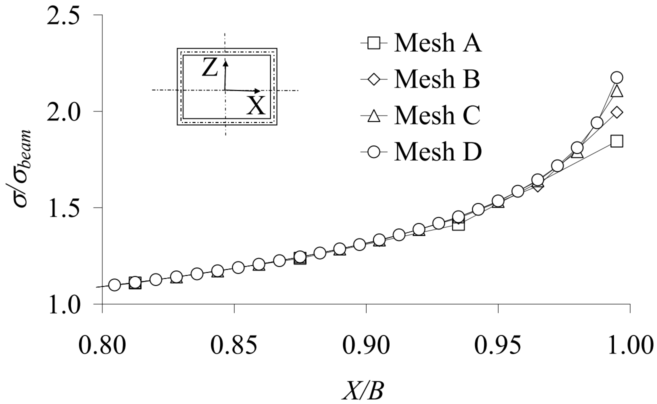


Figure 1. Longitudinal normal-stress distribution in flange.

In the present study, the objective is to clarify the shear-lag effect in a stiffened box girder. A simply supported box girder under uniformly distributed load is investigated by the three-dimensional finite element analysis using shell elements. The longitudinal normal-stress at the flange-web intersection in the mid-span cross section is of particular interest, since it is the largest stress. A parametric study is conducted, based on which empirical formulas for the stress concentration under uniformly distributed load are proposed. In all the analyses, the finite element program, MSC Nastran (2014), is used.

2 SHEAR LAG

Figure 1 shows an example of the longitudinal normal-stress distribution in the flange of a box girder. In the figure, X is the distance from the midpoint of the flange and B is a half of the flange width. The normal stress σ is normalized by the stress due to the beam theory σ_{beam}. This is the result by the finite element analysis with four different meshes: Mesh A is the coarsest mesh while Mesh D the finest. The stress distribution is not uniform obviously and the maximum stress occurs at the end of the flange, the flange-web intersection, illustrating the shear-lag phenomena.

The evaluation of the shear lag by the analytical analysis often employs some assumptions that help yield a solution, yet introduce more or less error inevitably. For example, the parabolic distribution of normal stress in a flange and the linear distribution in a web were assumed in the work of Reissner (1941) and Reissner (1946). The assumption is a source of the discrepancy in the shear-lag effect evaluation.

The finite element method can yield a solution with a much more realistic model. However, the result would be significantly dependent on the mesh employed. The stress concentration is especially sensitive to the mesh. Figure 1 illustrates the point: the difference between the meshes is larger in the vicinity of the flange-web intersection where the stress concentration occurs. Thus the finite element mesh could be a source of the discrepancy in the evaluation of the shear-lag effect.

As Tenchev (1996) pointed out, a load in the beam theory can be modelled in various ways in the three-dimensional finite element analysis. However, the modeling of the load has not been paid much attention to, which could be yet another source of the discrepancy in the shear-lag effect evaluation.

In the present study, the maximum normal stress is to be obtained by the three-dimensional finite element method. The mesh and the modeling of the load are carefully treated.

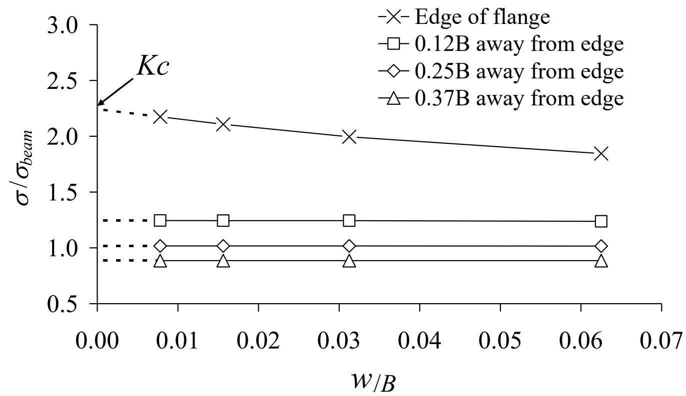


Figure 2. Variation of the normal stress with respect to the element size.

