分散制御における「独立設計」と「逐次設計」の統合

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はしがき

本研究『分散制御における「独立設計」と「逐次設計」の統合』は日本学術振興会科学研究費補助金 (基盤研究 (C) 課題番号 16560389) により, 平成 16 年度から平成 18 年度までの 3 年間に渡って実施されたものである. 本報告書は, その成果をまとめたものである.

本研究の基本課題は, 従来、まったく相異なる分散制御系の設計手順として提案されていた「独立設計」と「逐次設計」を, 統一的に扱える理論を構築することである. その第一歩として, 分散型安定化制御器のパラメトリゼーションについての成果をまとめたのが文献 [1] である. 安定化制御器のパラメトリゼーションは, 安定化制御器の本質的な自由度を表現し, 制御系の性能限界を明らかにする上でそれ自体重要である. このパラメトリゼーションの結果を基に, 「独立設計」と「逐次設計」を統合する「繰り返し独立設計」の基本的な枠組みを提案した. この「繰り返し独立設計」という枠組みにおいて, balancing factor を用いることでサブシステムごとに異なる設計の難易度を考慮できることを示した. さらに, balancing factor $\to 0$ (あるいは ∞) の極限で、「独立設計」と「逐次設計」を統一的に扱えることを理論的に示した. この成果をまとめたのが文献 [2] であり, 本研究課題の主要な成果である.

また, 関連研究として, 分散制御系が有利とされる耐故障性を考慮した制御器の構造についても研究を行った. 耐故障性を有する制御器は行列としてのランクが許容する故障の数に応じて低下する傾向があることを数値例を通して示した. また, 状態フィードバックに限定して、ある条件化で低ランクの制御器が得られることを理論的にも明らかにした. これらの成果をまとめたのが文献 [3〜7] である.

分散制御器は常にフルランクであり, 低ランク制御器の構造とは本質的に異なる. 文献 [3〜7] の成果は分散制御器の耐故障性における優位性を揺るがす成果でもある. 耐故障性と分散制御系の関連は理論的にも実用的にも重要な課題であり, 今後の進展が期待される研究課題である.

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[2] Noboru Sebe, Unification of Independent and Sequential Procedures for Decentralized Controller Design, *Automatica*, **43**-4, pp.707-713, 2007. (掲載予定)

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本報告書は, 以上の成果をまとめたものである.

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Explicit characterization of decentralized coprime factors ⋆

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Abstract

This paper is concerned with the parametrization of all the decentralized stabilizing controllers. The auxiliary diagonal system, which is defined by the diagonal elements of Bezout factors, plays important roles in the parametrization of decentralized controllers. This paper gives an explicit characterization of the auxiliary system.

Key words: Decentralized control, Coprime factorization, Parametrization, State-space realization, Transfer functions

1 Introduction

This paper is concerned with the parametrization of all the decentralized stabilizing controllers.

For centralized control systems, the parametrization of all stabilizing controllers is proposed by Youla *et al.* (1976). The parametrization has brought us great advantages in progress of control theory. It clarifies the structure of stabilizing controllers and the restrictions on the performance. It also helps the derivation of H_∞ controllers [6,5,7], and the derivation of the conditions for the strong stabilization problem and the simultaneous stabilization problem [18], etc. The parametrization helps to develop design procedures not only for problems with frequency domain specifications but also for those with time domain specifications. Boyd and Barratt (1991) propose to design a free parameter $Q(s)$, which appears affinely in the parametrization, by the convex optimization. It should be noted that the state space representation of the Bezout factors [12] has also achieved the progress in control theory.

For the decentralized control systems, the decentralized Bezout identity and its stable factors, called "d-coprime factors," have been proposed [9,4]. The decentralized Bezout identity has a special structure and the diagonal parts of the coprime factors represent the auxiliary diagonal system. With this auxiliary diagonal system, parametrizations of decentralized controllers have been also proposed. Although the decentralized Bezout identity and the auxiliary diagonal system play important roles in the parametrization of decentralized controllers, the explicit characterization has not been given.

From the viewpoint of controller design, the characterization of the auxiliary diagonal systems is very important. Unlike the centralized case, there are no practical methods to design decentralized controllers based on the parametrization. The first reason is that there are constraints on the parameters. The second reason is that the connection between the d-coprime factors and their state space representation is not clarified. Thus, there are no computer-oriented computational methods for the parametrization of decentralized controllers.

In this paper, a state space representation of decentralized stable factors is given and the characteristics of the diagonal parts of the decentralized stable factors, i.e., the auxiliary diagonal systems are clarified.

Notation.
In this paper, static matrices are used in state space representations of transfer function matrices. And transfer function matrices themselves are also used in this paper. For the sake of simplicity, the variable s is dropped in many cases. To avoid misunderstanding, matrices A, B, C, D, E, F, I and O denote static matrices (especially I and O denote an identity and a zero matrices respec-

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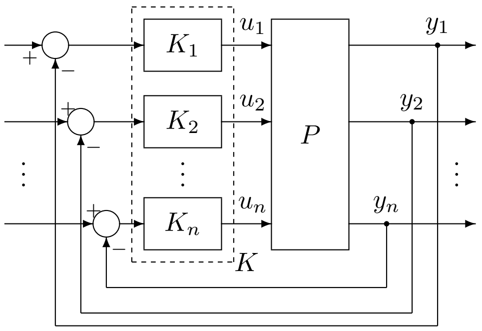

Fig. 1. Decentralized control system

tively) and the other matrices denote transfer function matrices throughout this paper.

2 Preliminaries

This paper considers the decentralized control of a linear time-invariant plant $P(s)$ with n control channels given by

$$\dot{x} = Ax + \sum_{i=1}^{n} B_i u_i, \tag{1}$$

$$y_i = C_i x + \sum_{j=1}^{n} D_{ij} u_j, \quad (i = 1, \ldots, n) \tag{2}$$

where x, u_i and y_i are the state, the i-th local inputs and the i-th local outputs of the plant, respectively. In this paper, packed matrix forms are used to represent realizations of systems, and the above realizations are represented by

$$P(s) = \left[\begin{array}{c|ccc} A & B_1 & \cdots & B_n \\ \hline C_1 & & & \\ \vdots & & D_{ij} & \\ C_n & & & \end{array}\right]. \tag{3}$$

The decentralized control problem is to find n local controllers

$$K_i(s) = \left[\begin{array}{c|c} A_{ki} & B_{ki} \\ \hline C_{ki} & D_{ki} \end{array}\right], \qquad (i = 1, \ldots, n) \tag{4}$$

to stabilize the given plant.

If a decentralized controller $K(s)$, defined as

$$K = \mathrm{diag}\{K_1, K_2, \ldots, K_n\}, \tag{5}$$

stabilizes the given plant $P(s)$, then, obviously $P(s)$ and $K(s)$ have the doubly coprime factorization [18]. Furthermore, according to the structure of the decentralized stabilizing controllers, some of the doubly coprime factorizations can be given as

$$P = NM^{-1} = \tilde{M}^{-1}\tilde{N}, \tag{6}$$

$$K = XY^{-1} = \tilde{Y}^{-1}\tilde{X}, \tag{7}$$

$$\begin{bmatrix} \tilde{Y} & -\tilde{X} \\ \tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & X \\ -N & Y \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix} \tag{8}$$

where

$$X = \mathrm{diag}\ \{X_1, X_2, \ldots, X_n\}, \tag{9}$$

$$Y = \mathrm{diag}\ \{Y_1, Y_2, \ldots, Y_n\}, \tag{10}$$

$$\tilde{X} = \mathrm{diag}\ \{\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n\}, \tag{11}$$

$$\tilde{Y} = \mathrm{diag}\ \{\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_n\}. \tag{12}$$

As the coprime factors of $K(s)$ have the diagonal structures (9)-(12), the next lemma holds.

