Analysis of Suitable Postures for Robot Manipulator Applying Force using Numerical Optimization Method

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Abstract – **This paper discusses suitable postures for a robot manipulator applying a force. In order to find out the suitable postures, a performance index to estimate the suitableness of a posture of a robot manipulator is defined, and then a constrained optimization problem is formulated; the performance index is minimized under a geometric condition. The suitable postures can be obtained by solving the constrained optimization problem numerically. Two case studies – obtaining and examining suitable postures for pushing and pulling robots – are conducted. The findings in the case studies reveal that the suitable postures can be classified into two types and the robot manipulators choose one of them depending on the magnitude of the effort force. A force control algorithm based on the results of the numerical analysis provides a capability to generate a large effort force for a robot manipulator. Moreover, by adoption of the force control algorithm, reduction of energy loss is expected.**

I. INTRODUCTION

In recent times, robots have become indispensable to the industrial world. Many robots have been employed to release workers from difficult, repetitive and dangerous labor, and to increase efficiency. For example, robot manipulators working for automobile companies have dramatically increase productivity.

Functions that a robot manipulator should be provided with are wide-ranging because of the diversity of the demanded tasks, and they are roughly categorized into two functions: positioning the end-effector and applying an instructed force to an object.

The authors are focusing on force control of a robot manipulator, and the aim of the study is to develop a control system that considers the suitableness of a work posture of a robot manipulator. As the first step in the development of the force control system, suitable postures for a robot manipulator applying a force are obtained and examined using a numerical optimization method.

Since an operational capability of a manipulator depends on its posture, the capability must be analyzed sufficiently before starting up the robot manipulator. To analyze a capability of a manipulator, it is necessary to choose an appropriate measure that evaluates the capability for executing a given task. For example, Asada extended the inertia ellipsoid, which is used to represent dynamic characteristics of a single rigid body, to a general ellipsoid for a series of rigid bodies in order to evaluate the manipulator dynamics[1]. Yoshikawa proposed a quantitative measure called manipulability[2]. The manipulability is used to examine the easiness of arbitrarily changing the position and orientation of the end-effector of a manipulator. Dynamic manipulability is also introduced by Yoshikawa[3]. Dynamic manipulability takes into account the manipulator dynamics that is completely ignored in the manipulability. Moreover, the concept of the manipulability was applied to several robot systems (e.g. [4, 5]).

Many other measures to evaluate a capability of a manipulator have been developed[6-12], but measures that satisfy requirements for the study discussed in this paper have not been found. The capability of generating a large force and the resulting energy loss must be evaluated. Therefore, the squared norm of the normalized joint torque is adopted as a performance index to measure the suitableness of a posture of a robot manipulator applying a force, and then a constrained optimization problem is formulated to find out suitable postures numerically.

The organization of this paper is as follows. In Section II, a performance index to estimate the suitableness of a posture of a manipulator is introduced, and a constrained optimization problem to find out suitable postures of a manipulator is formulated. In Section III & IV, the results of two case studies conducted to confirm the usefulness of the proposed method are reported. In Section V, this paper is concluded.

II. FORMULATION OF OPTIMIZATION PROBLEM

In this section, a suitable posture of a robot manipulator is discussed and a constrained optimization problem for obtaining the suitable posture is formulated.

A. Model of Manipulator Applying Force

Fig. 1 shows a model of an n DOF manipulator applying a force $\boldsymbol{f}$ to an object. The joint variables of the manipulator are denoted by q_i (i=1,2,…,n), and the joint vector is $\boldsymbol{q} = \left[q_1, \quad q_2, \quad \ldots, \quad q_n\right]^T$. The dynamics equation of the manipulator is described by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f} = \boldsymbol{\tau}, \qquad (1)$$

where $\boldsymbol{M}(\boldsymbol{q}) \in \boldsymbol{R}^{n\times n}$ is the matrix of inertia, $\boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \boldsymbol{R}^n$ is the centrifugal and Coriolis forces, $\boldsymbol{g}(\boldsymbol{q}) \in \boldsymbol{R}^n$ represents the gravity effect, $\boldsymbol{J}(\boldsymbol{q}) \in \boldsymbol{R}^{3\times n}$ is the Jacobian matrix, τ_i denotes the joint driving torque of the i-th joint, and $\boldsymbol{\tau} = \left[\tau_1, \quad \tau_2, \quad \ldots, \quad \tau_n\right]^T$. The vector $\boldsymbol{p}_e(\boldsymbol{q})$ is the position

