Characteristics of Slow Waves Propagating Along Annular Plasma Guides

— Part 1 —

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Abstract

The characteristics of the electromagnetic waves propagating along the plasma column of annular cross section without static magnetic field and collision losses is theoretically investigated by a quasistatic approximation. The annular plasma column placed in free space supports the forward waves and the backward waves ; the latter have oppositely directed phase and group velocities and exist at frequencies high compared with those of the former. The phase velocity of both types of wave is less than the velocity of light. The radial field distributions of these waves are distinctly different. When the plasma guide is surrounded with a perfect conductor, the forward waves vanish and only backward ones are found. Traveling wave interaction of a drifting electron beam with these modes is possible to be considered. It will be pointed out, however, that the problem of beam-wave interaction depends upon the magnitude of attenuation of waves.

I. Introduction

Since it was known that the bounded plasmas of finite transverse cross section can guide the electromagnetic waves, the features of the waves propagating along the bounded plasmas have been investigated by many authors. It is interesting that the plasma guides can support not only forward waves but backward waves and the phase velocity is smaller than that of light. There exist the backward waves in any case, where the cylindrical waveguide is partially filled with the magnetized plasma (Trivelpiece & Gould)[1], the isotropic plasma column is surrounded with a thin dielectric (Trivelpiece)[2], the cylindrical conductor is placed on the axis of plasma column (plasma Goubau line, Paik)[3], the plasma has the configuration of slab (Oliner & Tamir)[4] and it is inhomogeneous (Schumann)[5]. These plasma guide modes result from the propagation of space charge disturbance.

Ferrite is an another medium of interest, which can support the slow backward waves. The features of the waves along the ferrite rod placed in free space (Fletcher & Kittel)[6] and in a waveguide (Trivelpiece, Ignatius and Holscher)[7] are similar; the phase velocity is also lower than that of light and the smaller the radius of rod is in comparison with the free space wave length, the slower. These exist in the frequency region where the equivalent permeability of ferrite is negative.

The waves, described above, along the plasma or ferrite guides are the eigenmodes but not the spatial harmonics which occur on periodically constructed metal structures. These guides can be regarded as slow wave circuits; the traveling wave interaction between the waves and a drifting electron stream may be possible.

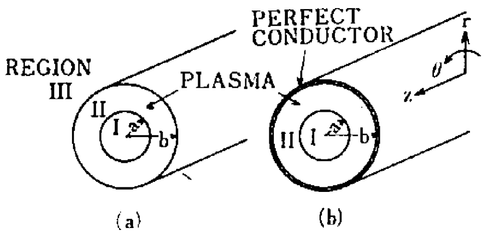


Fig. 1. Configurations of Annular Plasma Guides,
(a) Placed in Free Space,
(b) Surrounded with Perfect Conductor.

In this report, it is studied what characteristics the waves on the plasma guides have for the configurations of plasma column as shown Fig. 1 (a) and (b), which are presented for the purpose of an application of interation. When an electron stream passes through the hole, the interaction impedance is enhanced by surrouding the plasma with metal as shown Fig. 1 (b) instead of placing in free space. It will be indicated that if the annular plasma guide is in free space, the forward and backward waves are found, and only backward waves when surrounded with perfect conductor.

II. Theory of Annular Plasma Guide Waves

2.1 Introduction of Quasistatic Analysis

Since plasma is a complicated medium with various properties, it is necessary to simplify the expression of plasma without losing its essential properties. In a weakly ionized, low temperature plasma, it is appropriate under the high frequency field to specify the property of idealized plasma by equivalent dielectric constant, which is given by

$$\kappa_p = 1 - \omega_p{}^2/\omega^2 \tag{1}$$

where

$\omega_p = (n_e e^2/m\varepsilon_0)^{1/2}$: plasma frequency,

ω : radio frequency,

m and e : mass and electric charge of an electron, respectively,

n_e : electron density of plasma,

ε_0 : permittivity of free space.