Lemma 1 *[4] The coprime factors in (8) also satisfy*

$$\begin{bmatrix} \tilde{Y} & -\tilde{X} \\ \tilde{N}_d & \tilde{M}_d \end{bmatrix} \begin{bmatrix} M_d & X \\ -N_d & Y \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}, \tag{13}$$

where N, M, $\tilde{N}$ and $\tilde{M}$ are partitioned according to the sizes of inputs and outputs, and

$$N_d = \mathrm{diag}\ \{N_{11}, N_{22}, \ldots, N_{nn}\}, \tag{14}$$

$$M_d = \mathrm{diag}\ \{M_{11}, M_{22}, \ldots, M_{nn}\}, \tag{15}$$

$$\tilde{N}_d = \mathrm{diag}\ \{\tilde{N}_{11}, \tilde{N}_{22}, \ldots, \tilde{N}_{nn}\}, \tag{16}$$

$$\tilde{M}_d = \mathrm{diag}\ \{\tilde{M}_{11}, \tilde{M}_{22}, \ldots, \tilde{M}_{nn}\}. \tag{17}$$

Date and Chow (1994) call this doubly coprime factorization as "d-coprime factorization." With this doubly coprime factorization, they give a parametrization of decentralized stabilizing controllers, and show the connection to the other attempts on the parametrization [10,9,13].

To show the importance of N_d, M_d, $\tilde{N}_d$, $\tilde{M}_d$, another parametrization of decentralized controllers is reviewed here.

Lemma 2 *[15] All the decentralized controllers which stabilize the given plant $P(s)$ are parametrized as*

$$(\tilde{T}\tilde{Y} - \tilde{Q}\tilde{N}_d)^{-1}(\tilde{T}\tilde{X} + \tilde{Q}\tilde{M}_d), \tag{18}$$

where

$$\left(\begin{array}{l} \det(\tilde{T}\tilde{Y} - \tilde{Q}\tilde{N}_d) \not\equiv 0, \\ \tilde{T} + \tilde{Q}R \in \mathcal{U}, \quad \tilde{T}, \tilde{Q} \in \mathcal{D} \end{array} \right), \tag{19}$$

and

$$R = \tilde{N}M_d - \tilde{M}N_d. \tag{20}$$

($\mathcal{U}$ denotes the set of unimodular matrices and $\mathcal{D}$ denotes the set of stable block diagonal transfer function matrices.)

Definition 3 *For given $P(s)$ and $K(s)$, define the auxiliary diagonal system $P_d(s)$ as*

$$P_d = N_d M_d^{-1} \quad (= \tilde{M}_d^{-1}\tilde{N}_d), \tag{21}$$

where N_d, M_d, $\tilde{N}_d$, $\tilde{M}_d$ are defined in (14)-(17).

The auxiliary system $P_d(s)$ plays an important role in the parametrization of decentralized stabilizing controllers given by Lemma 2. If the parameter $\tilde{T}$ is fixed as $\tilde{T} = I$, then (18) becomes a parametrization of decentralized controllers which stabilize the auxiliary system $P_d(s)$. And the constraint (19) becomes $I + \tilde{Q}R \in \mathcal{U}$, which is the condition for the simultaneous stabilization (Vidyasagar, 1985). Thus the constraint (19) ensures that decentralized controllers stabilize both the actual system $P(s)$ and the auxiliary system $P_d(s)$ simultaneously.

It is easy to see that $\|\tilde{Q}\|_\infty < (\|R\|_\infty)^{-1}$ is a sufficient condition for $I+\tilde{Q}R$ to be an unimodular matrix, where $\|\cdot\|_\infty$ denote the H_∞-norm of $(\cdot)$. Thus, under the above condition on $\tilde{Q}$, it would be able to tune $\tilde{Q}$ similarly to the convex optimization methods for centralized control systems. Note that this procedure designs controllers for the auxiliary system $P_d(s)$. The auxiliary system $P_d(s)$ would be important for decentralized controller design.

Date and Chow (1994) shows that $P_d(s)$ is uniquely determined by given $P(s)$ and $K(s)$. But what is the system $P_d(s)$? Does $P_d(s)$ have a real meaning? The characteristics of the auxiliary system $P_d(s)$ has not been clarified yet. The reasons are

- The auxiliary system $P_d(s)$ is defined not only by the given $P(s)$, but also an initially given decentralized controller $K(s)$.
- The definition of $P_d(s)$ is complicated, extracting the diagonal elements from the coprime factors and reconstructing the system as a fraction of the elements.

The purpose of this paper is to reveal the property of the auxiliary system $P_d(s)$.

3 Main Results

Definition 4 *For given $P(s)$ and $K(s)$, define the auxiliary systems $H_i(s)$ as transfer functions from u_i to y_i,*

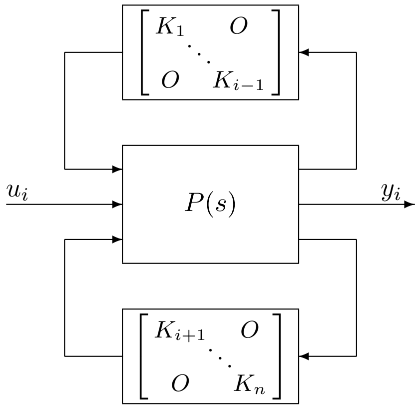

Fig. 2. Block diagram of $H_i(s)$

where all the local loops are closed by $K(s)$ except the i-th loop. The block diagram of $H_i(s)$ is shown in Fig. 2. With the systems $H_i(s)$ $(i = 1, \ldots, n)$, let us define the system $H(s)$ as

$$H = \text{diag}\,\{H_1, H_2, \ldots, H_n\}. \tag{22}$$

In most designs of decentralized control systems, especially in design procedures called "independent designs," controllers are designed for "non-interactive models" of plants, which consists of decoupled subsystems. For these design procedures, it is very important to evaluate the effect from the other loops to ensure the stability and/or the robust stability of closed-loop systems. Thus, the many classical and modern design procedures [14,1,8,17] use $H_i(s)$ to analyze the (robust) stability of closed-loop systems. Furthermore, these design procedures also use the (robust) stability conditions on the difference between $H_i(s)$ and $P_{ii}(s)$ as design specifications for each local loop. With the additional design specifications, designed controllers (robustly) stabilize the given plant. The auxiliary system $H(s)$ is important for designs of decentralized controllers.

Theorem 5 *For a given plant $P(s)$ and a decentralized stabilizing controller $K(s)$, let us define the auxiliary systems $P_d(s)$ and $H(s)$ as (21) and (22) respectively. Then,*

$$P_d(s) = H(s). \tag{23}$$

PROOF. See Appendix.

