Theoretical Study on Vortex Glass-Liquid Transition in Pinned Superconductors

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Abstract—**The vortex glass-liquid transition in pinned superconductors is studied theoretically from the viewpoint of thermal depinning of flux lines. It is clarified that this depinning phenomenon is a transition of the second order. This result is consistent with the fact that the scaling of the current-voltage curves is well explained by the theoretical model of flux creep and flow. It is also found that the degree of disorder of the flux line lattice decreases abruptly with elevating temperature above the transition temperature. This agrees qualitatively with the observation of flux lines using a Lorentz microscope.**

I. Introduction

The vortex glass-liquid transition (hereafter we shall call G-L transition)[1] was proposed to explain the drastic change in the current-voltage curve in high-temperature superconductors in the vicinity of the irreversibility line. It is assumed in the theory that flux lines are in a glass state for the temperature below the transition temperature, T_g, resulting in an effectively pinned state. Above T_g, on the other hand, flux lines are in a liquid state and the flux pinning is no longer effective. This change in the state of flux lines is expected to be a transition of the second order. This expectation was proved by the scaling of current-voltage curves in the vicinity of T_g [2].

On the other hand, the relaxation of the persistent current carried by flux pinning in the irreversible regime is well explained by the flux creep model [3]. This model also explains the flux dynamics in the resistive state in the low electric field regime, while the flux creep state shifts to the flux flow state in the high electric field regime. As a result, the irreversibility line is successfully explained by the flux creep-flow model [4],[5]. Hence, this trial to explain the drastic change in the current-voltage curve in the vicinity of the irreversibility line has been done.

Not only the scaling of current-voltage curves but also other characteristic features such as the transition temperature (T_g), the static critical index (ν) and the dynamic critical index (z) are well explained by this theoretical model [6]-[9]. That is, when the temperature dependence of the critical current density is expressed as $J_c \propto (T_g - T)^{\delta'}$ (or $J_c \propto (B_g - B)^{\delta'}$ with B_g denoting the transition field), ν is given by $\delta'/2$. This means that the important correlation length which determines the state of flux lines is the pinning correlation length, $l = (Ca_f/2\pi BJ_c)^{1/2}$, where C is the elastic modulus of flux lines and a_f is the flux line spacing. z is deeply associated with the distribution of flux pinning strength [9],[10]. That is, z decreases according to broadening the distribution width. The above speculation is also supported by the fact that T_g depends on the flux pinning strength similarly to the irreversibility temperature, T_i [7],[11],[12]. The above results show that the flux pinning is the more fundamental mechanism to determine the behavior of flux lines than the intrinsic property of themselves. In other words, it is speculated that the glass state is realized when the flux pinning is superior than the thermal activation of flux lines. In the above the dimensionality of superconductor is reflected on the pinning property through the flux bundle size [4],[13],[14]. Therefore, it is expected that the thermal depinning itself is the transition of the second order.

In this paper the G-L transition is investigated using a statistical theory of flux pinning. The disorder of the flux line lattice is also argued. Although the inhomogeneous distribution of pinning strength is not considered for simplicity in the theory, its effect on the current-voltage curve is discussed later.

II. Theory

Although it is known that the Larkin-Ovchinnikov theory [15] describes the pinning phenomena generally, it contains quantitative problems. On the other hand, the coherent potential approximation theory [16], which is a kind of statistical theory, explains quantitatively well the linear summation of elementary pinning forces for strong pinning centers such as normal precipitates. In addition, this theory is suitable for a statistical average. Hence, this theory is used in this paper.

It is assumed in this theory that long-range order does

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not exist in the practical flux line lattice. This is a quite important point by Larkin and Ovchinnikov. However, such long-range order was assumed in the Labusch theory [17] and this assumption resulted in a disagreement in the threshold level of the pinning strength [18]. We watch one flux line located at x which interacts with a pinning center. The flux line lattice is distorted locally around this pinning center, while it is in addition distorted on a much longer scale of the order of correlation length. The latter semi-macroscopic distortion comes from the flux pinning interactions by surrounding pinning centers. It is important to note that these interactions break the long-range order in the flux line lattice. Elastic restoring forces work corresponding to these distortions. For simplicity, we assume that the distortion is small enough that linear summations hold for the distortion and the elastic force. Then, if we virtually "switch off" the pinning interaction on the watched flux line, it would move to some position, x'. It was predicted by the Labusch theory that the long-range order was attained by this virtual treatment. However, this is not correct and brings about the serious problem on the pinning threshold.