The static magnetic field and electron collisions are not taken into accout. In the limitation of slow wave, where the phase velocity is much less than that of light, a quasistatic approximation is possible by neglecting the retardation effects. In this approximation, the electric fields are derived from a scalar potential

$$\mathbf{E} = -\nabla\phi \tag{2}$$

Since the plasma is regarded as a simple dielectric, there is no free charge, so that in plasma and free space

$$\nabla \cdot \mathbf{D} = 0 \tag{3}$$

This leads the Laplace's differential equation for the scalar potential

$$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial Z^2} = 0 \tag{4}$$

When it is assumed that waves are propagating in the direction of positive z with the phase constant β and so ϕ has a factor

$$F(\omega,\ n,\ \beta) = \exp j(\omega t - n\theta - \beta z), \tag{5}$$

Eq. (4) gives the differential equation satisfied with the modified Bessel functions.

2.2 The Case Placed in Free Space

When the annular plasma guide is placed in free space, the scalar potentials are given by

region

I : $\phi_1 = AI_n(\beta r) \cdot F(\omega,\ n,\ \beta)$ $\quad (0 \leqq r \leqq a)$ (6.1)

II : $\phi_2 = [BI_n(\beta r) + CK_n(\beta r)] \cdot F(\omega, n, \beta)$ $\quad (a \leqq r \leqq b)$ (6.2)

III : $\phi_3 = DK_n(\beta r) \cdot F(\omega,\ n,\ \beta)$ $\quad (b \leqq r)$ (6.3)

where A, B, C and D are the amplitude coefficients determined by the excitation and the boundary conditions, $I_n(x)$ and $K_n(x)$ are the n-th order modified Bessel functions of the first and second kinds, respectively. The boundary conditions lead to the dispersion equation:

$$\begin{aligned}\kappa_p^2 I_{na}K_{nb}(K'_{na}I'_{nb}-I'_{na}K'_{nb})\\ +\kappa_p[I_{na}K'_{nb}(I'_{na}K_{nb}-K'_{na}I_{nb})\\ +I'_{na}K_{nb}(I_{na}K'_{nb}-K_{na}I'_{nb})]\\ +I'_{na}K'_{nb}(K_{na}I_{nb}-I_{na}K_{nb})=0 \qquad (7)\end{aligned}$$

where I_{na}, K_{nb} ··· denote $I_n(\beta a)$, $K_n(\beta b)$ ···

For the azimuthally symmetric mode($n=0$) and the dipolar mode ($n=1$), the dispersion curves and the distributions of z-component of the electric field are illustrated in Fig. 2 and Fig. 3, respectively.

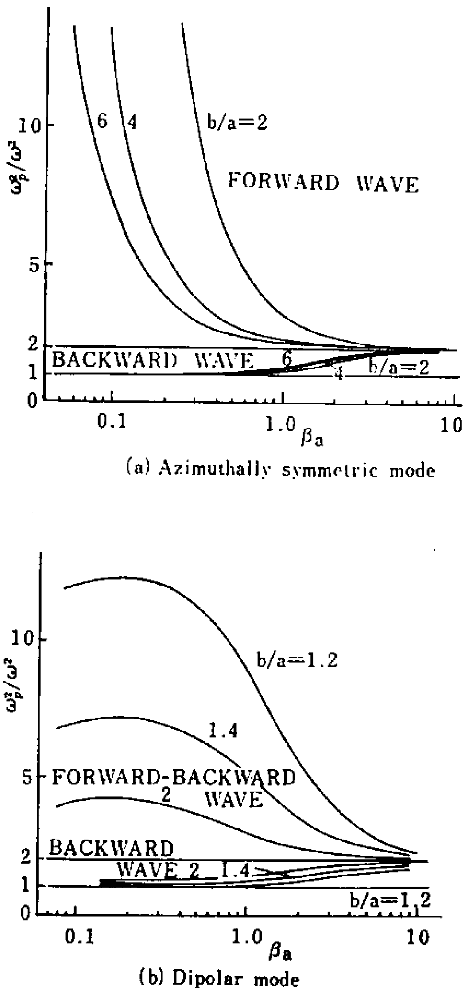


Fig. 2. Dispersion relations when placed in free space.