Now, let us discuss about designs of decentralized controllers based on the parametrization, and the meaning of Theorem 5. Date and Chow (1993) have suggested the direction of design of decentralized controllers based on the parametrization shown in Lemma 2. The procedures are basically designing local controllers for each subsystems of P_d, and are placed in the "independent design.” Unlike the ordinary (one-shot) “independent design” procedures, the design procedures based on the parametrization would be iterative design procedures, i.e., updating decentralized controllers iteratively. There have been some researches on the iterative independent design procedures (Date and Chow, 1994; Miyamoto and Vinnicombe,1997; Sebe,1998).

On the other hand, Theorem 5 shows that the P_d is the final step of a sequential design, i.e., the design procedures using P_d are deeply concerned with the 'sequential design.' According to these facts, Theorem 1 might be the key result to connect the independent and the sequential design procedures, even though these two design procedures are much different from each other. Consequently, based on Theorem 5, an iterative independent design procedure is proposed as a unification of these two design procedure [16]. This unified approach includes both the independent and the sequential design procedures as special cases of it. Furthermore, a modification of the iterative independent design procedure is also considered to take into account the H_∞ performance specifications [16].

The other contribution of Theorem 5 is the reduction of computational complexity and the improvement of accuracy on computation of P_d. As shown in the proof of Theorem 5 and the numerical example, the calculation of P_d by coprime factorization requires the reductions of unobservable modes. From the viewpoint of numerical computations, such reductions are not preferable. This numerical problem can be avoided by Theorem 5.

4 Numerical Example

In this section, a numerical example is given to verify the result. Let a given plant $P(s)$ and a decentralized (static) stabilizing controller $K(s)$ be

$$P(s) = \begin{bmatrix} \frac{s-3}{(s-1)(s-2)} & \frac{-1}{(s-1)(s-2)} \\ \frac{2}{(s-1)(s-2)} & \frac{s}{(s-1)(s-2)} \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right],$$

$$K(s) = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}.$$

From Definition 4, $H_1(s)$ and $H_2(s)$ are

$$H_1(s) = \frac{s+1}{s^2+s+2}, \quad H_2(2) = \frac{s+1}{s^2-2s-1}. \tag{24}$$

Assume the d-coprime factors of $K(s)$ be

$$\tilde{X} = X = \mathrm{diag}\{1,4\}, \quad \tilde{Y} = Y = \mathrm{diag}\{1,1\}. \tag{25}$$

Then, the d-coprime factors of $P(s)$, which satisfy Bezout identity (8) are given by

$$\begin{bmatrix} M \\ \hline N \end{bmatrix} = \left[\begin{array}{cc} \frac{s^2+s+2}{s^2+2s+3} & \frac{1}{s^2+2s+3} \\ \frac{-8}{s^2+2s+3} & \frac{s^2-2s-1}{s^2+2s+3} \\ \hline \frac{s+1}{s^2+2s+3} & \frac{-1}{s^2+2s+3} \\ \frac{2}{s^2+2s+3} & \frac{s+1}{s^2+2s+3} \end{array}\right]. \tag{26}$$

Extracting the diagonal elements from the above coprime factors, the elements of the auxiliary diagonal system $P_d(s)$ are given by

$$N_{11}M_{11}^{-1} = \frac{s+1}{s^2+s+2}, \tag{27}$$

$$N_{22}M_{22}^{-1} = \frac{s+1}{s^2-2s-1}. \tag{28}$$

The systems (27) and (28) are identical to $H_1(s)$ and $H_2(s)$ in (24) respectively. Please note that the stable unobservable modes $-1 \pm \sqrt{2}i$, which correspond to the poles of the closed-loop system, are reduced in (27) and (28). Theorem 5 enables to avoid the cancellations in the calculations of $H_i(s)$.

5 Conclusion

An explicit characterization of auxiliary diagonal systems which appears in the decentralized coprime factors is given in this paper. This characterization will provide additional insights into the parametrization of decentralized controllers.

The result also brings advantages in computational aspects, such as the reduction of computational complexity and the improvement of accuracy.

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Appendix

Lemma 6 *Assume state space representations of the given systems are*

$$K(s) = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right], \quad P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]. \tag{29}$$

And let a realization of the left coprime factors of $K(s)$ $(= \tilde{Y}^{-1}\tilde{X})$ be

$$[\tilde{Y} \quad \tilde{X}] = \left[\begin{array}{c|cc} A_k - F_kC_k & F_k & -(B_k - F_kD_k) \\ \hline -C_k & I & D_k \end{array}\right], \tag{30}$$

where $A_k - F_kC_k$ is a stable matrix. Then, a realization of the right coprime factors $\begin{bmatrix} M \\ N \end{bmatrix}$ of the givem plant $P(s)$ which satisfy Bezout identity

$$\tilde{Y}M + \tilde{X}N = I, \tag{31}$$

are given by

$$\left[\begin{array}{cc|c} A - BE^{-1}D_kC & -BE^{-1}C_k & BE^{-1} \\ B_k\tilde{E}^{-1}C & A_k - B_kDE^{-1}C_k & -F_k + B_kDE^{-1} \\ \hline -E^{-1}D_kC & -E^{-1}C_k & E^{-1} \\ \tilde{E}^{-1}C & -DE^{-1}C_k & DE^{-1} \end{array}\right], \tag{32}$$

where

$$E = I + D_kD, \quad \tilde{E} = I + DD_k \tag{33}$$

Note that this lemma is an extension of the result in Nett *et al.* (1984).

Proof of Theorem 5. Date and Chow (1994) have already mentioned that the d-coprime factors (8)-(17) are unique except the multiplication of block-diagonal unimodular matrices. As any block-diagonal unimodular matrices do not alter the auxiliary system $P_d(s)$, $P_d(s)$ is uniquely determined by the given plant and its stabilizing decentralized controller. Thus, the proof of Theorem 5 is only to perform the calculation shown in Date and Chow (1994) by state space representations, and to show that the auxiliary system $P_d(s)$ is identical to $H(s)$. For the sake of simplicity, we will develop the results for a 2-channel system. Results for the n-channel systems can be derive analogously, and hence, will be omitted here.

Let us assume

$$K(s) = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right] = \left[\begin{array}{cc|cc} A_{k1} & O & B_{k1} & O \\ O & A_{k2} & O & B_{k2} \\ \hline C_{k1} & O & D_{k1} & O \\ O & C_{k2} & O & D_{k2} \end{array}\right], \tag{34}$$

$$P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]. \quad (35)$$

Let us also assume

$$F_k = \mathrm{diag}\{F_{k1}, F_{k2}\}, \quad (36)$$

where $A_{ki} - F_{ki}C_{ki}$ are stable matrices.