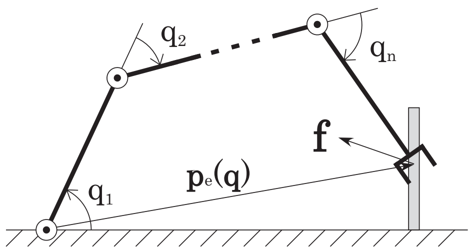

Fig. 1. Model of Manipulator

vector of the manipulator's end-effector.

It is assumed in this paper that the manipulator is stationary while it is applying a force to an object. Therefore, $\dot{\boldsymbol{q}} = \boldsymbol{0}$, $\ddot{\boldsymbol{q}} = \boldsymbol{0}$, and Equation (1) becomes

$$\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f} = \boldsymbol{\tau} \;. \tag{2}$$

It is realized from (2) that the joint driving torques consist of the torques for compensating the gravity effect and for generating the force at the end-effector in the situation. The analyses in this study are conducted based on this statics equation.

B. Suitable Postures for Manipulator Applying Force

The research purpose of this study is to obtain and examine suitable postures for a manipulator applying a force. Although "What does the suitable posture mean?" is a complicated problem, the following two items are listed here as essential qualifications for suitable postures of a manipulator applying a force:

- Capable of generating as large a force as possible
- Energy-saving

The reason for the first qualification is that a manipulator is frequently required to perform a difficult task, such as handling a heavy load. The reason for the second qualification is that exchanging and recharging batteries are time-consuming jobs and they make operational efficiency worse, especially for robots working outside.

A posture that fulfills the first qualification can be obtained by finding a posture that minimizes the joint driving torque when a desired effort force is given. As shown in (2), the joint driving torque to generate the desired effort force at the end-effector is proportional to the effort force through the Jacobian matrix, in which the elements depends on the posture of the manipulator. Therefore, if one can find a posture that minimizes the joint driving torque, the posture satisfies the first qualification.

As for energy-saving, it is difficult to accurately estimate the amount of actual energy consumption in a manipulator, because it depends on a variety of conditions: what kinds of motors are installed, how these motors are driven, the drive mechanisms and devices, and so on. Since it is unadvisable to fix a certain condition beforehand with respect to the mechanism and drive systems of the manipulator in this paper, we decide to ignore them and define a suitable posture as a posture that minimizes the joint driving torque (2), which is the output of the drive system. Minimizing the joint driving torque is expected to reduce energy loss.

In order to formulate a constrained optimization problem for finding out such a suitable posture, the squared norm of the normalized joint torque,

$$PI = \|\tilde{\boldsymbol{\tau}}\|^2 , \tag{3}$$

is adopted as a performance index to evaluate postures of a manipulator, where τ_{ri} is the rated torque of the motor installed in the i-th joint, $\tilde{\boldsymbol{\tau}}$ represents the joint torque vector normalized with the rated torques, and

$$\tilde{\boldsymbol{\tau}} = \boldsymbol{R}^{-1}\boldsymbol{\tau} , \tag{4}$$
$$\boldsymbol{R} = diag(\tau_{r1}, \tau_{r2}, \ldots, \tau_{rn}) \;. \tag{5}$$

From (3), (4) and (5), the performance index PI can be rewritten as follows:

$$PI = (\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f})^T \boldsymbol{W}(\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f}) , \tag{6}$$

where

$$\boldsymbol{W} = \boldsymbol{R}^{-T}\boldsymbol{R}^{-1} = diag(1/\tau_{r1}^2, 1/\tau_{r2}^2, \ldots, 1/\tau_{rn}^2) \;. \tag{7}$$

When the manipulator takes a posture that minimizes the performance index PI, the joint torques will be small and the posture must be suitable for a manipulator applying a force.

