# Theoretical Analysis of AC Transport Current Loss in Bulk Y-123

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**Abstract**

The AC-current loss in a QMG(Quench Melt Growth)-processed Y-123 bulk superconductor was found to fit well with the Norris's formula for a superconductor with an ellipsoidal cross-section based on the Bean's model, in spite of a rectangular cross-section and the magnetic field dependence of critical current density. The reason for this coincidence is generally argued by a theoretical analysis with respect to the effects of the magnetic field dependence of $J_c$, the geometry of effect of superconductor and the effect of $n$-value on the AC loss.

In this paper, finite element method(FEM) is used for the numerical analysis of the AC-current loss of the QMG bulk sample with taking account of the latter two effects. It is found that when the aspect ratio of rectangular superconductor increases, the loss becomes small. The AC loss takes on a larger value when the $n$-value decreases, suggesting a large effect of flux creep.

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## 1 Introduction

Norris's formula [1] is used for analysis of AC current loss, since it often agrees well with experimental results. However, the magnetic field dependence of critical current density is neglected in Norris's formula, while the field dependence is strong in high temperature superconductors. Therefore, it is necessary to take into account of the field dependence in the calculation of AC transport current loss. However, it was found that a deviation of a theoretical result based on Irie-Yamafuji's model [2,3] for $J_{\mathrm{c}}(B)$ from an experimental result for a QMG processed Y-based superconductor was larger than Norris's formula [4].

This deviation seems to originate from a geometrical effect of superconducting specimen and complex characteristics in high temperature superconductors, such as field-angle dependence of $J_{\mathrm{c}}$ and flux creep, etc. However, it is quite difficult to take into account these characteristics in the theoretical analysis, because the magnetic distribution in superconductor must be calculated self-consistently. For this purpose a finite element method(FEM) is useful. In a previous paper [5] the effect of field dependence of $J_{\mathrm{c}}$ on the current loss was investigated for a long superconducting cylinder by this method. The loss calculated by FEM based on Kim's model for $J_{\mathrm{c}}(B)$ agrees well to the theoretical result and the slope of loss vs. current amplitude, when the both are plotted in logarithmic scale, is steeper than the case of Bean's model.

In this study, the AC current loss of a superconductor with a rectangular cross-section was numerically analyzed by FEM in which the effects of the geometry of superconductor and the flux creep were considered. To investigate the effect

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of flux creep, the effect of $n$-value was examined by assuming the $n$-value model which approximately represents $E$-$J$ characteristics in an appropriate range of electric field. The result of FEM considering these effects was compared with simple Norris's formula.

## 2 Results and Discussion

The theoretical analysis was carried out using JMAG studio version 7 of Japan Research Institute. In the beginning, it was confirmed that not only the AC current loss but also the magnetic flux distribution inside a superconducting cylinder can be correctly calculated by the code as predicted by theories for Bean's and Kim's models [5].

To investigate the geometrical effect on the loss, we focus on the effect of aspect ratio of a rectangular superconductor. The cross-sectional area of the model of superconductor is $4.0 \times 10^{-6}$ m$^2$. For simplicity, we assumed Bean's model for $J_c(B)$ and the effect of flux creep was disregarded. Results are shown in Fig. 1. For a superconductor with a larger aspect ratio, the current loss is lower and the slope of the loss vs. current amplitude($I_p$), when the both are plotted in logarithmic scale, is more gentle. This is considered to be caused by a different distribution of magnetic field in the superconductor. That is, the penetration depth of magnetic flux becomes short due to a longer perimeter of the superconductor cross-section, according as the aspect ratio increases. In other words, the self field of the current decreases, resulting in the lower loss density. On the other hand, the loss becomes high around the edge of superconductor because of demagnetizing effect. However, this edge effect is not considered to be dominant because of a limited volume of the edge region.

In fact, it is seen from Fig. 1 that the loss becomes smaller with increasing aspect ratio. The reduction of the slope in Fig. 1 is attributed to the fact that the edge effect is prominent only at low self fields.