From Lemma 6, the right coprime factors of the given $P(s)$, which satisfy (8), can be given by (32). Extracting the $(1,1)$ blocks from M and N, the state space representation of M_{11} and N_{11} is

$$\begin{bmatrix} M_{11} \\ N_{11} \end{bmatrix} = \left[\begin{array}{cc|c} A - BE^{-1}D_kC & -BE^{-1}C_k & BE^{-1}\begin{bmatrix} I \\ O \end{bmatrix} \\ B_k\tilde{E}^{-1}C & A_k - B_kDE^{-1}C_k & -\begin{bmatrix} F_{k1} \\ O \end{bmatrix} + B_kDE^{-1}\begin{bmatrix} I \\ O \end{bmatrix} \\ \hline -[I \quad O]E^{-1}D_kC & -[I \quad O]E^{-1}C_k & [I \quad O]E^{-1}\begin{bmatrix} I \\ O \end{bmatrix} \\ [I \quad O]\tilde{E}^{-1}C & -[I \quad O]DE^{-1}C_k & [I \quad O]DE^{-1}\begin{bmatrix} I \\ O \end{bmatrix} \end{array}\right].$$

Then, the state space representation of $N_{11}M_{11}^{-1}$ is given by

$$N_{11}M_{11}^{-1} = \left[\begin{array}{ccc|c} A - B_2E_{22}^{-1}D_{k2}C_2 & O & -B_2E_{22}^{-1}C_{k2} & B\begin{bmatrix} I \\ -E_{22}^{-1}D_{k2}D_{21} \end{bmatrix} \\ * & A_{k1} - F_{k1}C_{k1} & * & * \\ B_{k2}\tilde{E}_{22}^{-1}C_2 & O & A_{k2} - B_{k2}D_{22}E_{22}^{-1}C_{k2} & B_{k2}\tilde{E}_{22}^{-1}D_{21} \\ \hline C_1 - D_{12}E_{22}^{-1}D_{k2}C_2 & O & -D_{12}E_{22}^{-1}C_{k2} & D_{11} - D_{12}E_{22}^{-1}D_{k2}D_{21} \end{array}\right] \quad (37)$$

where

$$E_{22} = I + D_{k2}D_{22}, \quad \tilde{E}_{22} = I + D_{22}D_{k2}. \quad (38)$$

Neglecting the unobservable modes associated with the eigenvalues of $A_{k1} - F_{k1}C_{k1}$, (37) can be reduced to

$$N_{11}M_{11}^{-1} = \left[\begin{array}{cc|c} A - B_2E_{22}^{-1}D_{k2}C_2 & -B_2E_{22}^{-1}C_{k2} & B_1 - B_2E_{22}^{-1}D_{k2}D_{21} \\ B_{k2}\tilde{E}_{22}^{-1}C_2 & A_{k2} - B_{k2}D_{22}E_{22}^{-1}C_{k2} & B_{k2}\tilde{E}_{22}^{-1}D_{21} \\ \hline C_1 - D_{12}E_{22}^{-1}D_{k2}C_2 & -D_{12}E_{22}^{-1}C_{k2} & D_{11} - D_{12}E_{22}^{-1}D_{k2}D_{21} \end{array}\right]. \quad (39)$$

It is easy to verify that the realization (39) is identical to that of $H_1(s)$. Similarly, $N_{22}M_{22}^{-1} = H_2(s)$ holds. ■

It should be noted that the above calculation requires not only the reduction of the unobservable modes which correspond to the eigenvalues of $A_{kj} - F_{kj}C_{kj}$ $(j \neq i)$, but also the stable pole-zero cancellations which correspond to the poles of the closed-loop system. From the viewpoint of numerical computations, such a reduction is not preferable. This numerical problem can be avoided by Theorem 5.

Unification of Independent and Sequential Procedures for Decentralized Controller Design

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Abstract

This paper is concerned with decentralized control problems. There are two typical procedures to design decentralized controllers, 'independent' and 'sequential' design procedures. As the concepts and the techniques in these two approaches are too different from each other, there have been no attempts to unify these approaches. This paper proposes an iterative independent design procedure for decentralized control systems, which is a unified approach to these conventional approaches.

Key words: Decentralized control, Multivariable control systems, Coprime factorization, Parametrization, H_∞ control, Interactions, Structured singular value.

1 Introduction

Decentralized control systems are the systems with the constraints on the controller structure. This paper focuses on the decentralized control systems with block diagonal controllers. To design decentralized controllers, there are two typical procedures called 'independent' and 'sequential' design procedures. Although there are some attempts to design decentralized controller base on matrix inequality approaches (Zhai et al., 2001; D'Andrea and Dullerud, 2003; Ebihara et al., 2004), these two design procedures are still very useful for theoretical analysis and practical design for decentralized control systems. In this paper, a unified approach of these two design procedures is proposed.

The independent design procedure constructs local controllers for corresponding local subsystems or diagonal approximation of the given plant. Each local controller is designed independently to the other local controllers (Rosenbrock, 1969; Araki and Nwokah, 1975; Grosdidier and Morari, 1986; Skogestad and Morari, 1989). Generally, the procedure only uses local loop information, thus designed controllers might be simple and easy to tune. Furthermore, the complexity of controllers is proportional to that of local subsystems. On the other hand, it is not easy to attain the best performance.

In the sequential design procedure, local loops are closed step by step (Davison and Gesing, 1979; Bernstein, 1987; Chiu and Arkun, 1992). Each local controller is designed with the information of previously designed local controllers. Thus, the sequential design would attain better performance than independent design does. The drawback is complexities of local controllers. The later the local controller is designed, the more complex it becomes.

As the concepts and techniques in these two approaches are too different from each other, there have been no attempts to unify the approaches. In this paper, an iterative independent design procedure for decentralized control systems is proposed. The proposed procedure has parameters to balance the difficulty and significance of local loops. With particular selections of these parameters, both the independent and sequential design procedures can be implemented. Furthermore, a modification to H_∞ controller design is also discussed in this paper.

This paper is organized as follows. In Section 2, a parametrization of plants and controllers are stated. In Section 3, the stability condition with balancing weights is derived. Section 4 contains the proposed design procedures with balancing weights. Design examples are given in Section 5.

Notations. M_d denotes the block diagonal part of a matrix M, where the block diagonal structure is compatible with a decentralized controller. $\bar{\sigma}(M)$ denotes the maximum singular value of a matrix M. $\mu_\Delta(M)$ denotes the structured singular value of a matrix M with respect to the block diagonal structure Δ (Doyle, 1982).

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Throughout this paper, the diagonal structure Δ is same as that of a decentralized controller, and Δ is dropped in many cases for the sake of simplicity. For matrices M and N partitioned as M_{ij}, N_{ij}, $(i, j = 1, 2)$, the upper linear fractional transformation (LFT), the lower LFT, and the star product of M and N are defined as

$$\mathcal{F}_u(N, M_{22}) = N_{22} + N_{21}M_{22}(I - N_{11}M_{22})^{-1}N_{12},$$
$$\mathcal{F}_l(M, N_{11}) = M_{11} + M_{12}N_{11}(I - M_{22}N_{11})^{-1}M_{21},$$
$$M \star N = \begin{bmatrix} \mathcal{F}_l(M, N_{11}) & M_{12}(I - N_{11}M_{22})^{-1}N_{12} \\ N_{21}(I - M_{22}N_{11})^{-1}M_{21} & \mathcal{F}_u(N, M_{22}) \end{bmatrix},$$

respectively. The inverse LFT of M is also defined as

$$M^{-} = \begin{bmatrix} O & I \\ I & O \end{bmatrix} M^{-1} \begin{bmatrix} O & I \\ I & O \end{bmatrix}.$$

Note that $M \star M^{-} = \begin{bmatrix} O & I \\ I & O \end{bmatrix}$, $M^{-} \star M = \begin{bmatrix} O & I \\ I & O \end{bmatrix}$. $\mathcal{RH}_\infty$ denotes the set of stable transfer function matrices, and $\|G(s)\|_\infty$ denotes the H_∞ norm of $G(s)$.

