Numerical Calculation of Critical Current Density and Third Harmonic Voltage in Superconducting Films

H. Wada [a], M. Migita [a], E.S. Otabe [a], M. Kiuchi [a],
T. Matsushita [a,1], Y. Mawatari [b], H. Yamasaki [b],
Y. Nakagawa [b]

[a] *Faculty of Computer Science and Systems Engineering, Kyushu Institute of Technology, 680–4 Kawazu, Iizuka 820–8502, Japan*

[b] *National Institute of Advanced Industrial Science and Technology, Tsukuba, Ibaraki 305–8586, Japan*

Abstract

When an AC magnetic field of a suitable magnitude is applied locally normal to a wide surface of a superconducting film, a third harmonic voltage is induced in a pick-up coil. This measurement is used for an evaluation of the critical current density of the film. Mawatari *et al.* analyzed the relationship between the third harmonic voltage amplitude (V_3) and the driving current amplitude for the field (I_0) by using the critical state model and a simple assumption that the magnetic field is locally parallel to a flat surface. In this study, this relationship is numerically analyzed by Finite Element Method (FEM) for a condition similar to the experiment by Mawatari *et al.* This result is compared with the above theoretical and experimental results. It is found that the scaling of V_3-I_0 curves holds and I_0 at which V_3 starts to appear is proportional to the critical current density, as prediction by Mawatari *et al.* The effect of imperfections in the film on the V_3-I_0 curve is also investigated.

Keywords: Critical current, Superconducting films, FEM
PACS: 74.25.Sv, 74.76.-w, 02.70.Dh

---

[1] Corresponding author.
Postal address: Department of Computer Science and Electronics, Kyushu Institute of Technology, 680–4, Kawazu, Iizuka 820–8502 Japan
Phone: +81–948–29–7661
Fax: +81–948–29–7661
E-mail address: matusita@cse.kyutech.ac.jp

Preprint submitted to FDP–48/ ISS2002 *November 11, 2002*

1 Introduction

A simple contactless measuring method of critical current density J_c in superconducting thin films or coated tapes is required. Claassen *et al.* [1] proposed an inductive method to measure a third harmonic voltage of amplitude V_3 generated by a current induced in a superconducting thin film due to an external AC magnetic field, which was generated by a sinusoidal driving current $I_0 \cos\omega t$ flowing in a small coil placed close to the surface of the film. It is empirically known that the local J_c is proportional to a threshold value of I_0 (I_c0) at which V_3 starts to appear.

The relationship between I_0 and V_3 was theoretically given by Mawatari *et al.* [2] by using the critical state model with an assumption that the magnetic field is parallel to the surface due to a shielding effect where magnetic flux penetrates the superconducting thin film. According to their theoretical analysis, V_3 is almost zero when I_0 is smaller than I_c0, which is proportional to $J_\mathrm{c}d$, with d denoting the film thickness. When $I_0 > I_\mathrm{c0}$, on the other hand, the third harmonic voltage is expressed as $V_3\exp(-\mathrm{i}\theta_3) = \omega I_\mathrm{c0} G(I_0/I_\mathrm{c0})$, where $G(x)$ is a scaling function determined by a configuration of the coil and θ_3 is a phase of the third harmonic voltage. However, the magnetic field is not exactly parallel to the surface as assumed in [2], especially around the central area of the coil where the magnetic field is strong. Hence, the validity of the above approximation should be clarified.

In this paper, the magnetic field distribution in a superconducting film was numerically calculated by Finite Element Method (FEM) to estimate the third harmonic voltage and the scaling of V_3-I_0 curves was examined. This result is compared with the above theoretical and experimental results by Mawatari *et al.* In addition, the effect of locally deteriorated region in a film on the signal was also investigated to examine the applicability of this measuring method for characterization of superconducting thin films.