Notice that the manipulator is possibly unable to realize its maximum capability if the unnormalized torque vector is adopted as a performance index. This issue is discussed in Section IV.

C. Constrained Optimization Problem

As mentioned above, the postures that minimize the performance index PI are considered to be the suitable ones for a manipulator applying a force. Therefore, if the following constrained optimization problem is solved, the suitable posture is obtained.

Minimize:

$$PI = (\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f}_d)^T \boldsymbol{W}(\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f}_d) \tag{8}$$

Subject to:

$$\boldsymbol{p}_e(\boldsymbol{q}) - \boldsymbol{p}_a = \boldsymbol{0} \tag{9}$$

The vector $\boldsymbol{f}_d$ represents the desired effort force at the end-effector. Equation (9) is a geometrical constraint for that the manipulator's end-effector applies an effort force to a desired place, where the vector $\boldsymbol{p}_a$ is the position vector of the desired place. This constrained optimization problem is numerically solved by means of Lagrange's method and Newton's method.

III. CASE STUDY I: PUSHING ROBOT

In this section, suitable postures for a robot manipulator pushing an object are calculated and examined by means of the method described in the previous section.

Fig. 2 shows a model of the pushing robot. The robot has a 3-link manipulator, which is installed on a vehicle, and the assigned task is to push an object. It is assumed that the vehicle has the capability to make itself stationary and the object is fixed on the ground while the manipulator is applying a force. In order to simplify this case study, the vehicle does not actively assist the manipulator in doing the pushing task. The direction of the pushing force is horizontal and its magnitude is $f(N)$. The physical parameters of the manipulator are listed in TABLE I. The mass density of the links is uniform, so the distance from the proximal joint to the center of mass of the link is half of the length, $0.1m$. The origin of a coordinate system is placed at the position of the first joint of the manipulator.

The constrained optimization problem to obtain the suitable postures of the pushing robot is solved with the condition that the manipulator exerts an effort force at the places indicated by the cross markers in Fig. 3. Fig. 4 shows values of the performance index for the pushing robot when the effort force f is $1N$, $10N$, $30N$ and $50N$. The X and Y axes indicate the position of the end-effector of the manipulator.