2 Preliminaries

In this section, some basic results concerned with the parametrization of stabilizing controller are given.

Lemma 1 *(Youla et al., 1976) For a given plant P, there exists K_g such that all the controllers which stabilize P are parametrized as $\{K \mid K = \mathcal{F}_l(K_g, Q), Q \in \mathcal{RH}_\infty\}$.*

Corollary 2 *If a controller K is given, then there exists P_g such that all the plants which are stabilized by K are parametrized as $\{P \mid P = \mathcal{F}_u(P_g, R), R \in \mathcal{RH}_\infty\}$.*

Then, we have the following theorem.

Theorem 3 *Let P and K be a given plant and its stabilizing controller, respectively. Then there exist P_g and K_g such that*

(i) The (1,1) element of K_g is K and K_g gives the parametrization of all the controllers in Lemma 1.
(ii) The (2,2) element of P_g is P and P_g gives the parametrization of all the plants in Corollary 2.
(iii)
$$P_g \star K_g = \begin{bmatrix} O & I \\ I & O \end{bmatrix}. \tag{1}$$

PROOF. Suppose a doubly coprime factorization (Vidyasagar, 1985) of P and K over $\mathcal{RH}_\infty$ is given by

$$\begin{bmatrix} \tilde{Y} & -\tilde{X} \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & X \\ N & Y \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}, \tag{2}$$

$$P = \tilde{D}^{-1}\tilde{N} = ND^{-1}, \quad K = \tilde{Y}^{-1}\tilde{X} = XY^{-1}. \tag{3}$$

Then, one of the pairs (P_g, K_g) which satisfy the three conditions is given by

$$P_g = \begin{bmatrix} -\tilde{X}\tilde{D}^{-1} & D^{-1} \\ \tilde{D}^{-1} & P \end{bmatrix}, \; K_g = \begin{bmatrix} K & \tilde{Y}^{-1} \\ Y^{-1} & -\tilde{N}\tilde{Y}^{-1} \end{bmatrix}. \tag{4}$$

3 Main results

In this section, the parametrization of all decentralized stabilizing controllers is reviewed, and the stability condition for decentralized control systems is derived.

Let us consider a block diagonal decentralized controller,

$$K = \mathrm{diag}\{K_1, K_2, \ldots, K_n\}. \tag{5}$$

If a decentralized controller K stabilizes a given plant P, then a doubly coprime factorization (2) exists. According to the structure (5), we can choose the factors of K in (2) to be block diagonal. Then, the next lemma holds.

Lemma 4 *(Date and Chow, 1994) Let P and K be a given plant and its decentralized stabilizing controller, and their doubly coprime factorization be given by (2), (3), where the coprime factors X, Y, $\tilde{X}$, $\tilde{Y}$ are block diagonal. Then the coprime factors in (2) also satisfy*

$$\begin{bmatrix} \tilde{Y} & -\tilde{X} \\ -\tilde{N}_\mathrm{d} & \tilde{D}_\mathrm{d} \end{bmatrix} \begin{bmatrix} D_\mathrm{d} & X \\ N_\mathrm{d} & Y \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}. \tag{6}$$

Let us define the auxiliary plant H as

$$H = N_\mathrm{d}D_\mathrm{d}^{-1} = \tilde{D}_\mathrm{d}^{-1}\tilde{N}_\mathrm{d}. \tag{7}$$

This auxiliary plant H plays very important role in the parametrizations of decentralized stabilizing controllers (Manousiouthakis, 1989; Gündeş and Desoer, 1990; Özgüler, 1990; Sebe, 2004). With this auxiliary plant, we have the following theorem.

Theorem 5 *Let P and K be a given plant and its decentralized stabilizing controller. Assume H be defined by (7). Then there exist P_g and K_g which satisfy the conditions below.*

(i) *The set* $\{\hat{K} \mid \hat{K} = \mathcal{F}_l(K_g, Q), Q \in \mathcal{RH}_\infty\}$ *gives the parametrization of all the controllers which stabilize* H.

(ii) *The set* $\{\hat{P} \mid \hat{P} = \mathcal{F}_u(P_g, R), R \in \mathcal{RH}_\infty\}$ *gives the parametrization of all the plants which are stabilized by* K.

(iii) P_g *and* K_g *have the forms that*

$$P_g = \begin{bmatrix} * & * \\ * & H \end{bmatrix}, \qquad K_g = \begin{bmatrix} K & * \\ * & * \end{bmatrix} \tag{8}$$

where all the elements denoted by $*$ *are block diagonal. Furthermore,* P_g *and* K_g *satisfy (1).*

(iv) *There exists* $R \in \mathcal{RH}_\infty$ *such that*

$$P = \mathcal{F}_u(P_g, R), \quad R_{\mathrm{d}} = O. \tag{9}$$

PROOF. With the coprime factors defined in (6), P_g and K_g which satisfy the conditions (i), (ii) and (iii) can be given by

$$P_g = \begin{bmatrix} -\tilde{X}\tilde{D}_{\mathrm{d}}^{-1} & D_{\mathrm{d}}^{-1} \\ \tilde{D}_{\mathrm{d}}^{-1} & H \end{bmatrix}, \; K_g = \begin{bmatrix} K & \tilde{Y}^{-1} \\ Y^{-1} & -\tilde{N}_{\mathrm{d}}\tilde{Y}^{-1} \end{bmatrix}. \tag{10}$$

From (1), (2) and (6),

$$\begin{aligned} R &= \mathcal{F}_u(P_g^-, P) = \mathcal{F}_u(K_g, P) \\ &= -\tilde{N}_{\mathrm{d}}\tilde{Y}^{-1} + Y^{-1}ND^{-1}(I - \tilde{Y}^{-1}\tilde{X}ND^{-1})^{-1}\tilde{Y}^{-1} \\ &= -\tilde{N}_{\mathrm{d}}\tilde{Y}^{-1} + Y^{-1}N \\ &= -\tilde{N}_{\mathrm{d}}\tilde{Y}^{-1}(\tilde{Y}D - \tilde{X}N) + (\tilde{D}_{\mathrm{d}}Y - \tilde{N}_{\mathrm{d}}X)Y^{-1}N \\ &= -\tilde{N}_{\mathrm{d}}D + \tilde{D}_{\mathrm{d}}N \qquad (\in \mathcal{RH}_\infty). \end{aligned} \tag{11}$$

From (6) and the diagonal structure of $\tilde{N}_{\mathrm{d}}$ and $\tilde{D}_{\mathrm{d}}$,

$$R_{\mathrm{d}} = -\tilde{N}_{\mathrm{d}}D_{\mathrm{d}} + \tilde{D}_{\mathrm{d}}N_{\mathrm{d}} = O. \tag{12}$$

This completes the proof. □

The matrix K_g gives the parametrization for the auxiliary plant H, not for the real P. Accordingly, unlike the centralized case in Boyd and Barratt (1991), a candidate controller $\mathcal{F}_l(K_g, Q)$ does not always stabilize P. Thus, we have to clarify the stability conditions.