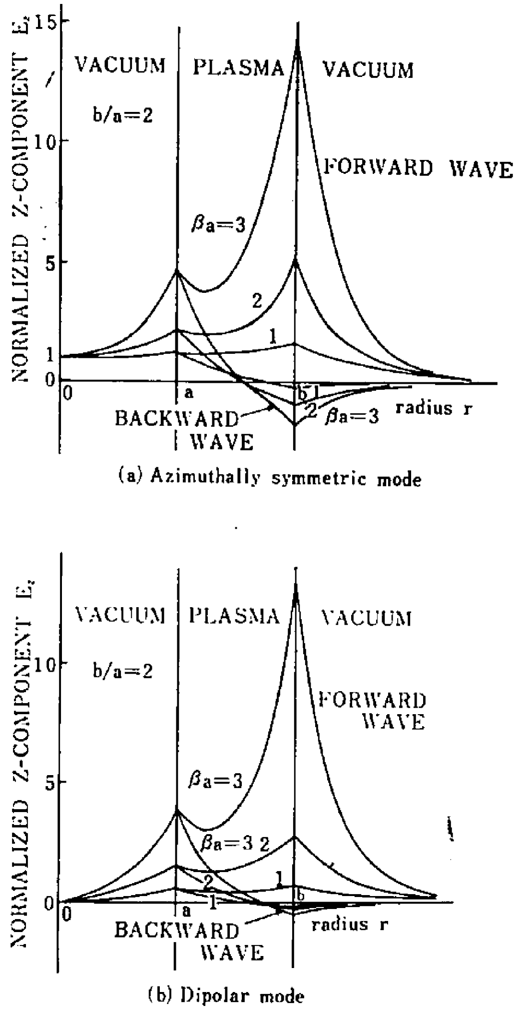


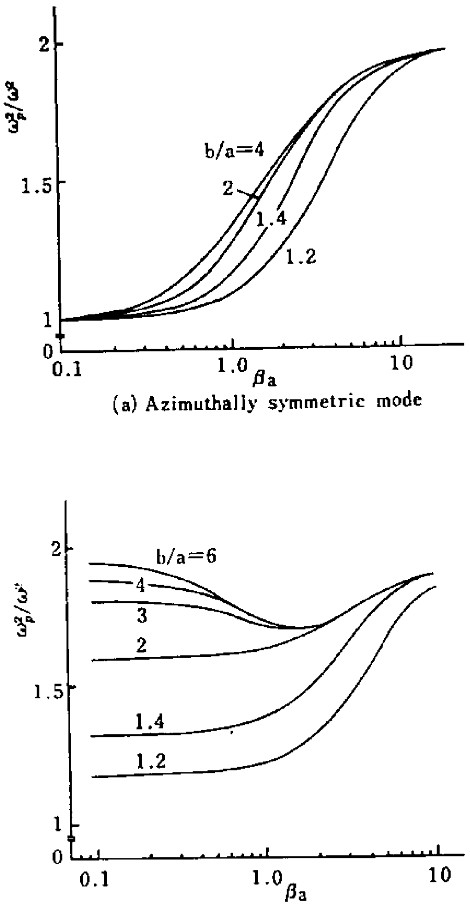
Fig. 3. Field distributions when placed in free space.

2.3 The Case Surrounded with Perfect Conductor

The scalar potentials are given by the equation (6. 1) and (6. 2). From the continuity at $r=a$ and vanishing of tangential electric field at $r=b$, the next simple characteristic equation is obtained:

$$\frac{\omega_p^2}{\omega^2}=\frac{I_{nb}}{\beta a I_{na}(I'_{na}K_{nb}-K'_{na}I_{nb})} \qquad (8)$$

