RECURSIVE NONLINEAR IMAGE ENHANCEMENT —VECTOR PROCESSING

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SYNOPSIS

An approach to design of a recursive image enhancer is introduced. A vector nonlinear dynamical model is derived to represent the statistics of the processor output when several lines of the picture are processed simultaneously. Based on the vector model, a kalman filter is designed and utilized to recursively enhance the image.

The application of a kalman filter to image enhancement is first proposed by Nahi. His method, however, has many practical problems such as a decision of a parameter on the correlation of the image.

In this paper, it is showed that these problems are solved by assuming a nonstationary mean of the image and hence the application of a Kalman filter to a nonlinear image enhancement becomes more practical.

1. INTRODUCTION

The image enhancement describes the classical problem of statistical estimation and filtering, where one basically attempts to filter out the noise from an observation. The most general and computationally efficient procedures available at present to perform this filtering operation are the recursive estimation methods of kalman filtering and thier nonlinear extensions.[1] However, here it is necessary that the observation be a function of one independent variable, such as time, in contrast to an image which is defined on a plane.

The converting the planer statistical information of the image and the noise into a form suitable for application of recursive estimation procedures is sucessfully made by Nahi,[2] under the condition that the statistical natures of the image and the noise are known. In general, however, there are few cases that the statistical natures such as the autocorrelation of the image are completely known especially in the case that the observation includes a nonliner function.

The purpose of this paper is to derive autocorrelation of the image approximately from the blured image and to apply Kalman filter to nonlinear image enhancement. The effectiveness and computational simplicity of this method for image enhancement will be clearly demonstrated.

2. KALMAN FILTERING FOR NONLINEAR IMAGE ENHANCEMENT

The image is scanned M lines at a time where $M>1$. Let $\boldsymbol{f}(t)$ be a M dimensional vector random variable which represents an ideal image with an enseble average denoted dy $\bar{\boldsymbol{f}}$, where $\bar{\boldsymbol{f}}$ is not assumed uniform. Thus the random process $\boldsymbol{f}(t)$ is nonstationary. The covariance function is given by

$$E[f_k(t_1)-\bar{f}_k(t_1)][f_\ell(t_2)-\bar{f}_\ell(t_2)] = Ke^{-\mu_h|t_2-t_1|}\cdot\zeta(\ell-k), \tag{1}$$

where f_k and f_ℓ are kth and ℓth element of vector $\boldsymbol{f}$ respectively. E[·] is an expectational operator. μ_h is a positive constant and $\zeta(\ell-k)$ is an arbitrary function of $(\ell-k)$.

Defining $z(t)$ as

$$z(t)=\boldsymbol{f}(t)-\bar{\boldsymbol{f}}(t), \tag{2}$$

Eq.(1) becomes

$$Ez_k(t_1)z_\ell(t_2) = Ke^{-\mu_h|t_2-t_1|}\cdot\zeta(\ell-k). \tag{3}$$

The state-space model for the scanner output, an M dimensional vector denoted by $\boldsymbol{x}(t)$, is given by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (4)$$
$$z(t) = Cx(t)$$
$$A = -\mu_h$$
$$B = \sqrt{2\mu_h}$$
$$CC' = H$$
$$Eu(t)u'(t+\tau) = K \cdot I\delta(\tau)$$
$$Ex(0)x'(0) = I$$
$$\{I : \text{identity matrix}\}$$

where A and B are scalars and H is an $M \times M$ matrix with i, jth element, h_{ij}, given by

$$h_{ij} = \zeta(j-i) \quad (5)$$

and ζ is given by Eq.(1)[1]. C' is a transpose matrix of C.

If the observation is linear function of f, then this is all. The desired estimator can be designed straightforwardly. But if the observation is a nonlinear function of $f(t)$ such that

$$g(t) = S(f(t)) + v(t) \quad (6)$$

then it is needed to modify the estimator, where g and v are M dimensional vectors and $S(\cdot)$ is a nonlinear operator. $v(t)$ is a observation noise with zero mean and covatiance given by

$$Ev(t_1)v'(t_2) = L\delta(t_2 - t_1), \quad (7)$$

where L is an $M \times M$ matrix.