2 Simulation

JMAG studio version 7 of Japan Research Institute was used for the calculation. For comparison with experimental results, a similar circumstances to experiment was assumed: A single coil of internal and external diameters of 2 mm and 5 mm and a height of 1 mm is mounted at a position of 0.2 mm apart from the surface of a large-area superconducting film of thickness 0.6 μm. The number of windings of the coil is 400 turns. The coil axis is directed normal to a wide surface of a superconducting film. The magnetic flux density distribution is calculated assuming the critical state model inside the superconducting film.

Because of the azimuthal symmetry of the system, we calculated the field and current distributions inside the region of 1 degree, 1/360, of the whole system. This region was divided into 60 parts in radial direction and 56 layers in axial direction: 12 layers in the film and 34 and 10 layers in the space above and below the film.

The numerical analysis was carried out using the Bean model in the absence of external magnetic field. On the other hand, the value of J_c was changed in the range of 1.0×10^9 to 1.0×10^{10} A/m^2. This was equivalent to the application of external magnetic field as in experiment, since the superposition of uniform DC magnetic field does not influence the current distribution. The distributions of magnetic field and current inside the superconductor were determined, and the voltage induced in the driving coil was derived from the magnetic flux which interlinked the driving coil. The amplitude V_3 and phase θ_3 of third harmonic voltage were derived by Fourier analysis.

V_3-I_0 curves were derived for the critical current density in the range of $1.0 \times 10^9 - 1.0 \times 10^{10}$ A/m^2. The predicted scaling of V_3-I_0 curves is examined and the proportionality between I_{c0} and J_c is checked. In addition, to investigate a modification of the induced thrid harmonic voltage V_3 by an imperfection of the film, a deteriorated region of a ring shape was introduced in the center of the coil. The width of the ring was 0.1 mm and the inner radius was 0.5, 1.1 and 2.0 mm. The critical current density in the ring was assumed to be 10 % of the value of uniform region, 1.0×10^{10} A/m^2.

3 Results and Discussion

Fig. 1 shows the drive-current amplitude I_0 dependence of harmonic voltage amplitude V_3. As I_0 increases beyond some threshold current I_{c0}, V_3 starts to appear. I_{c0} is the current at which the magnetic field penetrates through the film as indicated by the magnetic field distribution. In fact, Fig. 2(a) and (b) show the magnetic field distributions just below and above I_{c0}, respectively. In addition, Fig. 3 shows that I_{c0} is proportional to J_c. These results show that the electromagnetic response becomes nonlinear when the magnetic field penetrates through the film. Fig. 4 shows calculated curves of V_3/I_{c0} vs I_0/I_{c0}. It is seen that these curves meet on a single curve. Such a scaling behavior agrees with theoretical and experimental results by Mawatari *et al.* It means that an assumption by Mawatari *et al.* that the magnetic field is parallel to the surface where the magnetic flux penetrates the film is correct. When I_0

is just above I_{c0}, the magnetic flux in the region, where the flux is almost parallel to the surface, penetrates through the film and the magnetic flux in other regions does not penetrate through the film, resulting in no contribution to V_3. This is considered as the reason for the correct hypothesis by Mawatari *et al.*

Fig. 5 shows V_3-I_0 curves when a damaged region of a ring shape exists in the film. It is seen that the curves are appreciably modified by the existence of such defects. This suggests that this measuring method is useful also for a detection of such damage in the film. Hence, a further detailed analysis is needed for finding the correlation of the type of damage and the V_3-I_0 curves.

4 Summary

The magnetic field distribution in a superconducting film was numerically calculated by Finite Element Method (FEM) to estimate the third harmonic voltage. This result was compared with the theoretical and experimental results by Mawatari *et al.* In addition, the effect of locally deteriorated region in a film on the third harmonic voltage was also investigated to examine the applicability of this measuring method for characterization of films.

1. It is shown that V_3 starts to appear at the threshold current I_{c0} where the AC magnetic field penetrates through the film. In addition, I_{c0} is found to be proportional to J_c. Hence, the critical current density of the film can be estimated from I_{c0}.
2. It is shown that the calculated curves of V_3/I_{c0} vs I_0/I_{c0} meet on a single curve. Such scaling behavior agrees with the theoretical and experimental results by Mawatari *et al.* It means that the simple assumption by

Mawatari *et al.* is suitable.