On the other hand, it was reported that the slope of loss vs. current amplitude curve in a superconducting strip of aspect ratio of about 5000 is steeper($\simeq 4$) [6]. This seems to be contradictory to an extrapolation of the present speculation. It is necessary, therefore, to investigate the AC current loss in the thin limit of the superconductor.

Secondly the effect of flux creep on the AC current loss is investigated for a cylindrical superconductor of radius of $1.128 \times 10^{-3}$ m assuming the $n$-value model that where the electric field is expressed as

$$E = E_{\mathrm{c}}(J/J_{\mathrm{c}})^n. \tag{1}$$

In the above $E_{\mathrm{c}}$ is the electric field criterion for the determination of the critical current density. In this paper, $E_{\mathrm{c}} = 10^{-4}$ V/m and $J_{\mathrm{c}} = 10^9$ A/m$^2$ are assumed corresponding to the usual transport measurement. Hence, $J_{\mathrm{c}}$ does not depend on the magnetic field as in Bean's model. The analysis of FEM is carried out for the frequency of 10 mHz, and the corresponding electric field at the surface is about $2.0 \times 10^{-7}$ V/m. Fig. 2 shows the AC transport current loss for various $n$ values at $i \equiv I_{\mathrm{p}}/I_{\mathrm{c}} = 0.09$. As the $n$-value increases, the loss becomes small and finally approaches Bean's model ($n = \infty$). In other words, when the effect of flux creep is strong, the AC loss increases appreciably. Fig. 3 shows the AC transport current loss for $n = 10, 20$ and 30. It is seen that the loss is almost proportional to a third power of $I_{\mathrm{p}}$ and only its value changes with the $n$ value. This suggests that the loss can simply be explained only by an effective critical current density.

Eq. (1) shows that the current of the density,

$$J = J_{\mathrm{c}}(E/E_{\mathrm{c}})^{1/n}, \tag{2}$$

flows when the electric field is $E$. This current density is regarded as the effective critical current density $J_{\mathrm{c}}'$. Hence, if we concentrate to the region of low current amplitude, the loss density can be simply given by

$$W = \frac{\mu_0}{6\pi S^2}\frac{I_{\mathrm{p}}^3}{J_{\mathrm{c}}'} = \frac{\mu_0 I_{\mathrm{p}}^3}{6\pi S^2 J_{\mathrm{c}}}\left(\frac{E_{\mathrm{c}}}{E}\right)^{1/n}, \tag{3}$$

where $S$ is the cross section of superconductor and $\mu_0 I_{\mathrm{p}}^3/6\pi S^2 J_{\mathrm{c}} \equiv W_0$ is the prediction of Bean's model in the limit of $n \to \infty$. The losses estimated from Eq. (3) at $E = 2.0 \times 10^{-7}$ V/m for $n$=10, 20, and 30 are shown by the symbols in Fig. 2. It is found that although the simple theoretical estimation explains qualitatively the result of FEM, a quantitative defference exists between them. Hence, a further investigation is needed to find the reason for this quantitative difference. It is also necessary to investigate the AC loss under the situation where the field dependence of $J_{\mathrm{c}}$, the geometrical effect and the effect of flux creep are simultaneously considered.

## 3 Conclusion

In this work, the AC transport current loss was analyzed by the finite element method from a viewpoint of the effects of geometry of superconductor and the flux creep. The following results are obtained:

1. When the aspect ratio of rectangular superconductor increases, the loss becomes small. This is explained by the fact that the self field of the current decreases in a superconductor with a large aspect ratio.

2. The AC loss takes on a larger value when the $n$-value decreases, suggesting a large effect of flux creep. The $n$-value dependence of AC loss is qualitatively explained by a difference of flowing current at the level of given electric field. However, there still exists a quantitative disagreement.