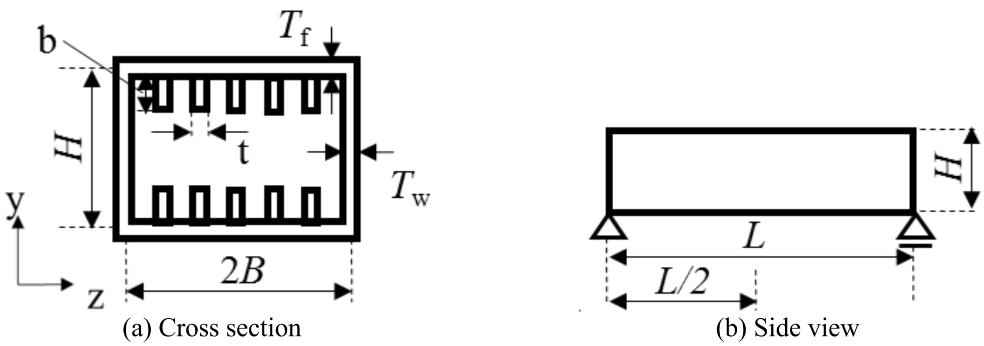


Figure 3. Structural geometry of box girder.

Figure 2 shows that the normal stress at the flange-web intersection in the mid-span cross section changes with the element size *w*. Cook *et al*. (1989) state that the strain error is linearly dependent on the element size. In the linear analysis, "strain" can be read as "stress." Therefore, by the linear extrapolation, the stress for an infinitesimally small element size can be estimated, as indicated by the dotted lines in Figure 2. From the stress ratio σ/σ_{beam} thus obtained, the value of the stress concentration factor *Kc* can be cmputed, which is the target in the present study. This is the so-called multimesh extrapolation method. It has been used successfully indeed in the previous studies of the authors' group: the results have been found close to those by the adaptive finite element method (Lertsim *et al*. 2004).

The present study deals with the uniformly distributed load. Its modelling is presented in the next section.

3 BOX GIRDER MODEL

The analysis of simply supported box girders subject to uniformly distributed load is conducted. Figure 3 shows the symbols for describing the geometry of the girder.

Following the work of Lertsima *et al*. (2004) and Lertsima *et al*. (2005), two loading models are employed (Figure 4): uniformly distributed load is applied to the centerline of each web in Load D-1, while the whole web is subject to uniformly distributed load in Load D-2.

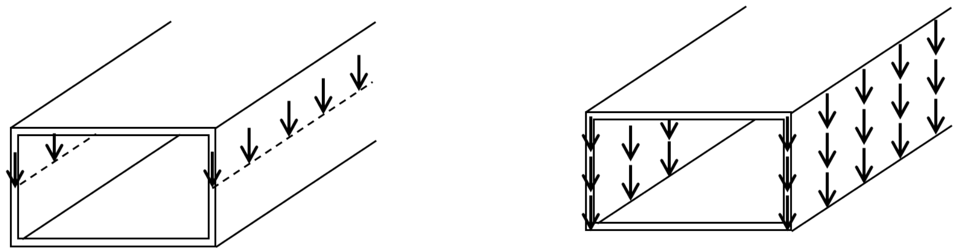


Figure 4. Distributed load.

4 STRESS CONCENTRATION FACTOR

Some of the typical results are shown in Figure 5. It is observed that as the values of the parameters including As/A_f increase, the stress concentration factor Kc becomes larger. From the numerical results obtained in the present study, the following empirical formulas for the stress concentration factor Kc are proposed as shown in Eq. (1-9).

Load D-1

$$K_c = F_{D1} \times \left[1 + \left\{1.253\left(\frac{H}{L}\right) + 0.021\left(\frac{B}{H}\right)\right\}\left(\frac{A_s}{A_f}\right)\right] \tag{1}$$