It may be allowed to assume that, if we virtually switch off all other pinning interactions, the long-range order appears in the flux line lattice. The position of the watched flux line in this case is represented by Δ. Then, the simplified one-dimensional force balance equation on the watched flux line is formally described as [16]

$$k_f'(\Delta - x) + f(x) = 0, \tag{1}$$

where the first and second terms are the elastic force and the pinning force on the flux line, respectively. In the above k_f' is the effective spring constant for the displacement of the flux line from the equilibrium position, Δ, to x. This situation is schematically illustrated in Fig. 1. As a result, the effective spring constant is given by

$$\frac{1}{k_f'} = \frac{1}{k_f} + \frac{1}{K}, \tag{2}$$

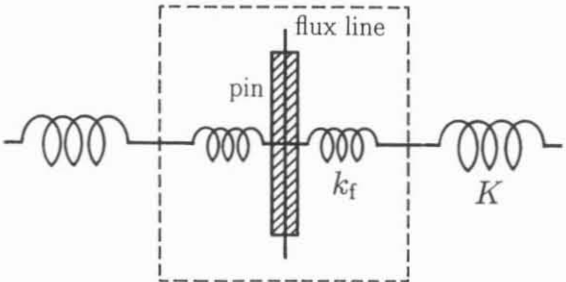

Fig. 1. Schematic illustration of the deformation of a flux line lattice. Restoring forces due to local and semimacroscopic deformations are represented using spring constants, k_f and K, respectively.

where $k_f = G^{-1}(0)$ is the spring constant for a local deformation of the flux line lattice derived by Labusch [17] and K is a spring constant for a semi-macroscopic displacement of flux lines representing the reaction from surrounding pinning centers. K is proportional to the Labusch parameter, α_L, which is used as a parameter representing the strength of the coherent pinning potential. In usual situations k_f is much larger than K, and hence, k_f' is dominantly determined by K. Since the threshold level, f_{pt}, is proportional to k_f', it is approximately proportional to the elementary pinning force, f_p, through α_L. In the creep-free case f_{pt} is always smaller than f_p and the threshold problem does not arise [16].

To consider the effect of thermal activation, we should treat the Langevin equation considering hopping flux motions under the thermal activation. However, since this is not easy, we will use an elegant method employed by Yamafuji *et al.* [19] that the effect of thermal activation can be approximately introduced by making the pinning potential shallow.

It rarely happens that, just at the instance when the watched flux line is thermally agitated, surrounding pinned flux lines are agitated simultaneously. This means that the watched flux line experiences the interactions from surrounding pins represented by K whenever it attempts to hop. Thus, f_{pt} is not influenced by the thermal activation. As a result, the effective elementary pinning force, f_p, decreases much faster than f_{pt} with increasing temperature. The flux pinning becomes ineffective at all at the transition temperature, T_g. Hence, it can be expected that f_p is reduced to f_{pt} at $T = T_g$. Thus, it is reasonable to assume as

$$f_p - f_{pt} \propto T_g - T \tag{3}$$

in the vicinity of T_g. In fact the scaling of current-voltage curves predicted by the vortex glass-liquid transition theory can be achieved also by the percolation flux flow model [20] which is based on the theoretical approach of [19].

Here we assume the periodic pinning force versus the position of flux line [21] shown in Fig. 2. This is described

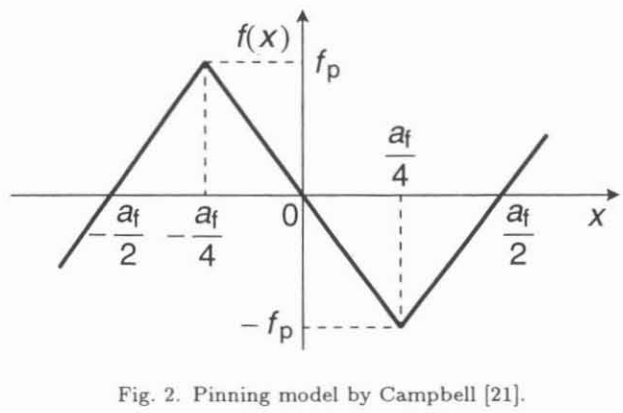

Fig. 2. Pinning model by Campbell [21].