3. It is found that the V_3-I_0 curve is appreciably modified when a damaged region of a ring shape exists in the film. This suggests that this measuring method is useful also for a detection of such damages in the film.

References

[1] J.H. Claassen, M.E. Reeves, R.J. Soulen Jr., Rev. Sci. Instrum. **62** (1991) 996.

[2] Y. Mawatari, H. Yamasaki, Y. Nakagawa, Appl. Phys. Lett. **81** (2002) 2424.

Figure captions

Fig. 1. Driving current amplitude I_0 dependence of third harmonic voltage amplitude V_3.

Fig. 2. Distribution of magnetic field at (a) $I_0/I_{c0}= 0.85$ (b) $I_0/I_{c0}= 1.125$.

Fig. 3. Threshold current I_{c0} vs critical current density J_c of film.

Fig. 4. Scaled curves of V_3/I_{c0} vs I_0/I_{c0}.

Fig. 5. V_3-I_0 curves when a damaged region of ring shape exists in the film.

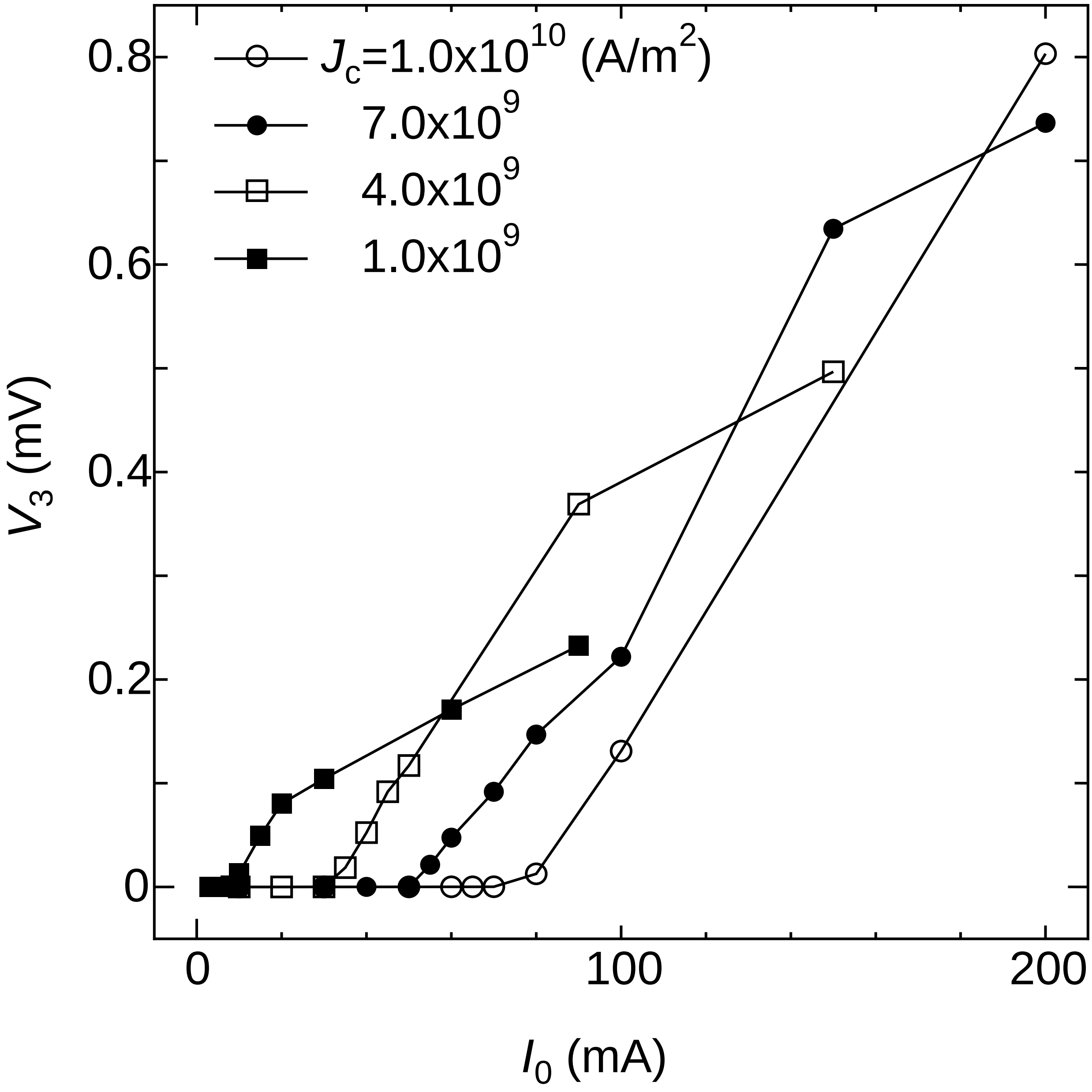

Fig. 1: H. Wada *et al.*/$FDP-48/ISS2002$