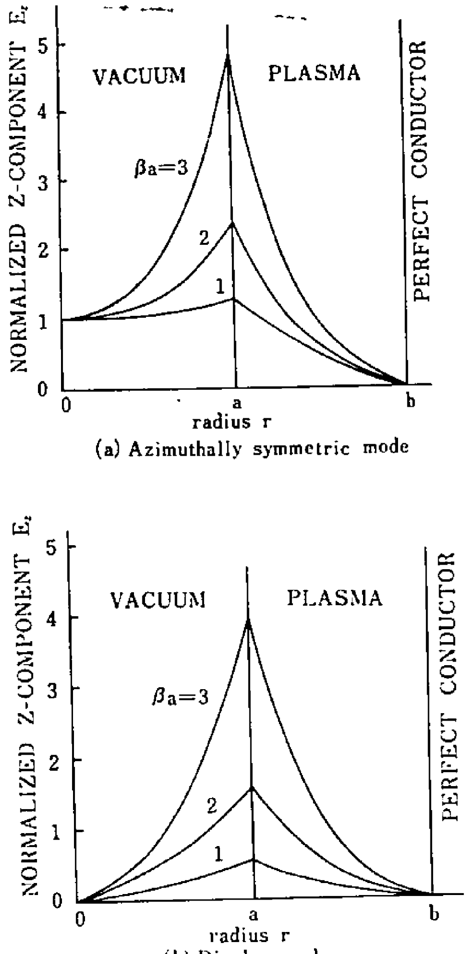
The dispersion curves and the field patterns are illustrated in Fig. 4 and Fig. 5, respectively.



(a) Azimuthally symmetric mode

(b) Dipolar mode

Fig. 4. Dispersion relations when surrounded with perfect conductor.



(a) Azimuthally symmetric mode

(b) Dipolar mode

Fig. 5. Field distributions when surrounded with perfect conductor.

III. Discussions

3.1 The Case Placed in Free Space

The plasma column with solid cross section in free space supports only the forward wave having the propagation band of $2 < \omega_p{}^2/\omega^2 < \infty$ for n=0 mode; on the other hand, the annular plasma guide in free space supports not only the forward wave with the same band but also the backward wave having the propagation band $1 < \omega_p{}^2/\omega^2 < 2$ for n=0 mode. There also exists the latter for n=1 mode. The dispersions for the forward waves is quite analogous to those of solid plasma column and the new modes (backward waves) are generated by the additional plasma-vacuum interface. For n =1, the upper mode in Fig. 2(b). indicates a property of backward-forward wave at small βa. The radial distribution of the tangential electric field Ez for each type of waves is markedly different as seen in Fig. 3. The fields of the forward wave strongly concentrate on the outer surface, on the contrary, those of the backward wave are rather on the inner surface and, moreover, Ez is in the opposite direction in the hole and outer space and has a vanishing point in the region of plasma. These features are never found in the case of solid plasma column. The energy flow of waves is in the opposite direction in free space and plasma region.

3.2 The Case Surrounded with Perfect Conductor

For both n=0 and n=1 mode, the forward waves disappear and only backward waves propagate along the annular plasma guide surrounded with metal. The propagation band width for n=1 mode is narrower than that for n=0 mode which has the band width of $1 < \omega_p{}^2/\omega^2 < 2$. As shown in Fig. 5, the fields concentrate on the vacuum-plasma interface.

3.3 Effect of Glass Tube

The value of $\omega_p{}^2/\omega^2$ for resonance ($\beta = \infty$), where the approximation gives the right results, is 2 for n=0 and n=1 modes when the outer surface of plasma contacts with free space. Actually, the plasma is contained in a glass tube, therefore, even if a metal covers the container, there can exist the forward and backward waves, and the resonance condition is given by $\omega_p{}^2/\omega^2 = 1 + \kappa_g$ (κ_g : relative dielectric constant of glass). The frequency band of the backward wave becomes wider for the higher value of κ_g. This effect of glass can be examined by making use of the asymptotic expansions of the modified Bessel functions for the dispersion equations.

3.4 Cutoff Condition

The rigorous field equation derived from the Maxwell's equations is given by Eq. (9) instead of Eq. (4) for the quasistatic approximation.

$$\frac{d^2Ez}{dr^2} - \frac{1}{r}\frac{dEz}{dr} - \left[\eta_i{}^2 + \frac{n^2}{r^2}\right] Ez = 0 \quad (9)$$

where in free space(i=1) $\eta_1{}^2 = \beta^2 - k^2{}_0$

in plasma(i=2)

$$\eta_2{}^2 = \beta^2 - k_o{}^2\ (1 - \omega_p{}^2/\omega^2)$$

$k_o = \omega/c$: free space wave number.