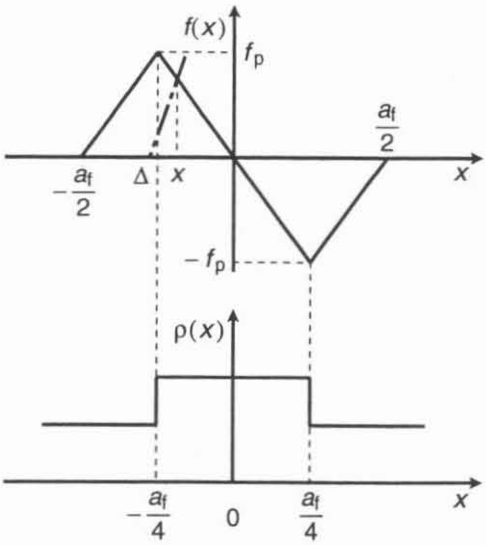

Fig. 3. Statistical distribution of flux lines for $f_p < f_{pt}$. The slope of the chained line in the upper figure gives k_f'

as

$$
\begin{aligned}
f(x) &= \frac{4f_p}{a_f}\left(x+\frac{a_f}{2}\right); \quad -\frac{a_f}{2} < x < -\frac{a_f}{4}, \\
&= -\frac{4f_p x}{a_f}; \quad -\frac{a_f}{4} < x < \frac{a_f}{4}, \\
&= \frac{4f_p}{a_f}\left(x-\frac{a_f}{2}\right); \quad \frac{a_f}{4} < x < \frac{a_f}{2}.
\end{aligned} \tag{4}
$$

The threshold value, f_{pt}, is determined by the condition, $\max|\partial f(x)/\partial x| = k_f'$, and we have

$$f_{pt} = \frac{k_f' a_f}{4}. \tag{5}$$

It is considered that the local position of flux lines in the virtual flux line lattice with the long-range order, which is obtained by switching off all the pinning forces, has no correlation with the position of randomly distributed pinning centers. Hence, if we take a statistical average, the probability to find such a flux line, the position of which is represented by Δ, around a pinning center is uniform. On the other hand, if the probability to find a flux line at x is denoted by $\rho(x)$, this is no longer uniform due to interactions between pinning centers and flux lines. This statistical distribution for $f_p < f_{pt}$, i.e. for $T > T_g$ is shown in Fig. 3. The distribution is symmetrical and this situation is unchanged even if the flux lines are displaced along the x-axis. Hence, it results in zero pinning force density. This can be derived also from the statistical average with respect to Δ. Under the present situation where the pinning force changes periodically with the period, a_f, the statistical average of an arbitrary function, g, is given by

$$\langle g \rangle = \frac{1}{a_f}\int_0^{a_f} g \mathrm{d}\Delta. \tag{6}$$

On the other hand, the statistical distribution has a vacant region for $f_p > f_{pt}$, i.e. for $T < T_g$. In this region flux lines are unstable to stay. This instability brings about the hysteretic nature of the pinning loss for a large displacement of flux lines and the reversible behavior with-