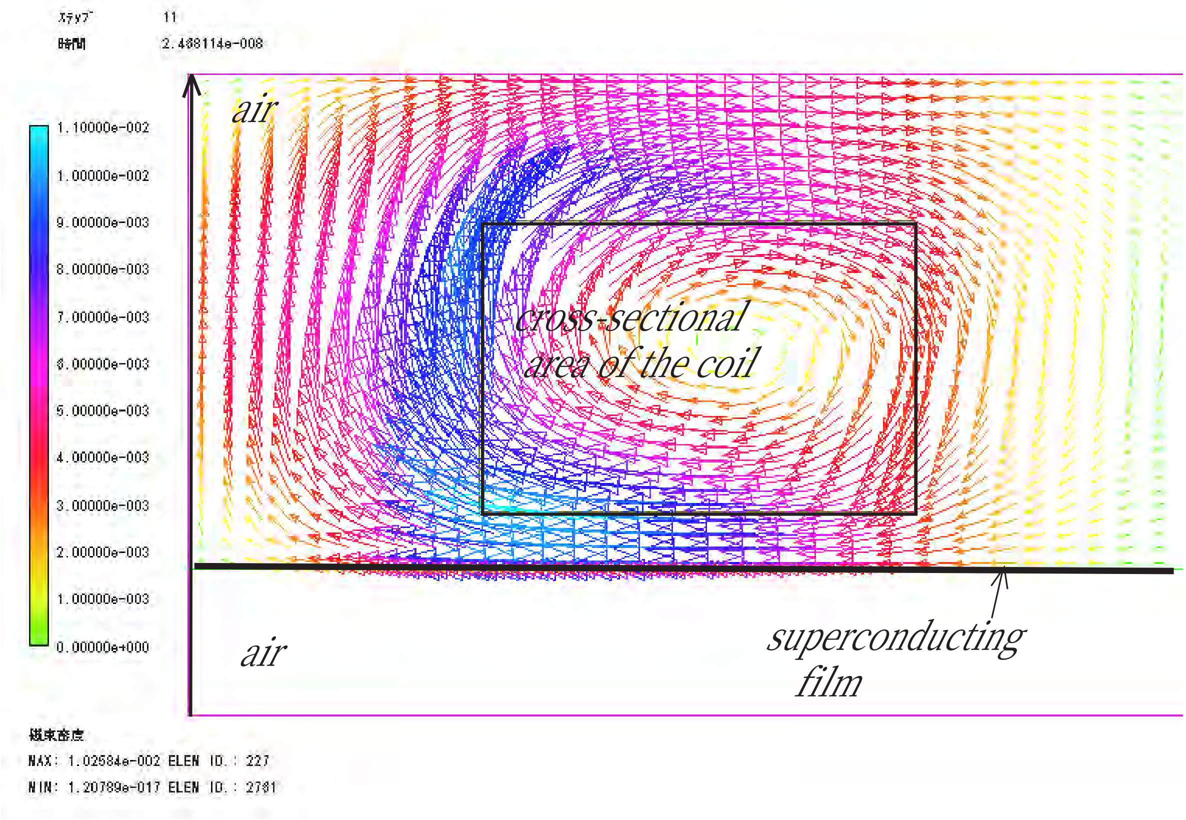

Fig. 2(a): H. Wada *et al.*/$FDP-48/ISS2002$

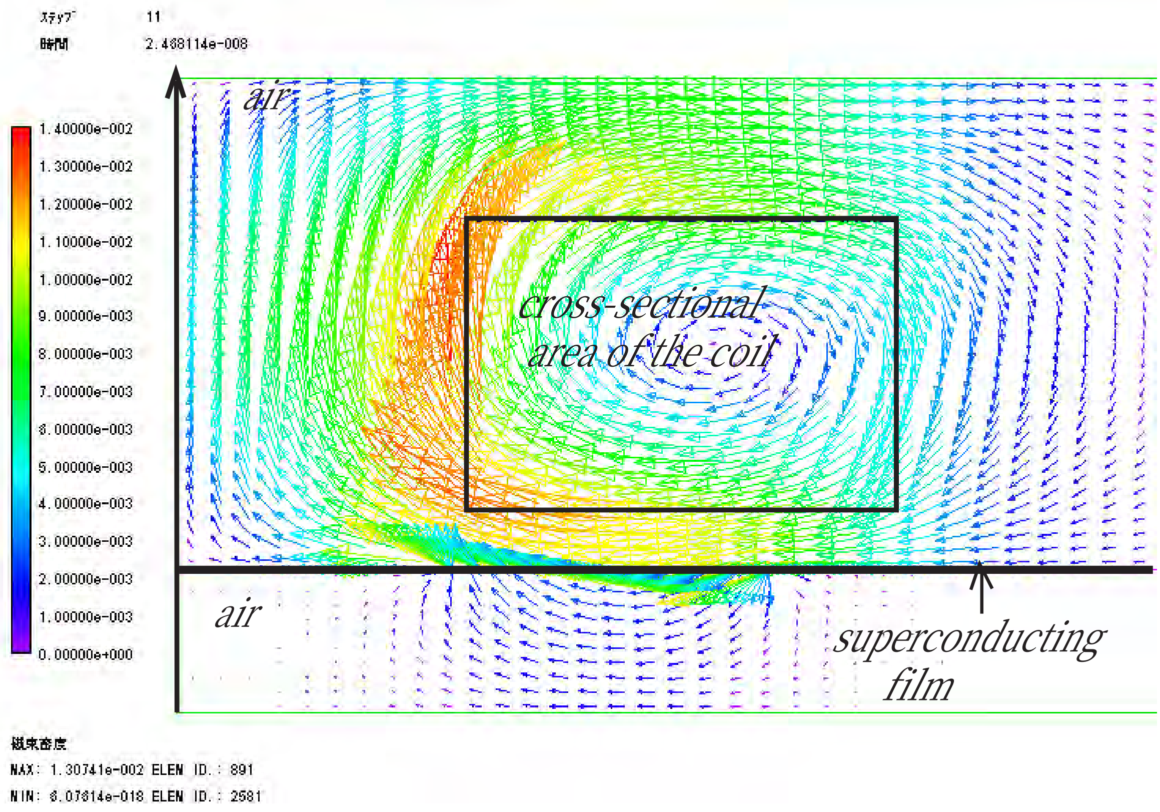

Fig. 2(b): H. Wada *et al.*/$FDP-48/ISS2002$

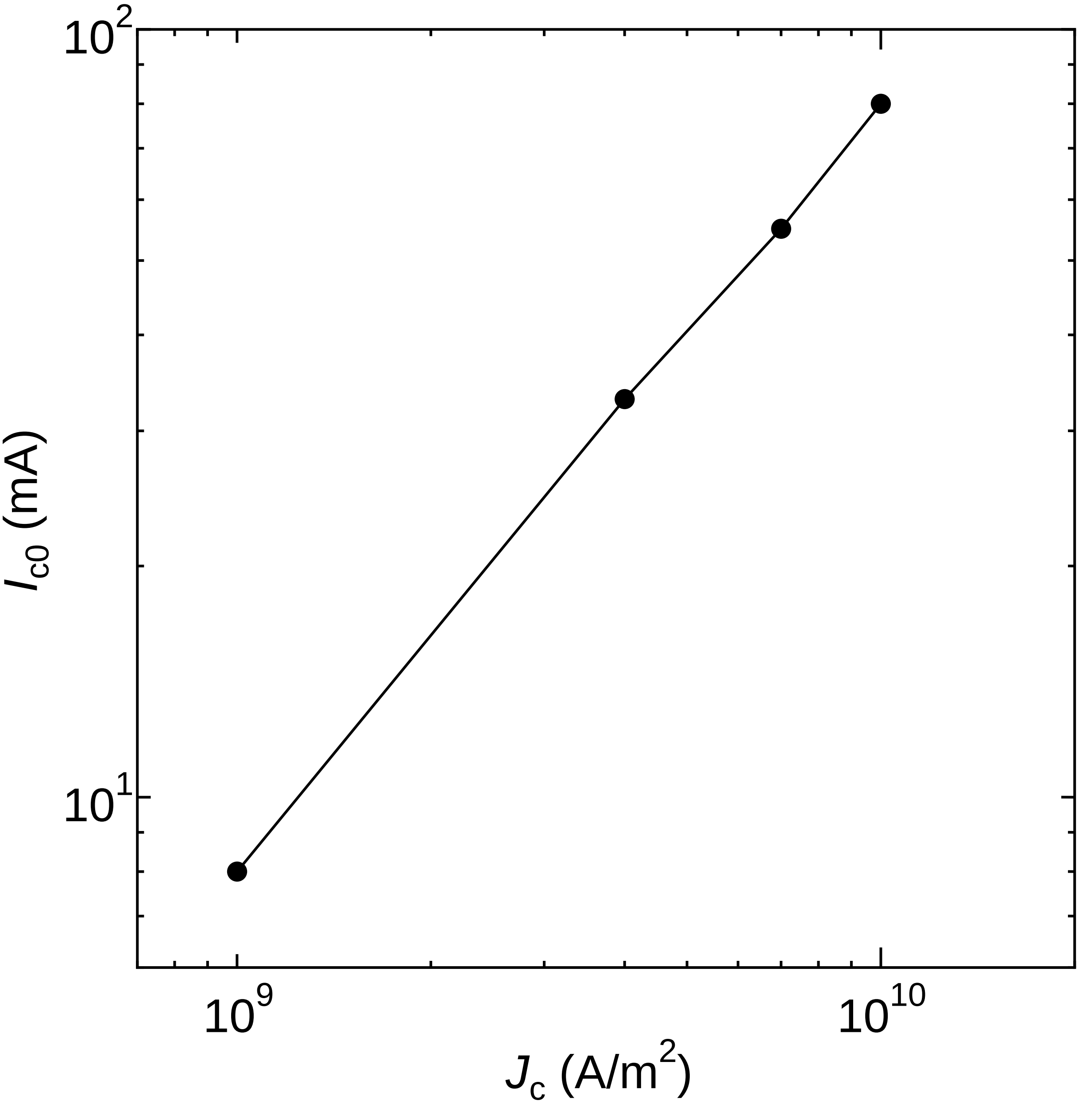