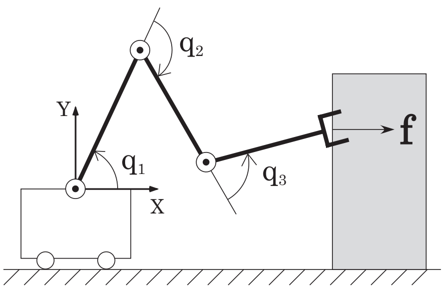

Fig. 2. Model of Pushing Robot

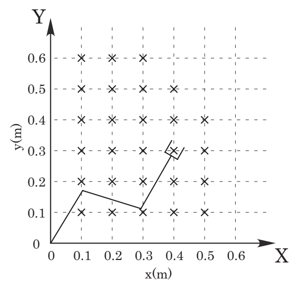

Fig. 3. Points of Application of Force

TABLE I
PHYSICAL PARAMETERS OF PUSHING ROBOT MANIPULATOR

	Link1	Link2	Link3
Length (m)	0.2	0.2	0.2
Mass (kg)	2.0	2.0	2.0

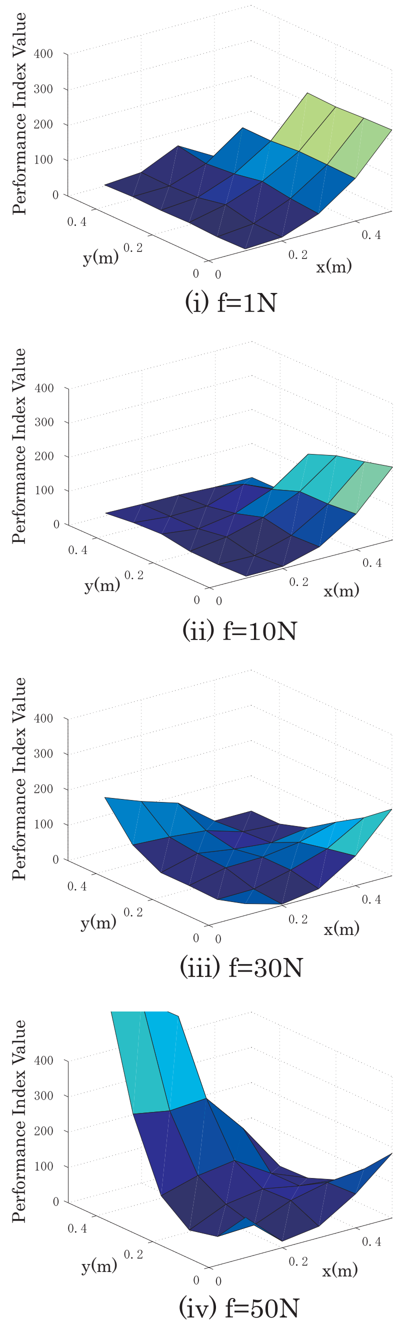

Fig. 4. Performance Index Value for Pushing Robot

As seen from the top graph in Fig. 4, the case of $f = 1N$, the pushing robot should approach the object as close as possible if the pushing force is small. On the other hand, if the pushing force is large, it is necessary to pay more attention to the performance index values. Fig. 4 shows that an increase of the pushing force does not always lead to deterioration of the performance index. For example, Fig. 5 indicates the performance index values for the manipulator pushing the object at (0.5, 0.2). In this case, the performance index values draw a downward convex curve, which has the minimum value when the manipulator pushes the object at about $60N$.

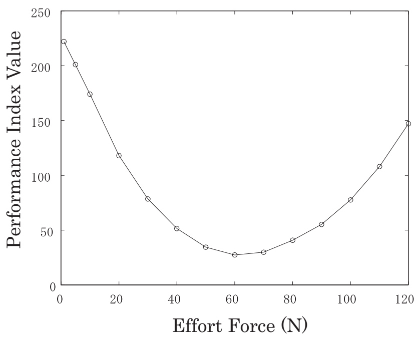

Fig. 5. Performance Index Value for Pushing Robot (p_e = (0.5, 0.2))

This effect is highly related to balance between the gravity effect torques and the torques to generate an effort force at the end-effector.

Fig. 6 and Fig.7 show the needed joint torques in the suitable postures for the effort force = 20*N* and 60*N*, respectively. The symbols in the figures are defined as $\boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f} = [J^Tf_1, J^Tf_2, J^Tf_3]^T$, $\boldsymbol{g}(\boldsymbol{q}) = [g_1, g_2, g_3]^T$, $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$. The J^Tf_1 increases for generating the larger effort force 60*N* at the end-effector, but the first joint driving torque τ_1 decreases because the torques J^Tf_1 and g_1 get balanced out considerably. As a result, when the effort force is 60*N*, the performance index becomes less despite the larger effort force.

Fig. 8 shows the suitable postures of the pushing robot manipulator on condition that the position of the end-effector is (0.5, 0.2) and the pushing forces are 1*N*, 20*N*, 40*N*, 60*N*, 80*N* and 100*N*. It is found in the figures that the manipulator changes the posture in response to the magnitude of the pushing force, and the postures can be classified into two types. We call the postures elbow-up type and elbow-down type respectively, as shown in Fig. 9. The manipulator type depending on the magnitude of the effort force. By examining the transition of the manipulator suitable posture thoroughly, it is clarified that the manipulator changes its posture discontinuously. Although it is significant to find out the points where a manipulator switches the posture to reduce the performance index value, at the moment we are unable to show how the switching points are found out without further numerical analysis.