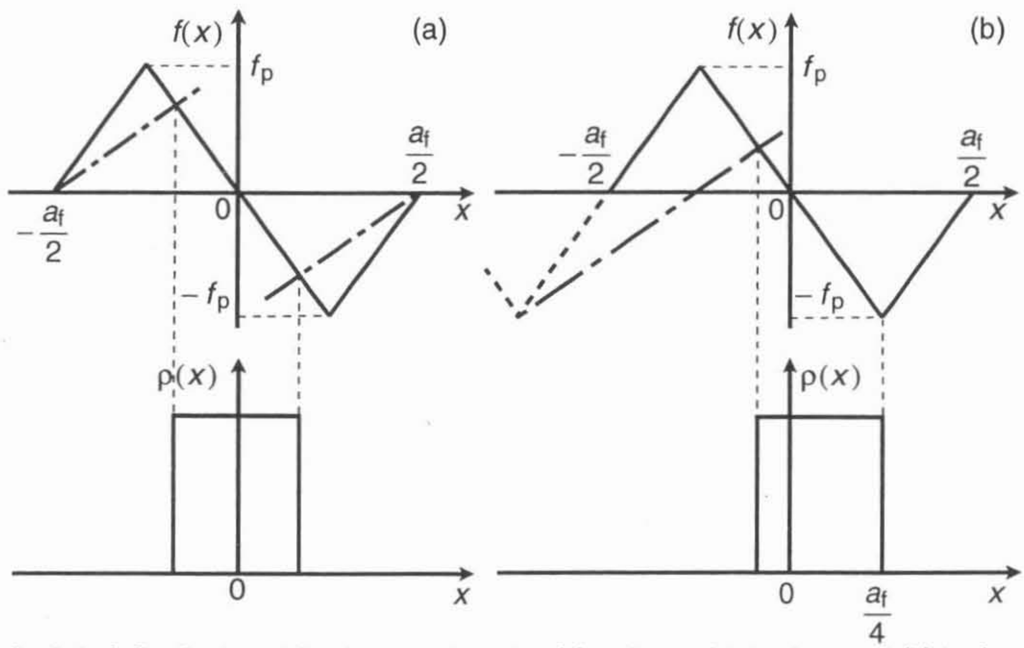

Fig. 4. Statistical distribution of flux lines for $f_p > f_{pt}$: (a) under no driving force and (b) in the critical state. The slope of the chained lines in the upper figures gives k_f'.

out loss for a small displacement inside pinning potentials. Figure 4(a) shows the statistical distribution of flux lines when the driving force is not applied and the distribution is symmetrical. When the driving force is applied, the flux lines are displaced. Figure 4(b) shows the distribution in the critical state under the driving force directed to the positive x-axis. The distribution is no longer symmetrical and a nonzero pinning force density is derived:

$$F_{\mathrm{p}} = N_{\mathrm{p}} f_{\mathrm{p}} \frac{f_{\mathrm{p}} - f_{\mathrm{pt}}}{f_{\mathrm{p}} + f_{\mathrm{pt}}}, \tag{7}$$

where N_{p} is the concentration of pinning centers.

Here we shall calculate the free energy. The free energy per pinning center is described as

$$F = U_{\mathrm{e}} + U'_{\mathrm{p}}, \tag{8}$$

where U'_{p} is the effective pinning energy under the influence of thermal activation of flux lines. It should be noted that the elastic energy, U_{e}, due to distortions of the flux line lattice contains a part which contributes to the Lorentz force, since the Lorentz force can be expressed as the elastic force using the uniaxially compressional modulus (C_{11}) and the tilt modulus (C_{44}) of the flux line lattice. The equilibrium condition, $\partial F/\partial x = 0$, leads to the force balance equation, (1).

Hence, U_{e} and U'_{p} are calculated from

$$U_{\mathrm{e}} = \frac{1}{2} k'_{\mathrm{f}} \langle (\Delta - x)^2 \rangle, \tag{9}$$

$$U'_{\mathrm{p}} = -\langle \int f(x) \mathrm{d}x \rangle. \tag{10}$$

After a simple but tedious calculation we have

$$\begin{aligned} F &= -\frac{a_{\mathrm{f}}}{24 f_{\mathrm{pt}}} f_{\mathrm{p}} (f_{\mathrm{p}} + 3 f_{\mathrm{pt}}); \qquad f_{\mathrm{p}} < f_{\mathrm{pt}}, \\ &= -\frac{a_{\mathrm{f}}}{24 f_{\mathrm{pt}}} \cdot \frac{f_{\mathrm{p}} (-3 f_{\mathrm{p}}^2 + 12 f_{\mathrm{pt}} f_{\mathrm{p}} - f_{\mathrm{pt}}^2)}{f_{\mathrm{p}} + f_{\mathrm{pt}}}; \\ & \qquad f_{\mathrm{p}} > f_{\mathrm{pt}}. \end{aligned} \tag{11}$$

