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STUDY ON VIBRATION AND STABILITY OF FUNCTIONALLY GRADED CYLINDRICAL SHELLS SUBJECTED TO HYDROSTATIC PRESSURE

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Abstract: Based on the Flügge's shell theory, the vibration characteristics and stability of submerged functionally graded (FG) cylindrical shell under hydrostatic pressure is examined. By means of conversion switch on axial wave number, the coupled frequency of submerged FG cylindrical shell with various boundary conditions is obtained, using wave propagation method and Newton method. Then the critical pressure of FG cylindrical shells is given by applying linear fitting method. Results are compared to known solutions, where these solutions exist. The natural frequency and critical pressure of FG cylindrical shell are illustrated. The effects of constituent materials, volume fraction, boundary condition and dimensions on the natural frequencies and critical pressures of submerged FG cylindrical shell are illustrated by examples.

Keywords: Natural Frequency; Critical Pressure; Functionally Graded Material; Cylindrical Shell; Hydrostatic Pressure

1 INTRODUCTION

Functionally graded materials, FGM for short, is a new kind of compound material structure with component and structure graded distribution along thickness. By using the new kind of functionally graded material, the requirements of special extreme environment such as ultra-temperature, larger temperature gradient and the strong thermal shock on FG cylindrical shell are satisfied. The liquid medium and the material properties of functionally graded material have significant impact on the vibration characteristics of cylindrical shell. And the structural analyses need to be carried out in the presence of critical pressure. Since the pioneer work of Junger [1] was published, a lot of theoretical investigations have appeared. Zhang et al. [2] used wave propagation method to consider the vibration characteristics of cylindrical shell with the impact of the fluid. Sheng and Wang [3] presented the report of an investigation into the vibration of FG cylindrical shells with flowing fluid by employing the first-order shear deformation theory. The input power flow for an infinite ring-stiffened cylindrical shell submerged in fluid was investigated by Liu et al. [4]. The hydrostatic buckling of shells with various boundary conditions is studied by Pinna and Ronalds [5].

Based on the Flügge shell theory, wave propagation method and Newton method, the coupled frequency of submerged FG cylindrical shell with various boundary conditions is derived. Then the critical pressure of FG cylindrical shells is given by applying linear fitting method. The effects of constituent materials, volume fraction, boundary condition and dimensions on the natural frequencies and critical pressures of submerged FG cylindrical shell are illustrated by examples. In numerical calculations, the functionally graded material has ceramic on its inner surface and has metal on its outer surface.

2 MECHANICAL MODEL

A FG cylindrical shell is considered, as shown in Fig.1. The shell is characterized by its inner radius R_i , outer radius R_0 , represented radius R , length L and thickness h . The x , θ and z are the axial coordinates, circumferential coordinates and radial coordinates, respectively.

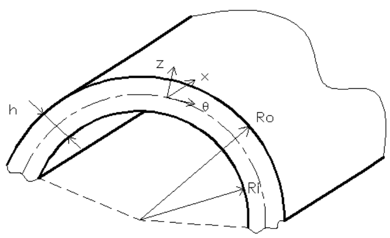

Fig.1. Geometry of a FG cylindrical

3 FUNCTIONALLY GRADED MATERIALS

The material property P of functionally graded material is the function of temperature and volume fraction, which is controlled by the volume fractions of material. The function can be defined as follows

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{1}$$

where P_0, P_{-1}, P_1, P_2, P_3 are the coefficients of temperature $T(\mathrm{K})$ and are unique to the constituent materials. For cylindrical shell with thickness h, the volume percentages can be given as

$$V_o = \left[\frac{z + h/2}{R_0 - R_i}\right]^N = 1 - V_i \tag{2}$$

where V_i and V_o are the volume percentages of the internal and external surfaces of the functionally graded material, respectively. N is the power-law exponent($0 \le N \le \infty$).So along the thickness of the shell, the Young's modulus E, Poisson ratio μ and the mass density ρ can be expressed as

$$\begin{cases} E = (E_o - E_i)\left[\dfrac{z + h/2}{R_0 - R_i}\right]^N + E_i \\ \mu = (\mu_o - \mu_i)\left[\dfrac{z + h/2}{R_0 - R_i}\right]^N + \mu_i \\ \rho = (\rho_o - \rho_i)\left[\dfrac{z + h/2}{R_0 - R_i}\right]^N + \rho_i \end{cases} \tag{3}$$