Theorem 6 *Suppose* P_g *and* K_g *are defined in Theorem 5 for a given* P *and* K. *Let* P_{gi} *consist of elements of* P_g *corresponding to the* i*-th local loop. Let unimodulars* U *and* V *be*

$$U = \mathrm{diag}\{U_1, U_2, \ldots, U_n\}, \tag{13}$$
$$V = \mathrm{diag}\{V_1, V_2, \ldots, V_n\}, \tag{14}$$

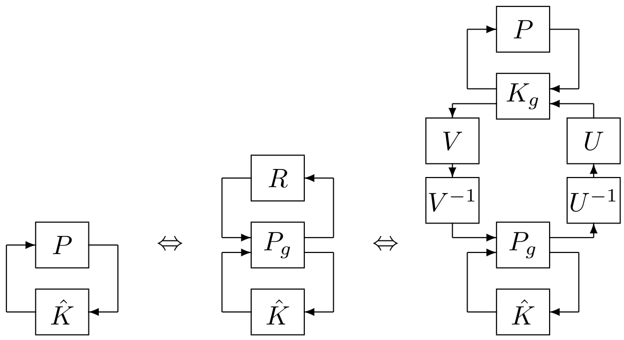

Fig. 1. Closed loop system with P and $\hat{K}$

which have diagonal structures compatible with K. *Then, a candidate controller* $\hat{K} = \mathrm{diag}\{\hat{K}_1, \hat{K}_2, \ldots, \hat{K}_n\}$ *stabilizes the given plant* P *if* $\mathcal{F}_l(P_{gi}, \hat{K}_i) \in \mathcal{RH}_\infty$ *and*

$$\bar{\sigma}\left(U_i^{-1}(j\omega)\mathcal{F}_l(P_{gi}(j\omega), \hat{K}_i(j\omega))V_i^{-1}(j\omega)\right) < 1/r(\omega) \qquad (\forall\omega, \forall i), \tag{15}$$

where

$$r(\omega) = \mu\left(V(j\omega)R(j\omega)U(j\omega)\right), \tag{16}$$
$$R = \mathcal{F}_u(K_g, P). \tag{17}$$

PROOF. By applying the small gain theorem to the system shown in Fig. 1 and taking account of the block diagonal structures of P_g, $\hat{K}$, U and V, a stability condition can be derived as

$$\bar{\sigma}\left(U^{-1}(j\omega)\mathcal{F}_l(P_g(j\omega), \hat{K}(j\omega))V^{-1}(j\omega)\right) < \mu^{-1}\left(V(j\omega)R(j\omega)U(j\omega)\right), \tag{18}$$

Furthermore, as from the diagonal structure,

$$\bar{\sigma}\left(U^{-1}\mathcal{F}_l(P_g, \hat{K})V^{-1}\right) = \max_i \bar{\sigma}\left(U_i^{-1}\mathcal{F}_l(P_{gi}, \hat{K}_i)V_i^{-1}\right). \tag{19}$$

□

The stability conditions (15) can be regarded as robust stability conditions for the perturbed local systems defined as

$$\mathcal{F}_u\left(P_{gi},\, V_i^{-1}\Delta_i U_i^{-1}\right), \tag{20}$$

where Δ_i are virtual uncertainties and $\bar{\sigma}(\Delta_i(j\omega)) < r(\omega)$ for all ω. Considering the virtual diagonal uncertainty $\{\Delta_1, \ldots, \Delta_n\}$ instead of the actual uncertainty R, Theorem 6 divides the overall stability condition into n

individual local robust stability conditions. This enables us to design local controllers independently.

In Theorem 6, the U and V act as weights to balance the difficulty and significance of local loops, as in Sebe and Kitamori (1995). Here, brief explanation about the 'balancing weights' U and V is given. With $V = I$ and $U = I$, the stability conditions are

$$\bar{\sigma}(\mathcal{F}_l(P_{gi}, \hat{K}_i)) < 1/\mu(R)$$

for all local loops. Let us modify U to

$$U_i = bI, \qquad U_j = I \quad (j \neq i),$$

where $b > 1$. Please note that

$$\mu(R) \leq \mu(R_b) \leq \mu(b \cdot R) = b \cdot \mu(R),$$

where $R_b = VRU$. Then the stability conditions become

$$\bar{\sigma}(\mathcal{F}_l(P_{gj}, \hat{K}_j)) < 1/\mu(R_b) \quad (\leq 1/\mu(R)) \qquad (j \neq i),$$
$$\bar{\sigma}(\mathcal{F}_l(P_{gi}, \hat{K}_i)) < b/\mu(R_b) \quad (\geq 1/\mu(R)).$$

The above inequalities imply that the virtual uncertainty of i-th local loop becomes smaller while those of the other local loops become larger. In other words, we can design $\hat{K}_i$ with less consideration of robustness against the virtual uncertainty. In Section 5, the numerical example demonstrates the advantage of the balancing weights.

However, the stability condition (15) depends on P_g, i.e., on H. As mentioned before, there exist decentralized controllers which stabilize P but do not stabilize H. This fact indicates that we have to find a preferable P_g for designing controllers. Accordingly, in the next section, iterative design procedures which systematically update P_g are proposed.

4 Design procedure

In this section, an iterative design procedure base on Theorem 6, is stated below. Here, P_g, U and V are not assigned explicitly. In subsection 4.1, the particular selections of $U^{(l)}$ and $V^{(l)}$ to implement the conventional independent and sequential design procedures are given. In subsection 4.2, with the modification to P_g, a design procedure for standard H_∞ control problem is given.

Design procedure.
Let $^{(l)}$ denotes the variables used at the l-th iteration.

(i) Assume that a decentralized stabilizing controller $K^{(0)}$ exists and is given.

(ii) With $K^{(l)}$, choose $P_g^{(l)}$, $K_g^{(l)}$ which have the diagonal structure defined by (8) and satisfy

$$P_g^{(l)} \star K_g^{(l)} = \begin{bmatrix} O & I \\ I & O \end{bmatrix}, \quad K_g^{(l)} = \begin{bmatrix} K^{(l)} & * \\ * & * \end{bmatrix}. \quad (21)$$

Also choose diagonal unimodulars $U^{(l)}$ and $V^{(l)}$.

(iii) Design local controller $K_i^{(l+1)}$ for each perturbed local system given by (20).

(iv) If $K^{(l+1)}$ attains sufficient performance, stop the procedure. Otherwise go to (ii).