From the relationship of (3) in the vicinity of the transition temperature, it is seen that the derivative with respect to T is proportional to the derivative with respect to f_{p} with an inversed sign. It is easy to show that the first derivative of F with respect to f_{p} is continuous at $f_{\mathrm{p}} = f_{\mathrm{pt}} (T = T_{\mathrm{g}})$. On the other hand, the second derivative is

$$\begin{aligned} \frac{\partial^2 F}{\partial f_{\mathrm{p}}^2} &= -\frac{a_{\mathrm{f}}}{12 f_{\mathrm{pt}}}; \qquad f_{\mathrm{p}} < f_{\mathrm{pt}}, \\ &= -\frac{a_{\mathrm{f}}}{12 f_{\mathrm{pt}}} \cdot \frac{-3 f_{\mathrm{p}}^3 - 9 f_{\mathrm{pt}} f_{\mathrm{p}}^2 - 9 f_{\mathrm{pt}}^2 f_{\mathrm{p}} + 13 f_{\mathrm{pt}}^3)}{(f_{\mathrm{p}} + f_{\mathrm{pt}})^3}; \\ & \qquad f_{\mathrm{p}} > f_{\mathrm{pt}}. \end{aligned} \tag{12}$$

This value goes to $a_{\mathrm{f}}/12 f_{\mathrm{pt}}$ in the limit of $f_{\mathrm{p}} \to f_{\mathrm{pt}}$ for $f_{\mathrm{p}} > f_{\mathrm{pt}}$. Thus, the second derivative is discontinuous at the transition temperature, and it can be concluded that thermal depinning is a transition of the second order.

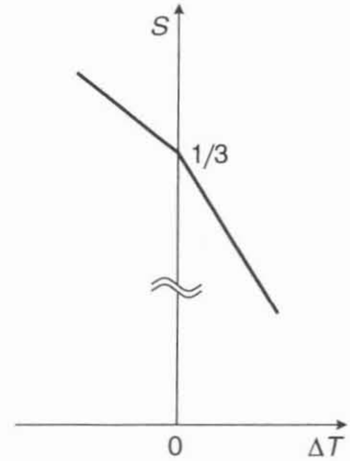

Fig. 5. Degree of disorder of flux line lattice vs temperature.

Next we shall estimate the degree of disorder in the flux line lattice. Since $x - \Delta$ is the deviation from the virtual lattice point in the flux line lattice with the long-range order, it seems to be reasonable to define the degree of disorder by

$$S = \left(\frac{4}{a_{\mathrm{f}}} \right)^2 \langle (x - \Delta)^2 \rangle. \tag{13}$$

After a simple calculation we obtain

$$\begin{aligned} S &= \frac{1}{3} \left(\frac{f_{\mathrm{p}}}{f_{\mathrm{pt}}} \right)^2; \qquad f_{\mathrm{p}} < f_{\mathrm{pt}}, \\ &= \frac{f_{\mathrm{p}}^2}{3 (f_{\mathrm{p}} + f_{\mathrm{pt}})^2} \left(3 \frac{f_{\mathrm{p}}^2}{f_{\mathrm{pt}}^2} - 6 \frac{f_{\mathrm{p}}}{f_{\mathrm{pt}}} + 7 \right); \\ & \qquad f_{\mathrm{p}} > f_{\mathrm{pt}}. \end{aligned} \tag{14}$$

If we write $\Delta T = T - T_{\mathrm{g}}$, (3) leads to

$$\frac{f_{\mathrm{p}}}{f_{\mathrm{pt}}} - 1 = -c \Delta T \tag{15}$$

with c denoting a constant. Hence, in the vicinity of the transition point where ΔT is sufficiently small, (14) is reduced to

$$\begin{aligned} S &= \frac{1}{3} (1 - c \Delta T)^2; \qquad \Delta T > 0, \\ &= \frac{(1 - c \Delta T)^2 (4 + 3 c^2 \Delta T^2)}{3 (2 - c \Delta T)^2}; \quad \Delta T < 0. \end{aligned} \tag{16}$$

Hence, when $|c \Delta T| \ll 1$, we have $S \simeq (1 - 2 c \Delta T)/3$ for $\Delta T > 0$ and $S \simeq (1 - c \Delta T)/3$ for $\Delta T < 0$. This result is shown in Fig. 5.