Expanding the first term of the right hand side of Eq.(6) in the vicinity of $f^*(t)$ called a nominal solution which is in the neighborhood of the true value $f(t)$ and dropping the nonlinear terms (i. e., terms of higher order than the first) yeilds

$$S(f(t)) = S(f^*(t)) + S_B \cdot (f(t) - f^*(t)), \quad (8)$$

where

$$S_B = \frac{\partial S(f)}{\partial f} = \begin{bmatrix} \left.\frac{\partial S(f_1)}{\partial f_1}\right|_{f_1 = f_1^*} & & \phi \\ & \ddots & \\ \phi & & \left.\frac{\partial S(f_M)}{\partial f_M}\right|_{f_M = f_M^*} \end{bmatrix} \quad (9)$$

From Eq.(2) and Eq.(4),

$$f - \bar{f} = z(t) \equiv Cx(t) \quad (10)$$

hence

$$S(f(t)) = S(f^*(t)) + S_B(Cx(t) + \bar{f}(t) - f^*(t)). \quad (11)$$

Substituting Eq.(11) into Eq.(6), we obtain

$$g(t) = S(f^*(t)) + S_B(Cx(t) + \bar{f}(t) - f^*(t)) + v(t). \quad (12)$$

Here, if we define new variable $y(t)$ as

$$y(t) = g(t) - S(f^*(t)) - S_B(\bar{f}(t) - f^*(t)), \quad (13)$$

then Eq.(12) becomes

$$y(t) = S_B Cx(t) + v(t) \quad (14)$$

Finally from Eq.(4) and Eq.(14) we get desired result;

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = S_B \cdot Cx(t) + v(t). \end{cases} \quad (15)$$

We want a discrete version of Eq.(15) suitable for digital computer processing.

It is easily obtained[1] and given by

$$\begin{aligned} x(k+1) &= A_1 x(k) + B_1 u(k) \\ y(k) &= S_B \cdot C_1 x(k) + v(k), \end{aligned} \quad (16)$$

where

$$A_1 = e^A = e^{-\mu_h}$$
$$B_1 = \sqrt{1 - e^{-2\mu_h}} \quad (* \text{see Appendix 1.}$$
$$S_B = \begin{bmatrix} \left.\frac{\partial S(f_1)}{\partial f_1}\right|_{f_1(k) = f_1^*(k)} & & \phi \\ & \ddots & \\ \phi & & \left.\frac{\partial S(f_M)}{\partial f_M}\right|_{f_M(k) = f_M^*(k)} \end{bmatrix},$$
$$C_1 = C = T\Lambda^{1/2}T'$$

where $\Lambda^{1/2}$ is a diagonal matrix whose entries are the square roots of the eigen values of H. T is the orthogonal matrix whose colums are the eigen vectros of H[3]. x, y, u and v are M dimensional vectors and A_1, B_1 are scalars, C_1 and S_B are $M \times M$ matrices.

The covariance of $u(k)$ and $v(k)$ are

$$\begin{aligned} Eu(k)u'(\ell) &= K \cdot I \cdot \Delta(\ell - k) \\ Ev(k)v'(\ell) &= L \cdot \Delta(\ell - k) \end{aligned} \quad (17)$$

The estimate of x is obtained directly by using Kalman filter[1]. S_B is a function of k and hence system is time variant;

$$\begin{aligned} \hat{x}(k+1) &= [A_1 I - F(k) S_B C_1]\hat{x}(k) + F(k)y(k) \\ F(k) &= A_1 P(k) C_1' S_B' [S_B C_1 P(k) C_1' S_B' + L]^{-1} \\ P(k+1) &= [A_1 \cdot I - F(k) S_B C_1] P(k) [A_1 \cdot I - F(k) S_B C_1]' + K B_1^2 \cdot I + F(k) L F'(k). \end{aligned} \quad (18)$$

If we decide to choose $f^*(k) = \hat{f}(k)$, then from Eq.(10), $\hat{f}(k+1)$ is gotten directly;

$$\hat{f}(k+1) = \bar{f}(k+1) + A_1(\hat{f}(k) - \bar{f}(k)) + C_1 F(k)(g - S(\hat{f}(k))) \quad (* \text{ see Appendix 2}) \quad (19)$$

$P(0)$ and $\hat{f}(0)$ are given as an initial condition.

3. DERIVATION OF THE AUTOCOVARIANCE OF AN IDEAL IMAGE

In general the parameter μ_h of the autocovariance in Eq. (1) is considered different for each image, hence it is necessary to find it out by some way.

It seems easy to get the parameter μ_h when the observation is a linear function of an ideal image[2,(3]. But in the nonlinear case the same method as a linear one can not be utilized.

In this section we consider the method of finding out the parameter μ_h in the nonlinear case.

Since the autocovariance of an ideal image is generally supposed to be separable, each parameter for a horizontal or a vertical direction can be derived independently. In the following we suppose that the form of a autocovariance for a vertical direction is also exponential like a horizontal one.