where

$$F_{D1} = \alpha\left(\frac{H}{L}\right)^2 + 1 \tag{2}$$

$$\alpha_{D1} = \beta_{D1}\left(\frac{B}{H}\right)^{2.368} \tag{3}$$

$$\beta_{D1} = 0.173\left(\frac{T_f}{T_w}\right) + 6.26 \tag{4}$$

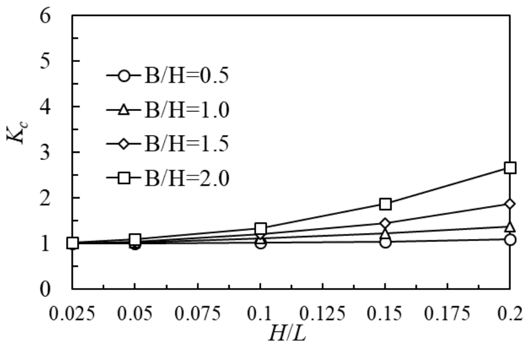
Load D-2

$$K_c = F_{D2} \times \left[1 + \left\{1.192\left(\frac{H}{L}\right) + 0.026\left(\frac{B}{H}\right)\right\}\left(\frac{A_s}{A_f}\right)\right] \tag{5}$$

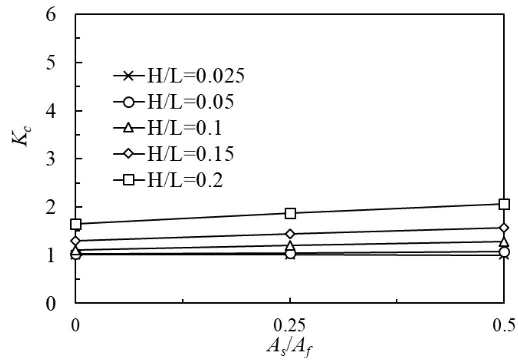
where

$$F_{D2} = \alpha_{D2}\left(\frac{H}{L}\right)^2 + 1 \tag{6}$$

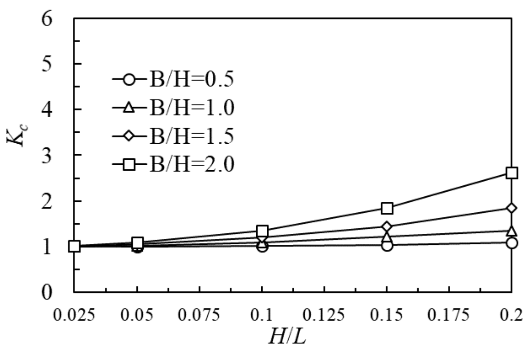
$$\alpha_{D2} = \beta_{D2}\left(\frac{B}{H}\right)^{\gamma} \tag{7}$$



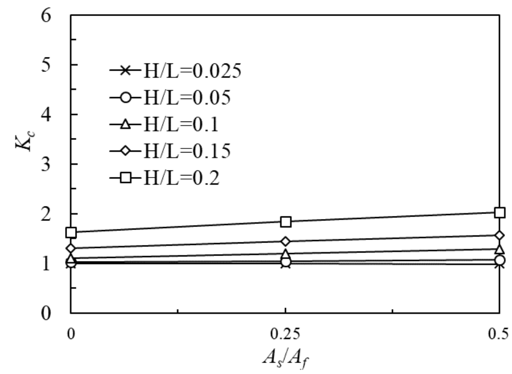
(a) Load D-1, $T_f/T_w = 1.5$, $A_s/A_f = 0.25$



(b) Load D-1, $B/H = 1.5$, $T_f/T_w = 1.5$



(c) Load D-2, $T_f/T_w = 1.5$, $A_s/A_f = 0.25$



(d) Load D-2, $B/H = 1.5$, $T_f/T_w = 1.5$

Figure 5. Influence of parameters on K_C.

$$\beta_{D2} = 0.127\left(\frac{T_f}{T_w}\right) + 6.275 \tag{8}$$

$$\gamma_{D2} = 0.063\left(\frac{T_f}{T_w}\right) + 2.235 \tag{9}$$

The average errors of Eqs. (1) and (5) are 3.1% and 3.2%, respectively. The equations are acceptable for a practical purpose.

5 CONCLUDING REMARKS

The shear lag effect in a stiffened box girder under uniformly distributed load was investigated by the there-dimensional finite element analysis. The discretization error was reduced by the multimesh extrapolation. As the values of the parameters that characterize the geometry of the box girder increase, the stress concentration factor was found to increase. The empirical formulas for the stress concentration factor in a stiffened box girder were then proposed. The accuracy of the formulas is satisfactory for practical use.

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