For the surface waves $\eta_i{}^2$ must be positive, that is, $\beta^2 > k_o{}^2$ (then $\eta_2{}^2 > 0$, since $\omega_p{}^2/\omega^2 > 1$). Therefore, the phase velocity of waves is always smaller than that of light. The approximation is valid in the case of $\beta^2 \gg k_o{}^2$ and then gives the right resonancc condition. However, at cutoff ($\beta \rightarrow k_o$) the approximation is destroyed and does not indicate the true cutoff condition. When β/k_o comes near 1, $\omega_p{}^2/\omega^2$ of n=0 mode (for simplicity we consider the case of Fig. 1 (b)) approaches 1 and swings upward rapidly when β/k_o comes very closely to 1, that is, the cutoff condition is not $\omega_p{}^2/\omega^2 = 1$. This situation is shown in Fig. 6 (ω/ω_p is taken for the ordinate instead of $\omega_p{}^2/\omega^2$.).

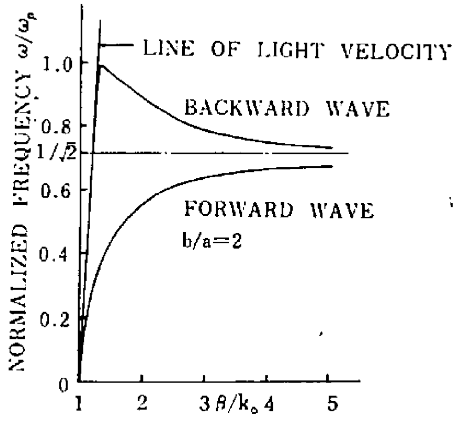


Fig. 6. Cutoff behavior for azimuthally symmetric mode when placed in free space.

3.5 Interaction Problem

One of the applications of the waves propagating along the plasma guides is as the plasma diagnostic technique.[8] By measuring the wave-length of wave on the positive column, the information of the average electron density is obtained, and it is interesting that the probing frequencies can be lowered compared with the plasma frequency. On the other hand, if the guides are regarded as slow wave circuits, the traveling wave interaction between a drifting electron stream and the plasma guide mode can be considered. As an electron stream passing through the hole interacts with the z-component of electric field Ez, n=0 mode having non-zero Ez on the axis of hole should be used. Moreover, the interaction impedance representing the characteristic of the slow wave circuits will be enhanced by covering the plasma guide with metal. The interaction with the back-ward wave of n=0, for instance, when $n_e=10^{11}\sim10^{12}$ cm^{-3}, occurs in the region of frequencies of $2.0\times10^{9}\sim8.7\times10^{9}$ c/s. This discussion is quite optimistic, since the attenuation of wave due to collision losses of electrons is neglected. The problem of interaction depends upon the magnitude of attenuation of waves.

IV. Conclusions

(1) There exist the forward and backward waves on the annular plasma guide in free space; the latter occur in the higher frequencies than the former. The fields concentrate on the surface for the forward waves. and on the inner surface for the other. The backward waves have a vanishing point of Ez in plasma.

(2) Only backward waves can be supported when the plasma guide is covered with metal. But actually, both types of wave occur for the presence of glass tube and the resonance condition is modified into $\omega_p{}^2/\omega^2=1+\kappa_g$, that is, the frequency band of the backward wave becomes wider.

(3) When $\beta\rightarrow k_0$, the quasistatic approximation is destroyed. The cutoff condition is modified into $\omega_p{}^2/\omega^2=\infty$ instead of $\omega_p{}^2/\omega^2=1$.

(4) The problem of beam-wave interaction depends upon the magnitude of attenuation of waves on the plasma guides.

The features of the axially magnetized annular plasma guides and the problems of interaction are under investigation.

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