4 FORMULATION

By using Flügge's [6] theory, the equations of motion for a cylindrical shell subjected to hydrostatic pressure are obtained.

$$\begin{cases} u_{xx} + \dfrac{1-\mu}{2}u_{\theta\theta} + \dfrac{1+\mu}{2}v_{x\theta} + \mu w_x + K(\dfrac{1-\mu}{2}u_{\theta\theta} - w_{xxx} + \dfrac{1-\mu}{2}w_{x\theta\theta}) \\ +T_1u_{xx} + T_2(u_{\theta\theta} - w_x) - \dfrac{\rho R^2(1-\mu^2)}{E}u_{tt} = 0 \\ \dfrac{1+\mu}{2}u_{x\theta} + \dfrac{1-\mu}{2}v_{xx} + v_{\theta\theta} + w_\theta + K(\dfrac{3(1-\mu)}{2}v_{xx} - \dfrac{3-\mu}{2}w_{xx\theta}) + T_1v_{xx} \\ +T2(v_{\theta\theta} + w_\theta) - \dfrac{\rho R^2(1-\mu^2)}{E}v_{tt} = 0 \\ \mu u_x - Ku_{xxx} + K\dfrac{1-\mu}{2}u_{x\theta\theta} + v_\theta - K\dfrac{3-\mu}{2}v_{xx\theta} + (1+K)w + Kw_{xxxx} + 2Kw_{xx\theta\theta} \\ +Kw_{\theta\theta\theta\theta} + 2Kw_{\theta\theta} - T_1w_{xx} - T_2(u_x - v_\theta + w_{\theta\theta}) + \dfrac{\rho R^2(1-\mu^2)}{E}w_{tt} = -\dfrac{R^2(1-\mu^2)}{Eh}\psi \end{cases} \tag{4}$$

where $T_1 = \dfrac{R}{2Eh}(1-\mu^2)P_o$, $T_2 = \dfrac{R}{Eh}(1-\mu^2)P_o$, $()_x = R\dfrac{\partial()}{\partial x}$, $()_\theta = \dfrac{\partial()}{\partial\theta}$, $()_t = \dfrac{\partial()}{\partial t}$, $K = \dfrac{h^2}{12R^2}$, P_o is the hydrostatic pressure and ψ is the acoustic pressure.

The displacements of the cylindrical shell can be expressed in the form of wave propagation.

$$\begin{cases} u(x,\theta,t) = U_m\cos(n\theta)e^{(i\omega t - ik_m x)} \\ v(x,\theta,t) = V_m\sin(n\theta)e^{(i\omega t - ik_m x)} \\ w(x,\theta,t) = W_m\cos(n\theta)e^{(i\omega t - ik_m x)} \end{cases} \tag{5}$$

where U_m, V_m, W_m are the wave amplitudes in the x, θ and z directions, ω is the natural angular frequency. The fluid of the cylindrical shell is assumed non-viscous which satisfy the acoustic wave equation. The equation of motion of the fluid can be written in the cylindrical co-ordinate system (x,θ,r) as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial^2 t} \tag{6}$$

where t is the time and c is the sound speed of the fluid. The associated form of the acoustic pressure field exterior of the shell, which satisfies the acoustic wave Eq.(6), is given as

$$\psi = \psi_m \cos(n\theta) H_n^{(2)}(k_r r) e^{(i\omega t - ik_m x)}, \quad \psi_m = \left[\omega^2 \rho_f / k_r H_n^{(2)'}(k_r R)\right] W_m \tag{7}$$

where $H_n^{(2)}()$ is the Hankel function of the second kind and order n, the prime on the $H_n^{(2)}()$ denotes differentiation with respect to the argument $k_r R$. The relationship between radial wave number k_r and axial wave number k_m is applied to $(k_r R)^2 = \Omega^2 (C_L / C_F)^2 - (k_m R)^2$.