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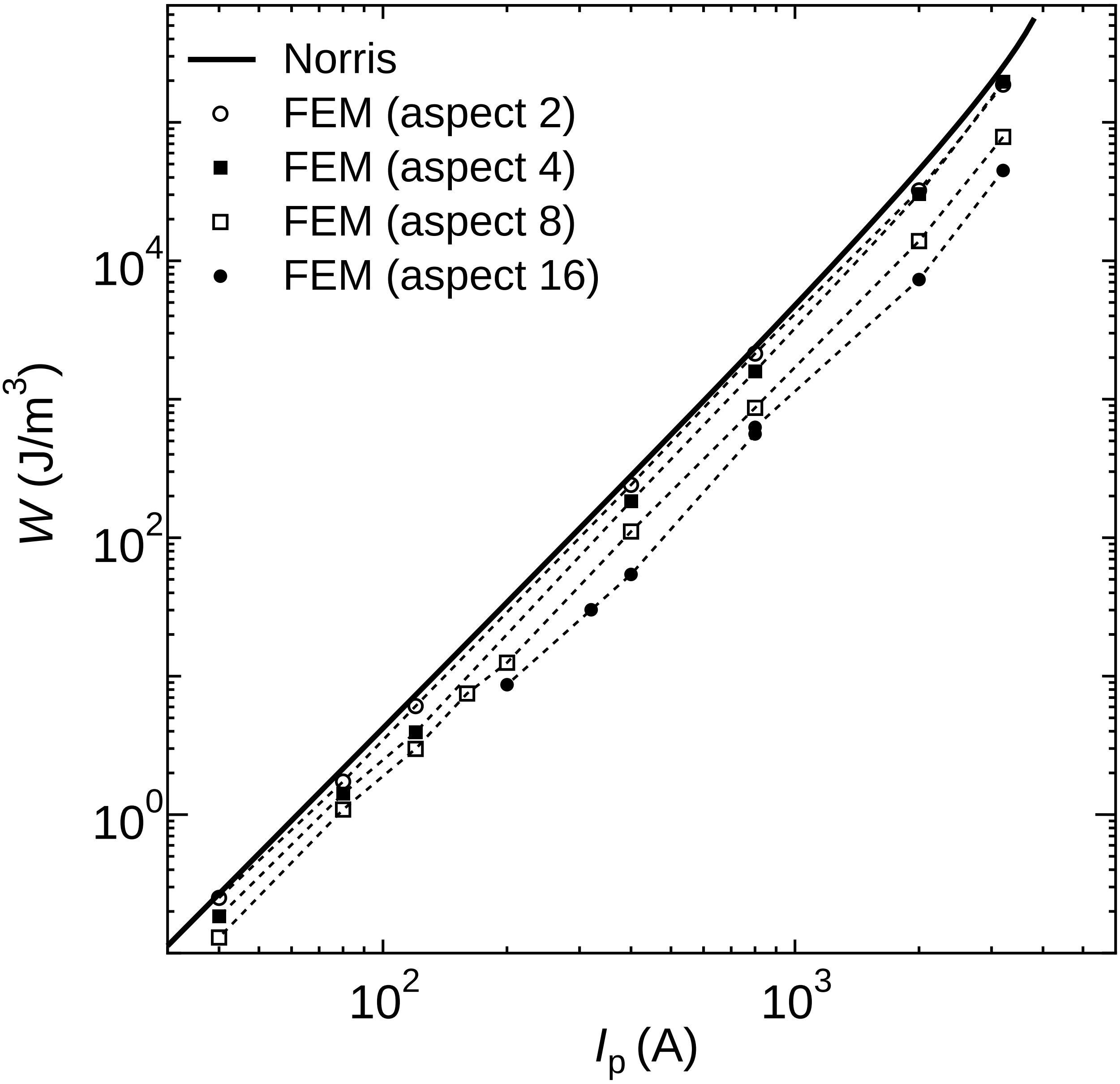
Figure captions

Fig. 1. AC current loss calculated by FEM with Bean's model for superconductor with different aspect ratio.

Fig. 2. AC current loss vs. $n$-value calculated by FEM for cylindrical superconductor at $i \equiv I_{\rm p} = 0.09$.

Fig. 3. AC current loss calculated by FEM with Bean's model for various $n$-value for cylindrical superconductor.