The autocovariance of an ideal image for a horizontal direction is given by

$$E[f_k(t_1)-\bar{f}_k(t_1)][f_k(t_2)-\bar{f}_k(t_2)] = R_{f_k}(\tau)_{\tau=t_2-t_1}, \tag{20}$$

where $f_k(t)$ is a scalar scanner output of kth line at time t. $\bar{f}_k(t)$ is an ensemble average of $f_k(t)$.

The autocovariance of $S(f_k(t))$ is not a function of $\tau(=t_2-t_1)$ but of t_1 and t_2 generally. However as long as S is not an extremely nonlinear function, it will be supposed that the autocovariance of $S(f_k(t))$ is a function of τ approximately. Hence we introduce $R_S(\tau)_{\tau=t_2-t_1}$ as the autocovariance of $S(f_k(t))$. That is

$$R_S(\tau)_{\tau=t_2-t_1} = E[S(f_k(t_1))-\bar{S}(f_k(t_1))][S(f_k(t_2))-\bar{S}(f_k(t_2))], \tag{21}$$

where

$$\bar{S}(f_k(t)) = ES(f_k(t)) = \int_{-\infty}^{\infty} S(f_k(t))\cdot P(f_k(t))\,df_k(t). \tag{22}$$

Expanding $S(f_k(t))$ in the vicinity of $\bar{f}_k(t)$ (nonstationary mean) and dropping off the higher order than second term, we get

$$S(f_k(t)) = S(\bar{f}_k(t))+S'(\bar{f}_k(t))(f_k(t)-\bar{f}_k(t)), \tag{23}$$

where

$$S'(\bar{f}_k(t)) = \frac{\partial S(f_k(t))}{\partial f_k(t)}\bigg|_{f_k(t)=\bar{f}_k(t)}.$$

Substituting Eq.(23) into Eq.(22), we obtain

$$\bar{S}(f_k(t)) = \int_{-\infty}^{\infty}\{S(\bar{f}_k(t))+S'(\bar{f}_k(t))\cdot(f_k(t)-\bar{f}_k(t))\}P(f_k(t))\,df_k(t) = S(\bar{f}_k(t)), \tag{24}$$

because the second term of the integrand is zero after integration from the definition of $\bar{f}_k(t)$.

Substituting this into Eq.(21) and using Eq.(23),

$$R_S(\tau)_{\tau=t_2-t_1} = E[S(f_k(t_1))-S(\bar{f}_k(t_1))][S(f_k(t_2))-S(\bar{f}_k(t_2))] = S'(\bar{f}_k(t_1))S'(\bar{f}_k(t_2))\cdot E[f_k(t_1)-\bar{f}_k(t_1)][f_k(t_2)-\bar{f}_k(t_2)]. \tag{25}$$

From Eq.(20),

$$R_S(\tau) = S'(\bar{f}_k(t_1))S'(\bar{f}_k(t_2))R_{f_k}(\tau)\,|_{\tau=t_2-t_1}. \tag{26}$$

Hence,

$$R_{f_k}(\tau) = [S'(\bar{f}_k(t_1))\cdot S'(\bar{f}_k(t_2))]^{-1}\cdot R_S(\tau). \tag{27}$$

Since by definition $R_{f_k}(\tau)$ is a function of τ, the term $S'(\bar{f}_k(t_1))\cdot S'(\bar{f}_k(t_2))$ must also be a function of τ. It is reasonable to think so because $S'(\bar{f}_k(t))$ is a deterministic value and a first derivative of $S(\bar{f}_k(t))$ and S is supposed not to be an extremely nonlinear function. Hence we introduce a time average for these products and denote it $R_{S'}(\tau)$.

$$R_{S'}(\tau)_{\tau=t_2-t_1} = S'(\bar{f}_k(t_1))\cdot S'(\bar{f}_k(t_2)), \tag{28}$$

where

$$R_{S'}(\tau) = \frac{1}{T-|\tau|}\int_0^{T-\tau}\frac{1}{N}\sum_{k=1}^{N} S'(\bar{f}_k(t_1))\cdot S'(\bar{f}_k(t_1+\tau))\,dt_1. \tag{29}$$

T is a traveling time of scanner spot for horizontal direction.