In the function, Ω is the non-dimensional frequency, C_L and C_F are the sound speed of the shell and fluid respectively. The fluid radial displacement and shell radial displacement must be equal at the interface of the shell inner wall and the fluid.

Substituting Eq.(5) into Eq.(4), which consideration of acoustic pressure on the shell and coupling Eq.(7), the equations of motion of coupled system in matrix form can be obtained.

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33}+F_L \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

where $L_{ij}(i, j = 1,2,3)$ are the parameters operated with the x and θ, F_L is the fluid loading term due to the presence of the fluid acoustic field.

$$F_L = \Omega^2 (\rho_f / \rho)(R/h)(k_r R)^{-1} \left[H_n^{(2)}(k_r R) / H_n^{(2)'}(k_r R)\right] \tag{9}$$

By substituting a boundary condition into k_r, the solution of Eq.(8) can be derived.

According to the classical theory, the function of elastic critical hydrostatic pressure is given in Шиманский [7]. However, the function can only analysis the elastic critical pressure of cylindrical shell for $m=1$ under simply supported (SS-SS) boundary condition. Because of linear relation between the frequency squared and hydrostatic pressure (Abramovich [8]), the critical hydrostatic pressure can be derived while the natural frequency is assumed to be zero. Therefore, the critical pressure can be obtained by using linear fitting method.

5 NUMERICAL RESULTS AND DISCUSSION

5.1 Vibration of FG cylindrical shell

In this paper, the metal materials are Stainless steel and Ti-6Al-4V, while the ceramic materials are Si_3N_4 and Zirconia. The material properties are taken into consideration the temperature dependency for the temperature of $T = 300K$ from Shariyat [9] and Kim [10].

The natural frequencies of FG cylindrical shell with simply supported ends are listed in Table 1. The functionally gradient material considered is composed of Stainless steel and nickel. The validity and feasibility of the study are verified by comparing results with those in Ref. [11].The geometric parameters of the shell are defined as: $E_N = 2.05098 \times 10^{11} N/\mathrm{m}^2$, $\mu_N = 0.31$, $\rho_N = 8900 kg/\mathrm{m}^3$, $L/R = 20$, $h/R = 0.002$, $m = 1$.

Table 1. Comparison of natural frequencies of FG cylindrical shell

	$N \to 0$		$N=1$		$N \to \infty$	
n	Present	Loy	Present	Loy	Present	Loy
1	13.548	13.548	13.211	13.211	12.894	12.894
2	4.5916	4.5920	4.4768	4.480	4.3687	4.3690
3	4.2628	4.2633	4.1523	4.1569	4.0484	4.0489
4	7.2247	7.2250	7.0354	7.0384	6.8574	6.8577
5	11.542	11.542	11.239	11.241	10.954	10.955

As an example, critical pressures for a simply supported ends isotropic cylindrical shell are studies in Table 2. Because of linear relation between the natural frequency squared and hydrostatic pressure, the equations of cylindrical shells for different conditions can be obtained by using linear fitting method, and then the critical pressure can be derived. Compared with results from Eq.(15), the feasibility of the study is proved. Some parameters are selected as: $E = 2.1\times10^{11} N/\mathrm{m}^2$, $\mu = 0.3$, $\rho = 7850 kg/\mathrm{m}^3$, h=0.01m , L=20m , R=1m , m=1 , n=2 .

Table 2. Comparison of critical pressures [MPa] of simply supported submerged isotropic cylindrical shell

h/R	L/R = 2		5		10	
	Present	Eq(15)	Present	Eq(15)	Present	Eq(15)
0.005	36.448	36.115	2.6929	2.6586	0.2079	0.2072
0.01	73.049	72.333	5.4377	5.3692	0.4610	0.4599
0.02	147.10	145.48	11.287	11.154	1.2832	1.2831

Because of the structure will create instability when the hydrostatic pressure is larger than critical pressure, so in this paper, the hydrostatic pressures should aim for under critical pressures. The natural frequencies of FG cylindrical shells under hydrostatic pressure for different volume fractions, axial half wave numbers and boundary conditions are studied in this paper, see Fig.2-4. Results given in these figures are obtained by setting $L/R = 20$, $h/R = 0.01$, $R = 1\mathrm{m}$, $N = 1$, m=1, $n = 2$ and $z = 0$.