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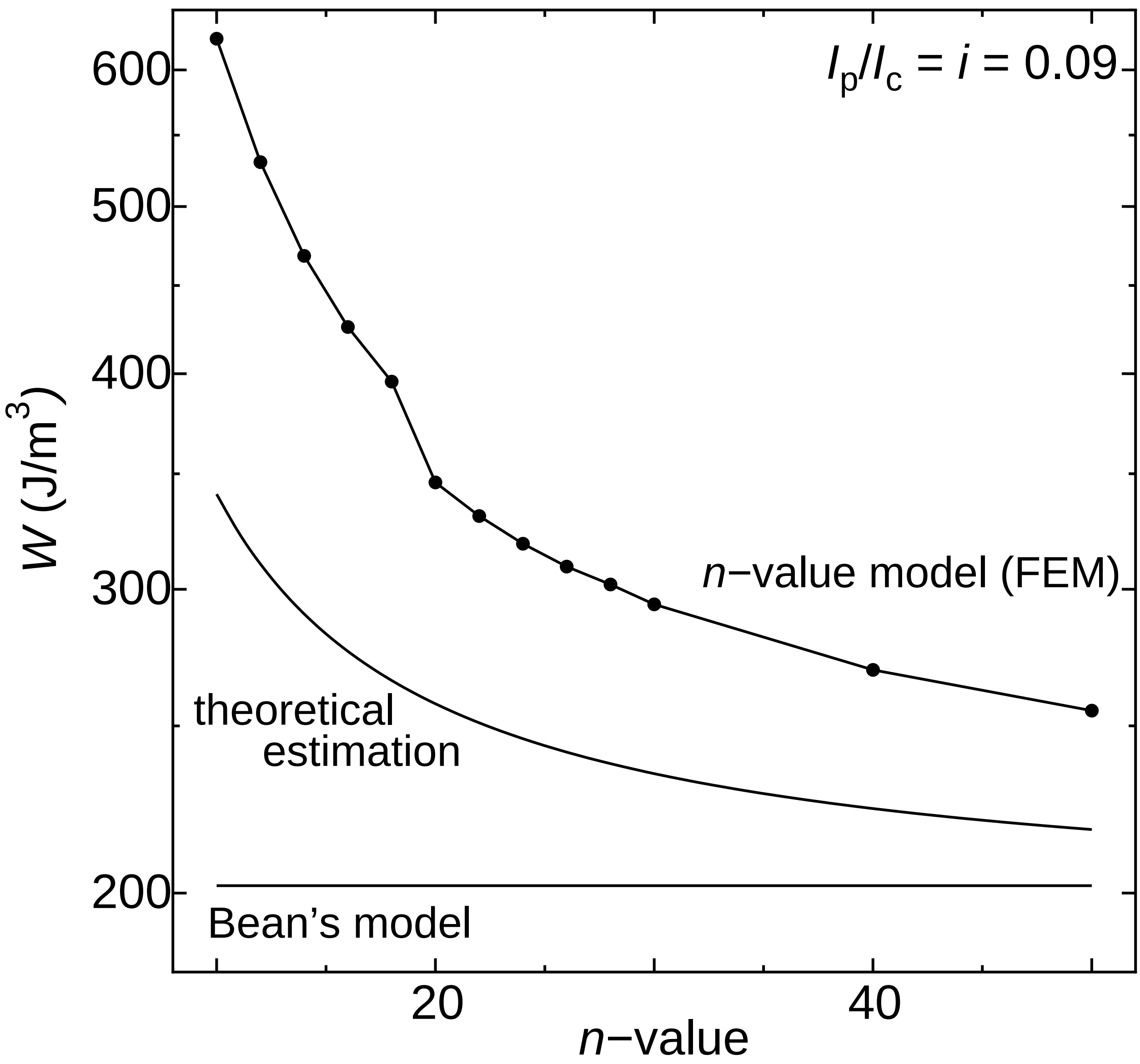