Consequently Eq.(27) becomes

$$R_{f_k}(\tau) = R_{S'}^{-1}(\tau)\cdot R_S(\tau) \tag{30}$$

Now we consider the relation between $R_S(\tau)$ and $R_g(\tau)$ which is defined as

$$R_g(\tau) = E[g_k(t_1)-\bar{S}(f_k(t_1))][g_k(t_2)-\bar{S}(f_k(t_2))]. \tag{31}$$

Since

$$g_k(t) = S(f_k(t))+v_k(t),$$

We have

$$R_g(\tau) = E[S(f_k(t_1))-\bar{S}(f_k(t_1))+v_k(t_1)]\cdot[S(f_k(t_2))-\bar{S}(f_k(t_2))+v_k(t_2)] = E[S(f_k(t_1))-\bar{S}(f_k(t_1))]\cdot[S(f_k(t_2))-\bar{S}(f_k(t_2))]+Ev_k(t_1)v_k(t_2) = R_S(\tau)+L\delta(\tau)\,|_{\tau=t_2-t_1},$$

because $v_k(t)$ is a white noise with zero mean and the autocovariance $L\delta(\tau)$, where L is a scalar constant.

Hence for $\tau \neq 0$,

$$R_g(\tau) = R_s(\tau). \tag{32}$$

Substituting this into Eq.(30), we obtain

$$R_{f_k}(\tau) = R_{s'}^{-1}(\tau) \cdot R_g(\tau) \qquad \text{for } \tau \neq 0. \tag{33}$$

Here it is also natulal that we use a time average for $R_g(\tau)$.

$$R_g(\tau) = \frac{1}{T-|\tau|}\int_{-\infty}^{\infty}\frac{1}{N}\sum_{k=1}^{N}[g_k(t_1)-\bar{S}(f_k(t_1))] \cdot [g_k(t_1+\tau)-\bar{S}(f_k(t_1+\tau))]dt_1. \tag{34}$$

In discrete form, it is written as

$$R_g(\ell) = \frac{1}{N(N-\ell)}\sum_{k_1=1}^{N-\ell}\sum_{k_2=1}^{N}[g(k_1, k_2) - \bar{S}(f(k_1, k_2))][g(k_1+\ell, k_2) - \bar{S}(f(k_1+\ell, k_2))]\ \ell=1, 2, \cdots N. \tag{35}$$

Similarly $R_{s'}(\tau)$ is also written as

$$R_{s'}(\ell) = \frac{1}{N(N-\ell)}\sum_{k_1=1}^{N-\ell}\sum_{k_2=1}^{N}S'(\bar{f}(k_1, k_2)) \cdot S'(\bar{f}(k_1+\ell, k_2)),\ \ell=0, 1, 2, \cdots N \tag{36}$$

Since it is supposed that

$$R_{f_k}(\tau) = K \cdot e^{-\mu_h|\tau|}, \tag{37}$$

we obtain

$$\frac{R_{f_k}(2)}{R_{f_k}(1)} = \sqrt{\frac{|R_{f_k}(2)\cdot R_{f_k}(4)|}{|R_{f_k}(1)\cdot R_{f_k}(3)|}} = e^{-\mu_h}.$$

Consequently we get the desired result;

$$\left.\begin{aligned} \mu_h &= -\ell_n\sqrt{\frac{|R_{f_k}(2)\cdot R_{f_k}(4)|}{|R_{f_k}(1)\cdot R_{f_k}(3)|}} \\ &\text{and} \\ K &= \frac{|R_{f_k}^2(1)|}{|R_{f_k}(2)|} = \sqrt{\frac{|R_{f_k}^2(1)\cdot R_{f_k}(2)|}{|R_{f_k}(4)|}} \end{aligned}\right\} \tag{38}$$

In the above equations, to determine the paramaters μ_h and K, $R_{f_k}(1)$, $R_{f_k}(2)$, $R_{f_k}(3)$ and $R_{f_k}(4)$ are used rather than $R_{f_k}(1)$ and $R_{f_k}(2)$, because of a smoothness of this covariance. These values are obtained through Eq.(33).

The parameter of a vertical direction is also gotten in similar manner.

4. COMPUTATIONAL PROCEDURE

From the above we can estimate f when an ensemble mean $\bar{f}$ is known. Generally there are few cases that $\bar{f}$ is known apriori when the observation includes a nonlinear function. Hence it is needed to derive it by some means. Here we derive it by the simplest method as follows.

First, the observation g is processed through a lowpass filter with a gaussian-like impulse response characteristic, so as to surpress the observation noise v.

Tnen the result is operated by a nonlinear operator $H^{-1}\cdot S^{-1}$. In this case we use a pseudo-inverse for H^{-1} in order to get a smoothed image.