Fig.2 describes the variations of natural frequencies of clamped-free (C-F) FG cylindrical shells for some different values of volume fraction, and the functionally graded material is made up of Stainless steel and Si_3N_4. It is shown in these figure that with the increasing of volume fraction, the natural frequency of FG cylindrical shell increases gradually. The effects of power-law exponent on natural frequencies are mainly reflected in the cases of low exponent.

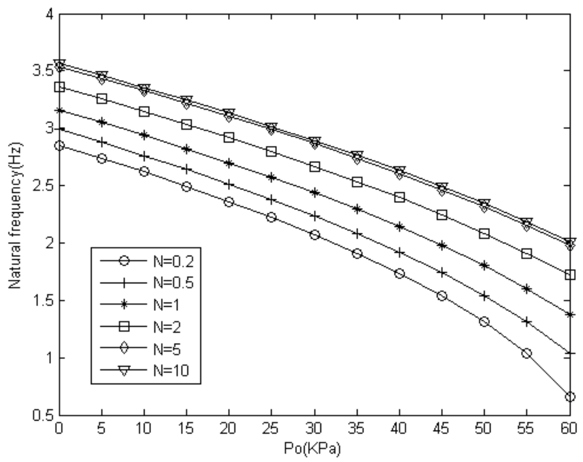

Fig.2. Variation of frequencies of submerged FG cylindrical shells for different values of volume fraction.

Fig.3 describes the change curves of natural frequencies of FG cylindrical shells for different support conditions. It is shown that the coupled frequencies for clamped-clamped (C-C) boundary condition are higher than those for other boundary conditions, the frequencies for free-sliding (F-S) boundary condition are lower than those for other boundary conditions. As the critical pressure approached, for clamped-free ends and free-sliding ends, the decline in natural frequency accelerated.

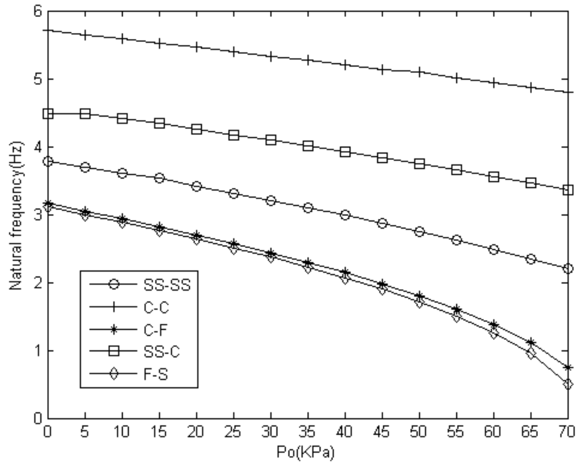

Fig.3. Variation of frequencies of submerged FG shells associated with various boundary conditions.

The variations of natural frequencies of clamped-free FG cylindrical shells for some different axial half wave numbers m under hydrostatic pressure are shown in Fig.4. It is shown that the natural frequency of FG cylindrical shell increased with the increasing of axial half wave number. With the increasing of axial half wave number, the range ability of the curves of natural frequency becomes smaller. Inferred by former results, with the increasing of axial half wave number, the critical pressure grows exponentially. When m comes to a certain degree, the change of the natural frequency is not significant.

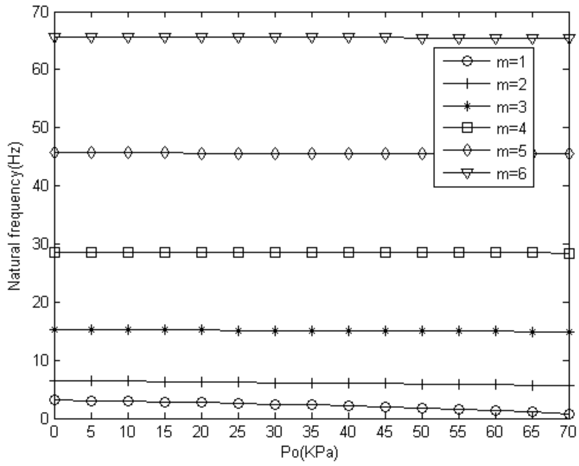

Fig.4. Variation of frequencies of submerged FG shells for some different axial half wave numbers.