Fig. 10, Fig. 11 and Fig. 12 are graphs that show the first, second and third joint torque squared, respectively; the pushing robot manipulator takes the suitable postures, or the elbow-up and elbow-down posture types are maintained. Both postures are solutions of the constrained optimization problem, but one of them is not optimal. As seen from these graphs, in the case discussed in this section, the manipulator posture is changed depending on the magnitude of the needed second and third joint torques in order to reduce the performance index value.

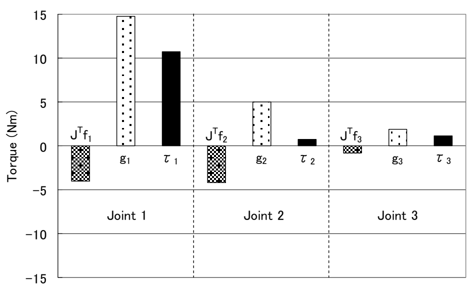

Fig. 6. Joint Torques (Effort Force = 20N)

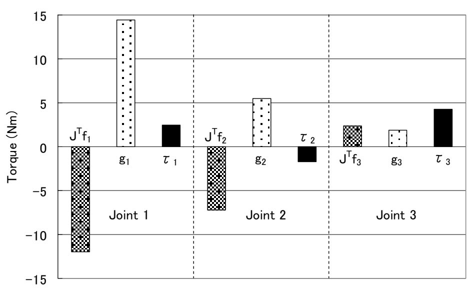

Fig. 7. Joint Torques (Effort Force = 60N)

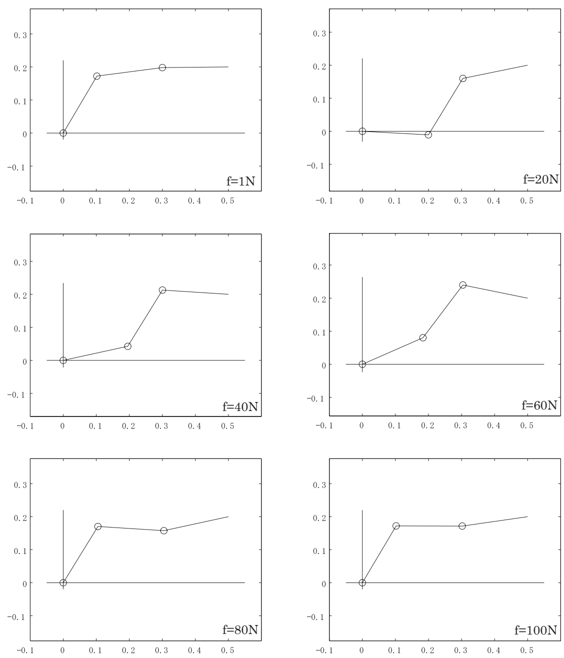

Fig. 8. Manipulator Postures: Point of Application of Force = (0.5, 0.2)