It will be sufficient that we take this result as $\bar{f}$ in practice.

An computational procedure is as follows.

(1) Choose $\bar{f}$ (e. g. using the above method.)
(2) Make the initial conditions $f(0)$ and $P(0)$ properly from the observation.
(3) Determine the covariance parameter such as μ_h by the method in section 3.
(4) Compute $S_{Bf=f(0)}$ given by Eq.(9).
(5) Using calculated values A_1, B_1 and C_1, $\hat{f}(1)$ is estimated along with Eq.s(18) and (19).
(6) Compute $S_{Bf=f(1)}$ like (4) and repeat (5) with $\hat{f}(2)$ instead of $\hat{f}(1)$. Similarly $\hat{f}(3)$, $\hat{f}(4)$ $\cdots\cdot$ $\hat{f}(k)\cdots\cdot$ are obtained by repeating (5) and (6).

5. CONCLUSIONS

The merit of using a nonstationary mean for an image restorotion is as follows.

(1) We see that in a stationary model, all the variability in the images must be conveyed by the covariance properties, since the ensemble mean has been fixed as a constant. This will make a poor result in modeling a class of images f. On the other hand, if the nonstationary mean is supposed, the covariance properties of the ensemble would represent the random perturbations about this structured mean of each individual image in the ensemble.

This property will make a good result in the modeling. Consequently the restored image will be better than that with the stationary mean.

(2) In the nonlinear case, though it is difficult to find the parameters of the covariance of f, it is easy to find them out approximately by assuming a nonstationary mean.

(3) It is sufficiently enough to use an exponential function as a first order approximation for the shape of the covariance with nonstationary mean.

(4) We can easily derive the nonstationary mean from the observation.

From the above reason, assuming a model which consists of a nonstationary mean will be helpful for an image restoration processing.

Finally we remark that applying kalman-filter to the restoration problem yeilds a computational efficiency.

APPENDIX

(1) From the transformation formula[1],

$$EB(k)u(k)u'(k)B'(k) = \int_0^1 e^{-\mu_k(1-s)}\cdot BK \cdot IB'e^{-\mu_k(1-s)}ds,$$

where $B(k) = \sqrt{2\mu_k}$ and I is an identity matrix,

$$= 2K\mu_k\cdot I\int_0^1 e^{-2\mu_k(1-s)}ds$$
$$= \sqrt{1-e^{-2\mu_k}}\cdot K\cdot I\cdot\sqrt{1-e^{-2\mu_k}}. \quad \text{(A-1)}$$

Hence

$$\begin{cases} B = \sqrt{1-e^{-2\mu_k}} \\ Eu(k)u'(\ell) = K\cdot I\cdot\Delta(\ell-k). \end{cases} \quad \text{(A-2)}$$

(2) Since

$$\bar{x}(k+1) = [A_1\cdot I - F(k)S_B C_1]\bar{x}(k) + F(k)y(k), \quad \text{(A-3)}$$

multiplying both sides by matrix C_1 from the left hand side,

$$C_1\bar{x}(k+1) = [A_1C_1\cdot I - C_1F(k)S_BC_1]\bar{x}(k) + F(k)y(k)$$
$$= [A_1 - C_1F(k)S_B]C_1\bar{x}(k) + C_1F(k)y(k). \quad \text{(A-4)}$$

Since

$$\hat{f}(k+1) - \bar{f}(k+1) = C_1\bar{x}(k+1) \quad \text{(A-5)}$$

and

$$y(k) = g(k) - S(f^*(k)) - S_B(\bar{f}(k) - f^*(k)), \quad \text{(A-6)}$$

Eq.(A-4) becomes

$$\hat{f}(k+1) - \bar{f}(k+1)$$
$$= [A_1 - C_1F(k)S_B][\hat{f}(k) - \bar{f}(k)] + C_1F(k)[g(k) - S(f^*(k)) - S_B(\hat{f}(k) - f^*(k))]$$
$$= A_1[\hat{f}(k) - \bar{f}(k)] - C_1F(k)S_B[\hat{f}(k) - \bar{f}(k)] + C_1F(k)S_B[f^*(k) - \bar{f}(k)] + C_1F(k)[g(k) - S(f^*(k))]. \quad \text{(A-7)}$$

If we choose $f^*(k) = \bar{f}(k)$, then we get the final result;

$$\hat{f}(k+1) = \bar{f}(k+1) + A_1[\hat{f}(k) - \bar{f}(k)] + C_1F(k)[g(k) - S(f^*(k))]. \quad \text{(A-8)}$$

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