III. Results and Discussion

The present theoretical result shows that thermal depinning of flux lines is a second-order transition. It is concluded, therefore, that the speculation described in I is correct. This result also explains the reason for the fact that E-J curves calculated using the flux creep-flow model are scaled similarly to experiments.

This agreement shows that the irreversibility line and the G-L transition line are similar characteristics determined by the mechanism of thermal depinning. The difference between them comes only from the difference of definition. That is, the irreversibility temperature is defined in an engineering sense by the temperature at which the critical current density obtained using a certain criterion reduces to a given threshold, while the transition temperature is defined in a more physical sense using scaling. Thus, it is concluded that the glass-liquid transition and the thermal depinning are equivalent.

It should be noted, however, that what determines the state of flux lines is not their intrinsic nature but the flux pinning. This can be most clearly seen in the fact that the transition temperature can be obtained only by the mechanism of thermal depinning. The static and dynamic critical indices are not universal but are strongly influenced by the flux pinning property. That is, if the correlation length which determines the state of flux lines is the pinning correlation length, ν is speculated to be given by $\delta'/2$ using the parameter, δ', describing the temperature or magnetic field dependence of the critical current density. The relationship between ν and $\delta'/2$ for various high-temperature superconductors is shown in Fig. 6 [8],[22]. The coincidence of these quantities proves that the above speculation is valid.

The dynamic critical index, z, depends largely on the distribution of the flux pinning strength. A recent observation [12] showed that the introduction of strong columnar defects into Y-123 thin films brings about the enhancement of the transition temperature normalized by T_c and simultaneously the enhancement of z. From the latter fact it can be understood that strong pins with a sharp distribution determine the thermal depinning.

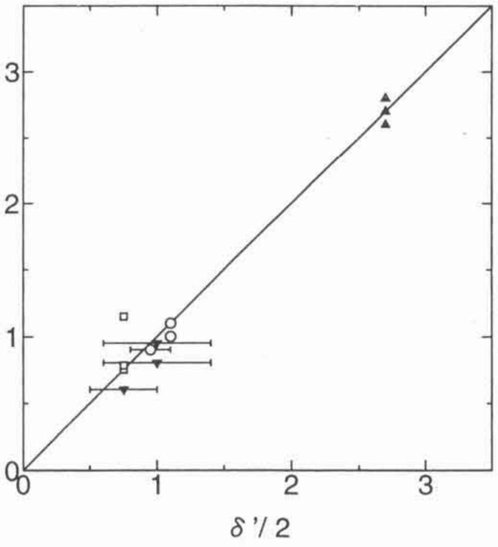

Fig. 6. Comparison between ν and $\delta'/2$ for Bi-2223 tapes (open circles), Bi-2212 tapes (solid triangles) and Y-123 thin films (open squares) [8],[22].

Thus, the present result is consistent with such experimental results which show that the scaling of current-voltage curves is not universal but depends on the flux pinning property.

Recently it was found [23] that the scaling of current-voltage curves is quite sensitive to the range of the electric field. That is, when the range of the electric field is lowered, T_g and ν decrease and z increases. This result also shows that the transition is not determined by the intrinsic property of flux lines. It is considered that such changes come from the difference of the diverging manner of the pinning correlation length, i.e. the difference of the temperature dependence of the critical current density, at a given level of the electric field.

In this paper it is assumed for simplicity that the pinning strength is uniform. However, the pinning strength is not uniform and this distribution influences largely the scaling parameters as mentioned above. In particular the distribution is very wide in practical high-temperature superconductors. That is, flux flow does not start globally but starts locally through percolating paths of weakly pinned regions. Hence, the scaling parameters are determined by the manner of development of percolating paths of flux lines with increasing temperature. Nevertheless, the present theoretical result that the thermal depinning is a second-order transition will not change essentially. That is, flux lines flow locally in weakly pinned regions to avoid an enhancement of the elastic energy, which contributes to the reduction in the total energy.

The present theory is based on the concept that the phase diagram of flux lines is determined by the elastic energy of flux lines, the thermal energy and the pinning energy. As a result, various transitions are explained to take place as argued in [24]. One of the important predictions from this theoretical result is that the irreversibility line below the melting transition observed for very weakly pinned specimens should also be a transition of the second order. Hence, it is expected that the scaling of current-voltage curves can be observed in this region also, although the critical indices may be different from the ones above the critical point.