5.2 Stability of FG cylindrical shell

As examples, based on the linear fitting method, critical pressures with different h/R, L/R, various boundary conditions, constituent materials and volume fractions are studied, see Fig.5 and Fig.6.

Fig.5 shows the curves of critical pressures of FG cylindrical shells for various values of power-law exponent. It is shown in the figure that the effects of power-law exponent on critical pressure are evident. The influences of power-law exponent on critical pressures are mainly reflected in the cases of small power-law exponent. With the increasing of power-law exponent, the functionally graded material is from a pure metal material to a pure ceramic material. This is due to the FG cylindrical shell is reduced to an isotropic cylindrical shell. Since critical pressure is material property dependent, use the FG cylindrical shell which composed of Stain steel and Zirconia as an example. Because of the critical pressure of pure Stain steel cylindrical shell is larger than pure Zirconia cylindrical shell, the critical pressure decreases with the increasing of power-law exponent.

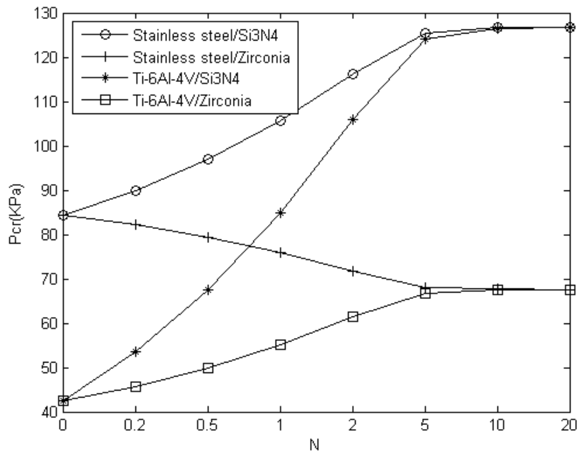

Fig.5. Variation of critical pressures of submerged shell for different values of power-law exponent.

The critical pressures of simply supported FG cylindrical shells for different h/R are shown in Fig.6. It is shown that with the increasing of h/R , the critical pressure increases rapidly. The influences of constituent material on critical pressure are mainly reflected in the cases of large h/R .

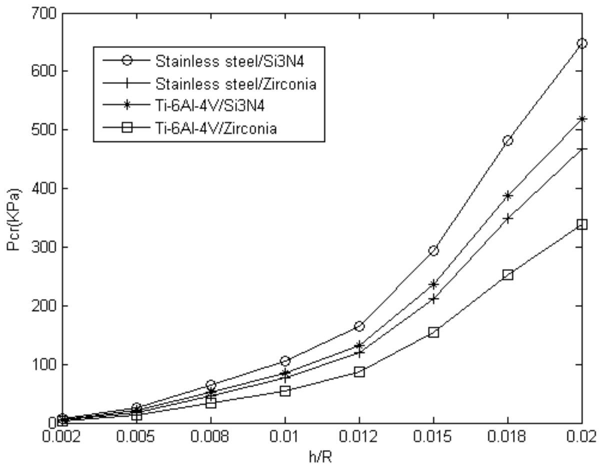

Fig.6. Variation of critical pressures of submerged cylindrical shell with different L/R ratios.

6 CONCLUSIONS

Based on the Flügge's shell theory, the natural frequencies and critical pressures of submerged FG cylindrical shells under hydrostatic pressure are studied. By means of conversion switch on axial wave number, the coupled frequency of submerged FG cylindrical shell with various boundary conditions is obtained, using wave propagation method and Newton method. Then the critical pressure of FG cylindrical shells is given by applying linear fitting method. The effects of the hydrostatic pressure, dimensions, constituent material, volume fraction and boundary condition on natural frequencies and critical pressure were investigated. Especially for the axial half wave number, the influence is obvious. In some cases, the change rules of natural frequency are similar to critical pressure. The characteristics of natural frequency and critical pressure will certainly provide guidance to the practical use and risk averse.

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