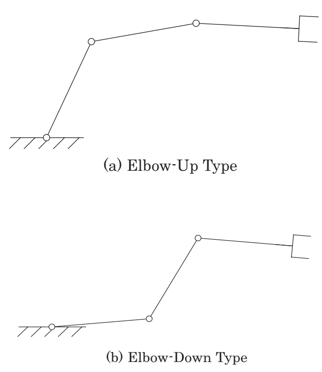

Fig. 9. Suitable Posture Types for Pushing Robot

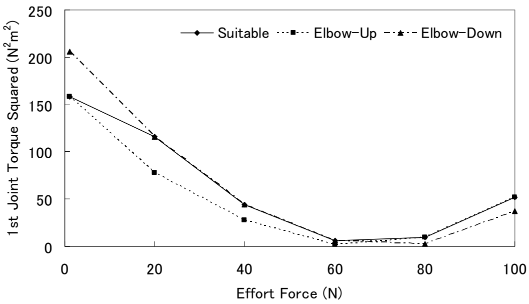

Fig. 10. First Joint Torque Squared

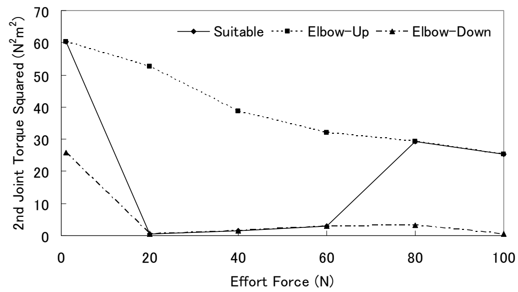

Fig. 11. Second Joint Torque Squared

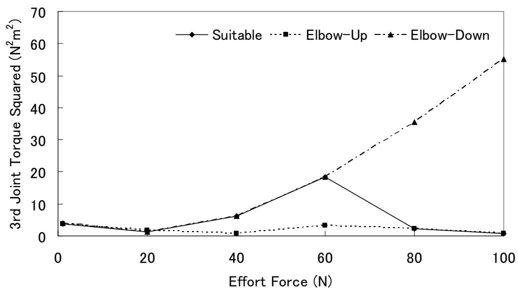

Fig. 12. Third Joint Torque Squared

IV. CASE STUDY II: PULLING ROBOT - WEEDING ROBOT -

A weeding robot is being developed in our laboratory[13]. The weeding robot does not cut weeds away, but pulls weeds out by the roots. Naturally, if the robot working outside performs weeding in an energy-conserving way, the operational efficiency will certainly be improved. Moreover, the weeding robot is frequently required to generate a large force to pluck out weeds, because some weeds have roots that spread deeply and tightly. In this section, we derive suitable postures for the weeding robot that meet the demands by means of the same method used for the pushing robot.

Fig. 13 depicts a model of the weeding robot. As with the pushing robot discussed in the previous section, the weeding robot has a 3-link manipulator on a vehicle; the vehicle has the capability to make itself stationary on the ground while the manipulator is weeding; the manipulator pulls weeds out vertically. In the case of exerting a force vertically, it may be difficult to efficiently utilize the gravity effect like the pushing robot. The physical parameters of the manipulator shown in TABLE II are selected based on the weeding robot under development in our laboratory.

First, suitable postures are calculated without the knowledge about the rated torques of the motors installed in the weeding manipulator, i.e. $\boldsymbol{W} = diag(1, 1, 1)$. Fig. 14 shows the suitable postures obtained. The distance from the first joint to the weed is 0.4*m*. The magnitude of the pulling force is varied from 1*N* to 30*N*. It can be seen from the figure that the suitable postures change slightly in response to the magnitude of the pulling force.

TABLE III shows the rated torques of the motors and the joint torques necessary to generate the pulling force 30*N* for the weeding robot in the calculated suitable posture. The manufacturer of the manipulator guarantees that the manipulator can generate 30*N* force at the end-effector. According to these figures, however, the necessary torque for the third joint torque is larger than the rated torque. This result indicates that the manipulator's best ability cannot be utilized in the calculated posture.

Next, suitable postures for the pulling robot are recalculated in consideration of the rated torques, i.e. $\boldsymbol{W} = diag(1/60^2, 1/30^2, 1/8^2)$. TABLE IV shows the necessary joint torques in the recalculated suitable posture. It is clear that the third joint torque is diminished and the problem stated above is solved.