Fig. 2: M. Migita *et al.*/$BSP-37/ISS2002$

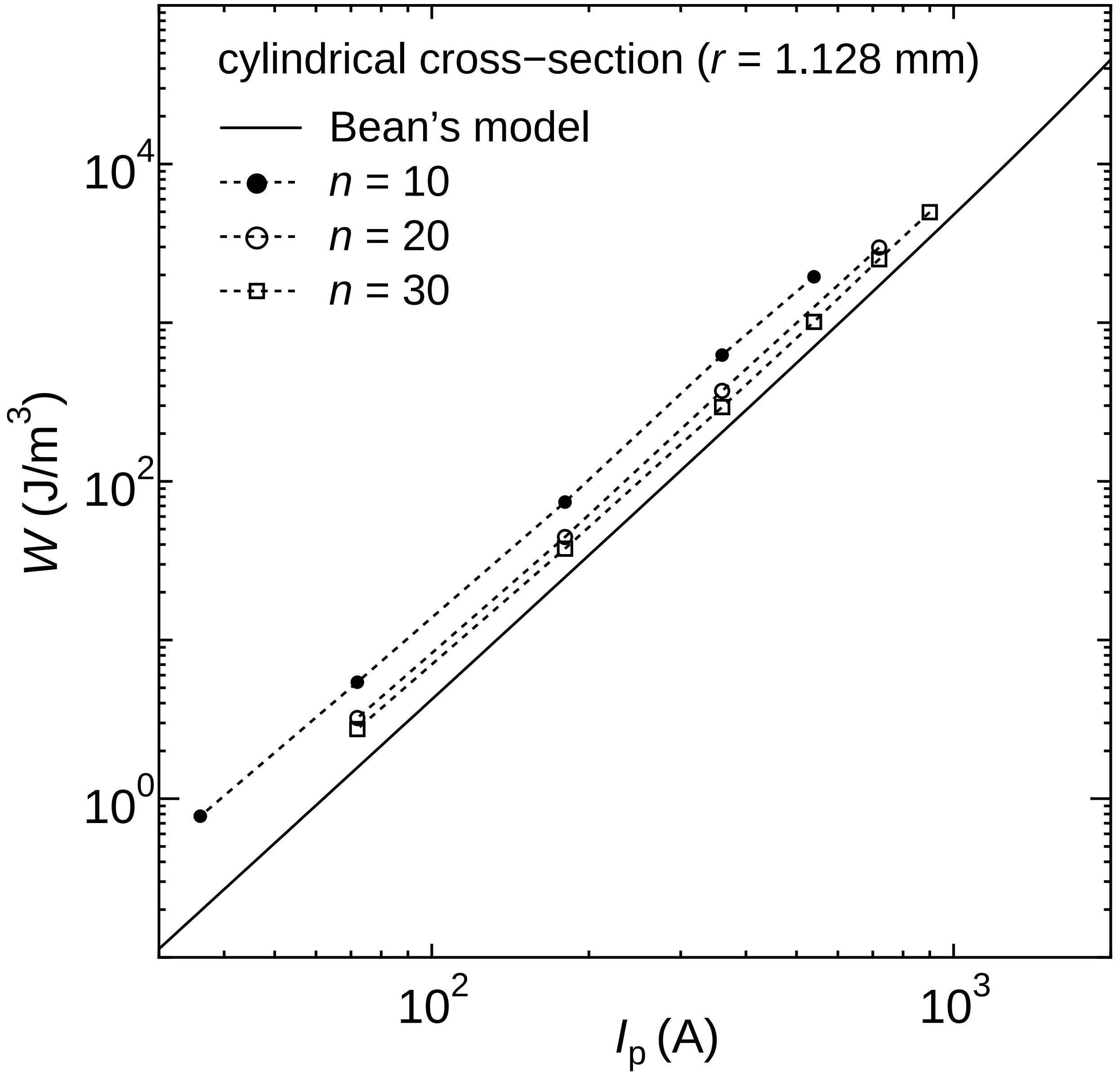


Fig. 3: M. Migita *et al.*/$BSP-37/ISS2002$