4.1 *Unification of independent and sequential designs*

Let us discuss about the conventional independent design. The stability conditions in Grosdidier and Morari (1986) and Skogestad and Morari (1989) can be derived by choosing the pairs $(P_g^{(0)}, K_g^{(0)})$ and $(U^{(0)}, V^{(0)})$ as

$$P_g^{(0)} = \begin{bmatrix} O & P_\mathrm{d} \\ I & P_\mathrm{d} \end{bmatrix}, \quad K_g^{(0)} = \begin{bmatrix} O & P_\mathrm{d}^{-1} \\ I & -I \end{bmatrix}, \quad (22)$$

$$U^{(0)} = I, \qquad V^{(0)} = I. \quad (23)$$

(Strictly, P_d must be a unimodular.) Note that the above choice of $(P_g^{(0)}, K_g^{(0)})$ means that $K^{(0)} = O$ (zero matrix), and the plant P is modeled as the nominal model P_d with the multiplicative uncertainty $R^{(0)}$. The independent design procedures proposed in those papers design decentralized controllers for the above perturbation model. Conversely, with (22) and (23), the first iteration of the above design procedure is exactly same as the conventional independent design procedures.

Now, let us consider the other particular selection below.

$$P_g^{(l)} = \begin{bmatrix} * & * \\ * & H^{(l)} \end{bmatrix}, \quad (24)$$

$$U_i^{(l)} = I, \quad U_j^{(l)} = \varepsilon I \quad (j \neq i), \quad (25)$$

$$V_i^{(l)} = I, \quad V_j^{(l)} = \varepsilon I \quad (j \neq i), \quad (26)$$

where $H^{(l)}$ is defined by (7) with P and $K^{(l)}$, and $\varepsilon(> 0)$ is sufficiently small. Let $\hat{Q}_i$ be the parameter which represents the candidate of i-th local controller $\hat{K}_i$, i.e.

$$\hat{K}_i = \mathcal{F}_l(K_{gi}^{(l)}, \hat{Q}_i) \;\text{ or }\; \hat{Q}_i = \mathcal{F}_l((K_{gi}^{(l)})^-, \hat{K}_i). \quad (27)$$

From Theorem 6, the stability conditions are

$$\bar{\sigma}(\hat{Q}_i) < 1/\mu\left(V^{(l)} R^{(l)} U^{(l)}\right) \quad (28)$$

$$\bar{\sigma}(\hat{Q}_j) < \varepsilon^2/\mu\left(V^{(l)} R^{(l)} U^{(l)}\right) \qquad (j \neq i) \quad (29)$$

for all ω. As $R_d^{(l)} = O$ and $\varepsilon \ll 1$,

$$\mu\Big(V^{(l)}R^{(l)}U^{(l)}\Big) \approx \varepsilon\,\mu\left(\begin{bmatrix} O & R_{1i}^{(l)} & O \\ & \vdots & \\ R_{i1}^{(l)}\cdots & O & \cdots R_{in}^{(l)} \\ & \vdots & \\ O & R_{ni}^{(l)} & O \end{bmatrix}\right). \quad (30)$$

From (30), the right hand side of (28) grows at $\mathcal{O}(1/\varepsilon)$, while that of (29) converges at $\mathcal{O}(\varepsilon)$. As $\varepsilon \to 0$, $\hat{Q}_i$ can be freely chosen while $\hat{Q}_j$ are fixed to O. In other words, as $\varepsilon \to 0$, the selection (24), (25), (26) makes the design procedure as a conventional sequential design procedure. Note that fixing the controllers $K_j^{(l)}$ permits $\varepsilon = 0$ and a complete sequential design can be implemented.

We give two particular selections of $P_g^{(l)}$, $U^{(l)}$ and $V^{(l)}$, which implement the conventional independent and sequential design procedures.

4.2 H_∞ controller design

In this subsection, a suitable selection of P_g is proposed for H_∞ control problem, as in Sebe (1998).

Design procedure for H_∞ control problem.

(i) Suppose $K^{(0)}$ is a decentralized stabilizing controller for P, and attains the performance index γ. Select a guaranteed upper bound $\gamma^{(0)}$ ($\geq \gamma$).

(ii) Assign $P_g^{(l)}$ to the parametrization matrix of all the plants which attain performance index $\gamma^{(l)}$ with $K^{(l)}$ (Doyle et al., 1989). Note that the $(1,1)$ element of $(P_g^{(l)})^-$ is $K^{(l)}$. Also select diagonal unimodulars $U^{(l)}$ and $V^{(l)}$, such that

$$\left\|(U_i^{(l)})^{-1}\right\|_\infty \leq 1, \quad \left\|(V_i^{(l)})^{-1}\right\|_\infty \leq 1, \quad (\forall i) \quad (31)$$

$$r^{(l)}(\omega) < \gamma^{(l)} \qquad (\forall\omega). \quad (32)$$

(iii) With a performance index $\gamma^{(l+1)}$ ($\leq \gamma^{(l)}$), design $K_i^{(l+1)}$ for the perturbed plant model given by (20). Note that this design step is solving the robust performance problem and it is hard to get exact solutions. However, $K^{(l)}$ attains performance index less than (or equal to) $\gamma^{(l)}$. And from (31) and (32),

$$\left\|(V_i^{(l)})^{-1}\Delta_i(U_i^{(l)})^{-1}\right\|_\infty < \gamma^{(l)}. \quad (33)$$

From this inequality and the fact that $P_g^{(l)}$ is the parametrization matrix for $K^{(l)}$, the controller $K^{(l)}$ is a tractable solution to the robust performance problem. Thus, this design step always improves the upper bound of the performance index $\gamma^{(l+1)}$, i.e. $\gamma^{(l+1)} \leq \gamma^{(l)}$. The basic concept of this performance improvement is proposed in Sebe (1999).

(iv) If $K^{(l+1)}$ attains sufficient performance, stop the procedure. Otherwise go to (ii).

Remark 7 *The procedure can be applied to problems where the performance measures have also the diagonal structure compatible with K (Sebe, 1998).*

Remark 8 *As P_g gives the parametrization of all plants which attain $\gamma^{(l)}$ with $K^{(l)}$, $\|R^{(l)}\| < \gamma^{(l)}$. This implies that $r^{(l)}(\omega) = \mu(R^{(l)}(j\omega)) < \gamma^{(l)}$, where $U_i^{(l)}$ and $V_i^{(l)}$ are selected as identity matrices. This ensures the existence of $U_i^{(l)}$ and $V_i^{(l)}$ which satisfy (31) and (32).*

Remark 9 *Although $R_{\rm d}^{(l)} \neq O$ in the above procedure, appropriate selections of U, V implement the sequential H_∞ controller design. Suppose $U_j = I$, $V_j = I$ ($j \neq i$), and select U_i and V_i such that $\bar{\sigma}(U_i^{-1}(j\omega))$ and $\bar{\sigma}(V_i^{-1}(j\omega))$ are as small as possible within (31) and (32). Then as $r(\omega) \to \gamma$, K_j ($j \neq i$) become the only admissible solutions to the robust performance problems at step (iii), and K_j are fixed. On the other hand, the constraint (33) becomes loose, and K_i can be updated more freely. This implements the sequential design procedure.*

5 Numerical example

Let us consider the mixed sensitivity minimization problem in Chiu and Arkun (1992). The plant, the weights for multiplicative uncertainties and those for the sensitivity functions are given below, respectively.