The present theory shows that the degree of disorder in the flux line lattice in Fig. 5 decreases significantly and the order recovers with increasing temperature above T_g, while it is large below T_g. This result seems to be natural, since the pinning effect which distorts the flux line lattice becomes weak with increasing temperature. In fact this prediction agrees well with the observation of flux lines with a Lorentz microscope [25]. That is, flux lines do not form a perfect lattice and many dislocations are included at low temperatures, but a beautiful flux line lattice is formed near T_g. This agreement suggests that the present theoretical result is reasonable.

IV. Summary

Thermal depinning is investigated using the coherent potential approximation theory in which the effect of thermal activation is approximately introduced by making the pinning potential shallow. The following results are obtained.

(1) It is shown that the thermal depinning of flux lines which occurs on the irreversibility line is a transition of the second order.

(2) The hypothesis that the vortex glass-liquid transition is identical with thermal depinning is supported by the effects of flux pinning on observed experimental results such as critical indices and by successful explanation of the scaling of current-voltage curves using the flux creep-flow model. Since the vortex glass-liquid transition is originally expected to be a second-order transition, the present theoretical result is reasonable.

(3) The degree of disorder of the flux line lattice decreases with increasing temperature and the order is recovered quickly above the transition temperature. This agrees qualitatively with observations using a Lorentz microscope.

References

[1] M. P. A. Fisher, "Vortex-Glass Superconductivity: A Possible New Phase in Bulk High-T_c Oxides," *Phys. Rev. Lett.*, vol. 62, pp. 1415–1418, 1989.

[2] See for example, R. H. Koch, V. Foglietti, W. Gallagher, G. Koren, A. Gupta and M. P. A. Fisher, "Experimental Evidence for Vortex-Glass Superconductivity in Y-Ba-Cu-O," *Phys. Rev. Lett.*, vol. 63, pp. 1511–1514, 1989; R. H. Koch, V. Foglietti and M. P. A. Fisher, "Comment on Experimental Evidence for Vortex-Glass Superconductivity in Y-Ba-Cu-O," *Phys. Rev. Lett.*, vol. 64, p. 2586, 1990.

[3] M. Kiuchi, H. Yamato and T. Matsushita, "Longitudinal Elastic Correlation Length of Flux Lines along the *c*-Axis in Superconducting Bi-2212 Single Crystal," *Physica C*, vol. 269, pp. 242–248, 1996.

[4] N. Ihara and T. Matsushita, "Effect of Flux Creep on Irreversibility Lines in Superconductors," *Physica C*, vol. 257, pp. 223–231, 1996.

[5] M, Kiuchi, E. S. Otabe, T. Matsushita, T. Kato, T. Hikata and K. Sato, "Pinning-Force Density of Superconducting Bi-2223 Tape Wires," *Physica C*, vol. 260, pp. 177–187, 1996.

[6] M. Kiuchi, K. Noguchi, T. Tagomori, T. Matsushita and T. Hasegawa, "Scaling of Current-Voltage Curves of Bi-2212 Tape Wires," *IEEE Trans. Appl. Supercond.*, vol. 7, pp. 2114–2117, 1997.

[7] M. Kiuchi, K. Noguchi, T. Matsushita, T. Kato, T. Hikata and K. Sato, "Scaling of Current-Voltage Curves in Superconducting Bi-2223 Silver-Sheathed Tape Wires," *Physica C*, vol. 278, pp. 62–70, 1997.

[8] T. Matsushita, M. Kiuchi and K. Noguchi, "Correlation Length of Flux Lines in the Vicinity of the Vortex Glass-Liquid Transition in High-Temperature Superconductors," *Physica C*, vol. 290, pp. 38–42, 1997.

[9] T. Kiss, T. Nakamira, K. Kuroda, M. Takeo, F. Irie and K. Yamafuji, "Pin Fluctuation and the Scaling of Transport Characteristics in Superconductors," *Czech Journal of Physics*, vol. 46, pp. 1719–1720, 1996.