Fig. 15 shows the derived suitable postures; the posture is drastically transformed as the desired effort force is changed

TABLE II
PHYSICAL PARAMETERS OF WEEDING ROBOT MANIPULATOR

	Link1	Link2	Link3
Length (m)	0.260	0.270	0.300
Mass (kg)	7.0	7.5	2.0

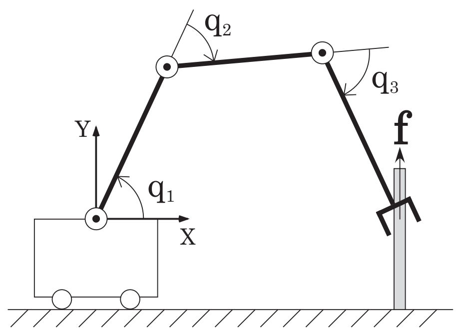

Fig. 13. Model of Pulling Robot

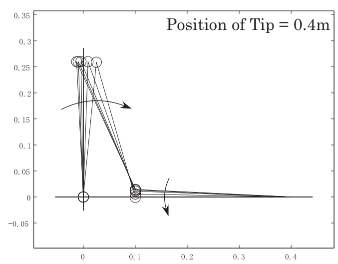

Fig. 14. Transition of Suitable Postures for Pulling Robot
(f = 1, 5, 10, 20, 30*N*; $\boldsymbol{W} = diag(1, 1, 1)$)

TABLE III
JOINT TORQUES (f = 30N; $\boldsymbol{W}$ = $diag$(1, 1, 1)):
The rated torques are not taken into consideration.

	Link1	Link2	Link3
Rated Torque (*Nm*)	60	30	8
Joint Torque (*Nm*)	22.36	18.41	11.94

TABLE IV
JOINT TORQUES (f = 30N; $\boldsymbol{W}$=$diag$($1/60^2$, $1/30^2$, $1/8^2$)):
The rated torques are taken into consideration.

	Link1	Link2	Link3
Rated Torque (*Nm*)	60	30	8
Joint Torque (*Nm*)	50.53	10.19	1.896

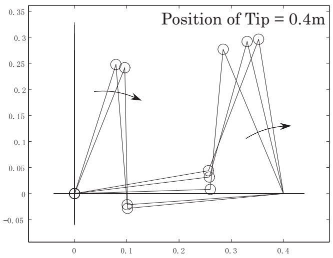

Fig. 15. Transition of Suitable Postures for Pulling Robot
(f = 1, 5, 10, 20, 30N; $\boldsymbol{W}$=$diag$($1/60^2$, $1/30^2$, $1/8^2$))

between 5N and 10N. This movement – swaying the third link up in order to approach the third link to the line of the effort force – makes the third joint necessary torque small. In the case of the relatively small effort forces, the weeding robot folds up the first and second links in order to draw the center of gravity aside so that the torques for compensating the gravity effect become small. In consequences, as with the pushing robot, the weeding robot has two kinds of suitable postures, and the posture of the manipulator is switched to one of the suitable postures depending on the magnitude of the pulling force in order to reduce the performance index value.

V. CONCLUSIONS

In this paper, the suitable postures for the pushing and pulling robots were calculated and examined by means of solving the constrained optimization problem. Because this method is a numerical approach, if a computer has a large computational power, this method can be readily applied to higher DOF (hyper-redundant) manipulators and interesting results may be obtained.

It has been found from the results of the two case studies that the suitable postures can be classified into two types and both of the robots choose one of the suitable posture types depending on the magnitude of the effort force. Moreover, the suitable postures that take into consideration the robot manipulator’s capability could be obtained by the adoption of the squared norm of the joint torque normalized with the rated joint torques as the performance index. Since the suitable posture is not always energy-saving due to the adoption of the normalization, we will experimentally verify the issue related to energy consumption in the near future.

In the case of the pushing robot, the question of whether or not it is possible to effectively utilize the gravity effect for generating a force is very significant. Therefore, if the weeding robot pulls a weed out at a slant, it would be possible to improve the operation efficiency because the manipulator can utilize the gravity effect to generate the effort force. The analysis of this is also left as future work.

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