$$P(s) = \begin{bmatrix} \frac{1.66}{1+39s} & \frac{-1.74(1-s)}{(1+4.4s)(1+s)} \\ \frac{0.34(2-s)}{(1+8.9s)(2+s)} & \frac{1.4(2-s)}{(1+3.8s)(2+s)} \end{bmatrix}, \quad (34)$$

$$l_i(s) = 0.07, \qquad W_{si}(s) = \frac{1+7s}{28s}. \quad (35)$$

Design 0 This design is an ordinary independent design where $R^{(0)}$ is modeled as an additve uncertainty, and is only for comparison. As the given plant is stable, let us choose $K^{(0)} = O$ and $\gamma^{(0)} = \infty$. Also choose an additive uncertainty model without balancing weights,

$$P_g^{(0)} = \begin{bmatrix} O & I \\ I & P_{\rm d} \end{bmatrix}, \quad K_g^{(0)} = \begin{bmatrix} O & I \\ I & -P_{\rm d} \end{bmatrix}, \quad (36)$$

$$U^{(0n)} = I, \qquad V^{(0n)} = I, \quad (37)$$

where ${}^{(0n)}$ denotes the first iteration without balancing weights. The magnitude of uncertainty $r^{(0n)}(\omega)$ defined by (16) is shown in Fig. 2. Then, local controllers are obtained by solving H_∞ optimization problems:

$$K_1^{(1n)} = \frac{687.5866(s+0.02564)(s+0.1597)}{s(s+80.89)}, \quad (38)$$

$$K_2^{(1n)} = \frac{140.8682(s+0.1597)(s+0.2632)(s+2)}{s(s+0.1116)(s+418.8)}. \quad (39)$$

As shown in Table 1, the achieved performance indexes are 1.22 and 0.3586. Although these indexes are achieved for the auxiliary plant P_d, the difference between these indexes suggests that the first local loop is much more difficult to control than the second local loop is. Note that the guaranteed performance with the decentralized controller $\mathrm{diag}\{K_1^{(1n)}, K_2^{(1n)}\}$ is $\gamma^{(0)} = \infty$. Thus, there is no inconsistency between the overall (real) index 0.7789 and the first local index 1.22.

Design 1 This design demonstrates the efficiency of the balancing weights. Instead of (37), let us choose

$$U^{(0)} = I, \quad V^{(0)} = \mathrm{diag}\{2.5, 1\}. \quad (40)$$

With these balancing weights, the magnitude of uncertainty $r^{(0)}$ is also shown in Fig. 2. Note that $0.4r^{(0)}(\omega) < r^{(0n)}(\omega) < r^{(0)}(\omega)$. These inequalities imply that the first local loop can be designed more freely than the second local loop. The designed local controllers are

$$K_1^{(1)} = \frac{286.2518(s+0.02564)(s+0.1597)}{s(s+19.49)}, \quad (41)$$

$$K_2^{(1)} = \frac{177.9925(s+0.1597)(s+0.2632)(s+2)}{s(s+0.0727)(s+382.5)}. \quad (42)$$

With the balancing weights (40), the difficulty in designing local controllers are balanced and the overall performance index is much improved from 0.7789 to 0.4568.

Design 2 This design demonstrates the efficiency of the iterative improvement and the implementation of the sequential design. Following Design 1, let us choose $\gamma^{(1)} = 0.5$ (> 0.4568). With this $\gamma^{(1)}$ and $K^{(1)} = \mathrm{diag}\{K_1^{(1)}, K_2^{(1)}\}$, assign $P_g^{(1)}$ to the parametrization matrix of plants which attains $\gamma^{(1)}$ with $K^{(1)}$, and define $K_g^{(1)} = (P_g^{(1)})^-$. The balancing weights are selected as

$$U^{(1)} = I, \quad V^{(1)} = \mathrm{diag}\{1.25, 1\}. \quad (43)$$

The magnitude of uncertainty $r^{(1)}(\omega)$ is shown in Fig. 3. Then, the designed local controllers are

$$K_1^{(2)} = \frac{286.5025(s+0.02888)(s+0.1623)}{s(s+19.37)}, \quad (44)$$

$$K_2^{(2)} = \frac{178.0887(s+0.1602)(s+0.2636)(s+2)}{s(s+0.0727)(s+382.1)}. \quad (45)$$

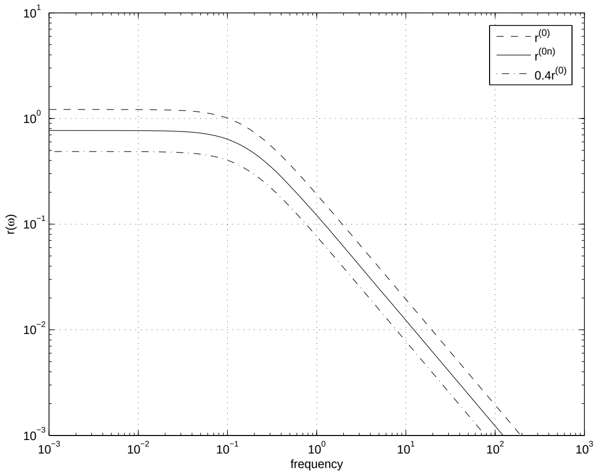

Fig. 2. Frequency responses of $r^{(0n)}$ and $r^{(0)}$.

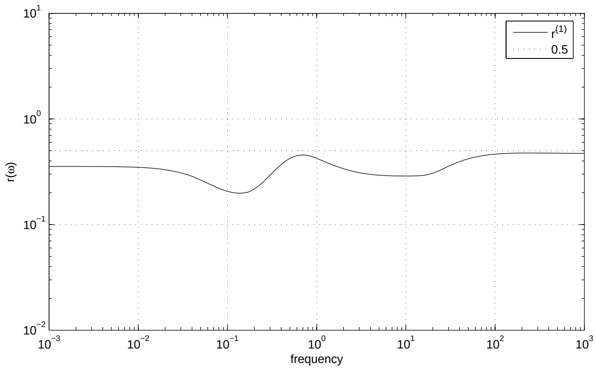

Fig. 3. Frequency responses of $r^{(1)}$.

Table 1
Achieved performances. (Performance indexes #1, #2, and overall index are the achieved indexes of the first and second auxiliary local loops and the overall system, respectively.)

Design	balancing weight U_2	performance index #1	performance index #2	overall index
0	{1, 1}	1.2200	0.3586	0.7789
1	{2.5, 1}	0.6007	0.6561	0.4568
2	{1.25, 1}	0.3833	0.4763	0.4318

The overall performance index is improved again. Notice that $K_2^{(1)}$ and $K_2^{(2)}$ are almost same. As $r^{(1)}(\omega) \approx \gamma^{(1)}$ and from Remark 9, this design step should update only the first local loop, and the sequential design procedure should be implemented.

6 Conclusions

This paper proposes an iterative independent design procedure for decentralized control systems, and introduces a new notion of balancing weights. The weights balance the difficulty and significance of local loops, and are very important and effective. Also, the balancing weights unify the independent and sequential design approaches. The design procedure which guarantees the improvement of H_∞ performance is also proposed.

Compared with approaches base on matrix inequalities, the proposed procedure has disadvantages in computation and attainable performance. Contrarily, as shown in the example, the procedure has advantages in identifying bottlenecks in control design, and making strategies for performance improvement. Note that the strategies are still important, as decentralized control problems are described by bilinear matrix inequalities.

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