[10] T. Matsushita, T. Tohdoh and N. Ihara, "Effects of Inhomogeneous Flux Pinning Strength and Flux Flow on Scaling of Current-Voltage Characteristics in High-Temperature Superconductors," *Physica C*, vol. 259, pp. 321–325, 1996.

[11] T. Matsushita, M. Tagomori, K. Noguchi, M. Kiuchi and T. Hasegawa, "Irreversibility and Vortex Glass-Liquid Transition Fields of Silver-Sheathed Bi-2212 Superconducting Tapes," *Adv. Cryog. Eng. Mater.*, vol. 44, New York: Plenum, 1998, in press.

[12] T. Sueyoshi, N. Ishikawa, A. Iwase, Y. Chimi, T. Kiss, T. Fujiyoshi and K. Miyahara, "Transport Characteristics near the Glass-Liquid Transition Temperature before and after Heavy-Ion Irradiation in a $YBa_2Cu_3O_{7-d}$ Thin Film," to be published.

[13] T. Matsushita, M. Kiuchi and Y. Nakayama, "Critical Current Density and Pinning Potential in Bi-2223 Superconductor," *Proc. Int. Works. on Critical Currents in Superconductors for Practical Applications*, L. Zhou, H. W. Weber and E. W. Collings, Eds., Singapore: World Scientific, 1998, pp. 40–45.

[14] T. Matsushita, M. Kiuchi and H. Yamato, "Comparison of Flux Bundle Size between Three-Dimensional Y-123 and Two-Dimensional Bi-2212 Superconductors," *Adv. Cryog. Eng. Mater.*, vol. 44, New York: Plenum, 1998, in press.

[15] A. I. Larkin and Yu. N. Ovchinnikov, "Pinning in Type II Superconductors," *J. Low Temp. Phys.*, vol. 34, pp. 409–428, 1979.

[16] T. Matsushita, "Pinning Force Density in Strongly Pinned Superconductors," *Physica C*, vol. 243, pp. 312–318, 1995.

[17] R. Labusch, "Calculation of the Critical Field Gradient in Type II Superconductors," *Crystal Lattice Defects*, vol. 1, pp. 1–16, 1969.

[18] E. J. Kramer, "Fundamental Fluxoid-Defect Interactions in Irradiated Superconductors," *J. Nucl. Mater.*, vol. 72, pp. 5–33, 1978.

[19] K. Yamafuji, T. Fujiyoshi, K. Toko, T. Matsuno, K. Kishio and T. Matsushita, "A Theory of Thermal-Fluctuation-Induced Depinning Line in Weakly Pinning High-T_c Superconductors," *Physica C*, vol. 212, pp. 424–434, 1993.

[20] K. Yamafuji and T. Kiss, "Current-Voltage Characteristics near the Glass-Liquid Transition in High-T_c Superconductors," *Physica C*, vol. 290, pp. 9–22, 1997.

[21] A. M. Campbell, "The Pinning of Flux Lines in Superconductors by Randomly-Spaced Pinning Centres," *Philos. Mag. B*, vol. 37, pp. 149–167, 1978.

[22] T. Kiss, T. Nakamura, K. Hasegawa, M. Inoue, M. Takeo, F. Irie and K. Yamafuji, "Transport Characteristics in High T_c Superconductors," *Proc. of 15th Int. Conf. on Magnet Technology*, Oct. 20–24, 1997, Beijing China, 1998, in press.

[23] T. Nakamura, T. Kiss, Y. Hanayama, T. Matsushita, K. Funaki, M. Takeo and F. Irie, "Transport Properties in $Y_1Ba_2Cu_3O_{7-\delta}$ Superconducting Thin Films over Wide Range of Electric Field," *Adv. Supercond. X*, Tokyo: Springer-Verlag, 1998, pp. 581–584.

[24] T. Matsushita and T. Kiss, "Thermal Depinning of Flux Lines in Superconductors," to be published.

[25] K. Harada, T. Matsuda, H. Kasai, J. E. Bonevich, T. Yoshida, U. Kawabe and A. Tonomura, "Vortex Configuration and Dynamics in $Bi_2Sr_{1.8}CaCu_2O_x$ Thin Films by Lorentz Microscopy," *Phys. Rev. Lett.*, vol. 71, pp. 3371–3374, 1993.