Fig. 3: H. Wada *et al.*/$FDP-48/ISS2002$

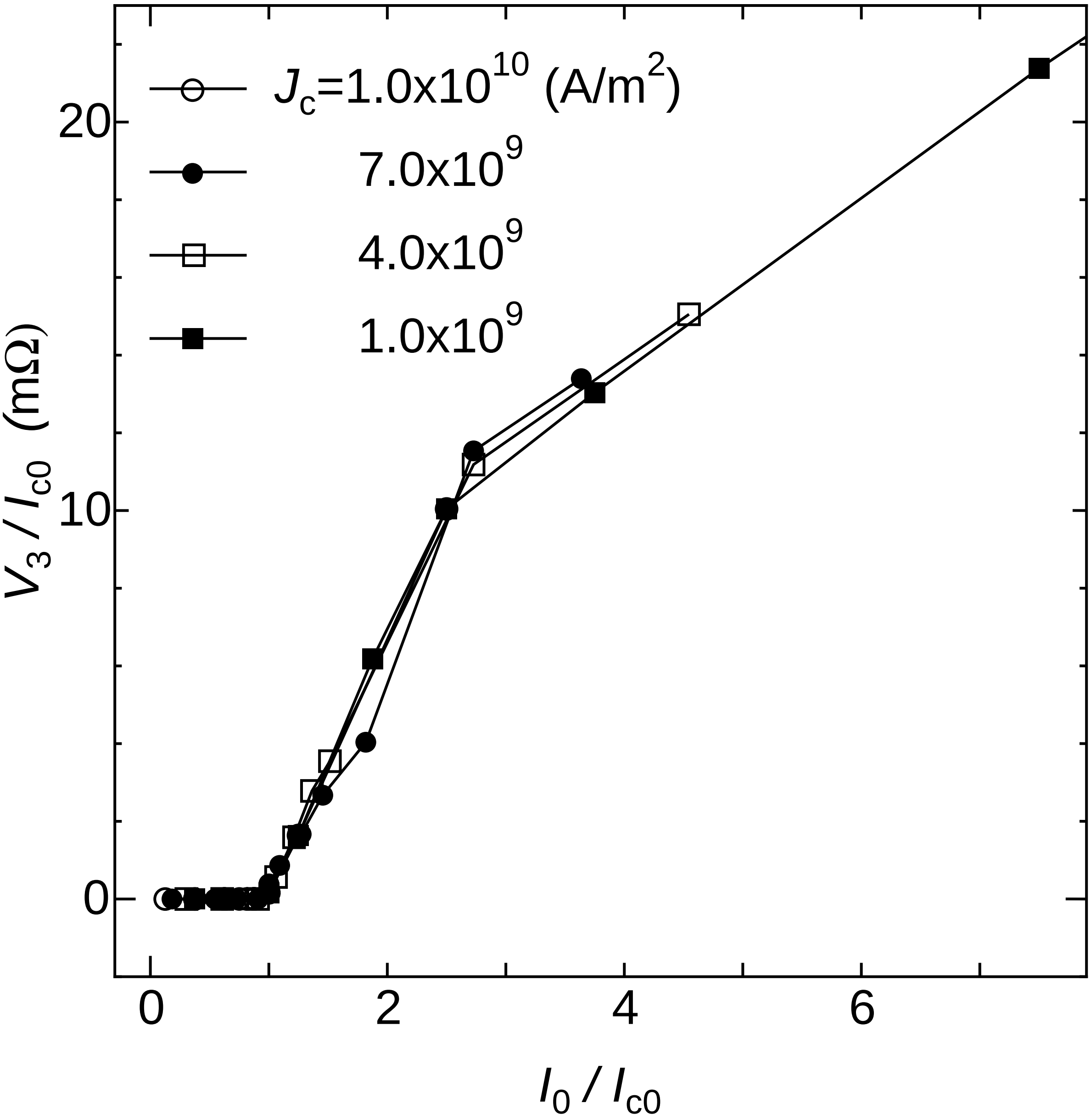

Fig. 4: H. Wada *et al.*/$FDP - 48/ISS2002$

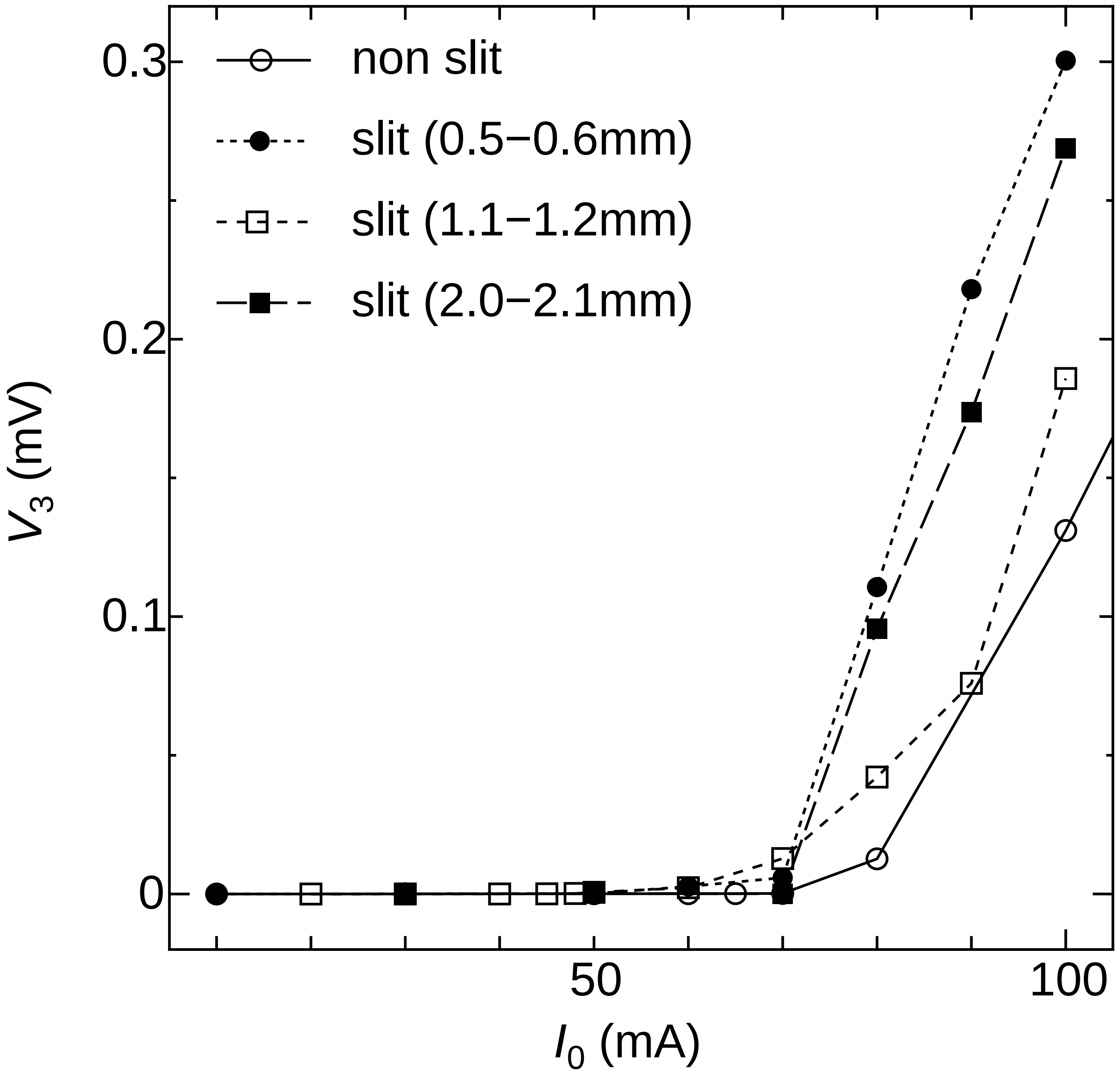

Fig. 5: H. Wada *et al.*/$FDP-48/ISS2002$