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Estimation of AC loss in cylindrical superconductor with ripple current

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Abstract

The loss energy density (AC loss) in cylindrical superconductor with ripple current based on Irie-Yamafuji model in which the magnetic fiel dependence of critical current density is taken into account is theoretically calculated for design of DC transmission cable system. It is confirme that the AC loss does not changed for the cases with and without DC current when the critical current does not depend on magnetic fiel which is corresponding to Bean-London model. On the contrary, it is found that there is current region where the AC loss becomes smaller than that for the case without DC current. The AC loss of ripple current is seems to be enough small in layered structure of DC transmission cable made by thin tape superconductor.

Keywords:
DC transmission cable, AC loss, ripple current, Irie-Yamafuji model, cuprate high temperature superconductor

1. Introduction

DC transmission cable by cuprate high temperature superconductor has been developed, since its AC loss is considerably smaller than that in AC transmission cable and it is considered to be suitable for long distance energy transmission[1, 2]. The size of the DC transmission cable becomes smaller than 3 phases AC transmission cable resulting in low cost of installation.

However, the existing conventional transmission system is based on AC, it is necessary to convert current from AC to DC and vice versa with very large inductance for installing DC transmission cable. In this case, AC is converted to DC and superposed AC (ripple current) and the energy loss density by ripple current is generated. Therefore it is desired to estimate the energy loss density (AC loss) for the case of DC current and superposed AC current for design of DC transmission cable system. If the AC loss of ripple current is enough small, the size of inductance for conversion can be decreased resulting in low cost of installation.

In this study, the AC loss of ripple current for cylindrical superconductor is theoretically estimated based on Irie-Yamafuji model in which the magnetic fiel dependence of the critical current density is taken into

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account [3]. Discussion is given for the ripple current amplitude dependence of the AC loss for the cases with and without DC current.

2. Theory

It is well known that the AC loss for cylindrical superconductor was derived by Hancox based on Bean-London model in which the critical current density is constant for magnetic fiel [4]. Norris revealed that the Hancox equation is the same to the result for superconducting wire with same area of elliptic cross section [5]. On the other hand, Irie-Yamafuji model has been widely used for the magnetic fiel dependence of the critical current density as

$$J_\mathrm{c} = \alpha_\mathrm{c} B^{\gamma-1}, \tag{1}$$

where α_c and γ are pining parameters [3].

Here, it is assumed that AC current $I(t) = I_\mathrm{m} \cos \omega t$ is applied to a straight cylindrical superconductor of radius R without external magnetic field

The critical current I_c of the superconductor is given by

$$I_\mathrm{c} = 2\pi \left(\frac{2-\gamma}{3-\gamma} \alpha_\mathrm{c} \mu_0^{\gamma-1} R^{3-\gamma} \right)^{1/(2-\gamma)}. \tag{2}$$

The self magnetic fiel at the surface of the superconductor is given by

$$H_\mathrm{IP} = \frac{I_\mathrm{c}}{2\pi R}. \tag{3}$$

The AC loss without DC current par cycle and per unit length Q_ac [J/m/cycle] based on Irie-Yamafuji model is given by

$$Q_\mathrm{ac} = Q_0 \int_0^{h_\mathrm{m}} \mathrm{d}h_\mathrm{I} h_\mathrm{I}^{2-\gamma} \left[-\int_{x_1}^1 \frac{1}{x} (1 + h_\mathrm{I}^{2-\gamma} - x^{3-\gamma})^{\frac{\gamma-1}{2-\gamma}} \mathrm{d}x + \int_{x_2}^{x_0} \frac{1}{x} (1 - h_\mathrm{I}^{2-\gamma} - x^{3-\gamma})^{\frac{\gamma-1}{2-\gamma}} \mathrm{d}x + \int_{x_0}^1 \frac{1}{x} (x^{3-\gamma} - 1 + h_\mathrm{I}^{2-\gamma})^{\frac{\gamma-1}{2-\gamma}} \mathrm{d}x \right], \tag{4}$$

where

$$Q_0 = \frac{\mu_0 I_\mathrm{c}^2}{\pi}, \tag{5}$$

$$x_0 = \left(1 - h_\mathrm{I}^{2-\gamma}\right)^{\frac{1}{3-\gamma}}, \quad x_1 = \left(1 - \frac{1}{2}\left(h_\mathrm{m}^{2-\gamma} - h_\mathrm{I}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}}, \quad x_2 = \left(1 - \frac{1}{2}\left(h_\mathrm{m}^{2-\gamma} + h_\mathrm{I}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}} \tag{6}$$

are braking points in the superconductor, and

$$h_\mathrm{m} = \frac{H_\mathrm{m}}{H_\mathrm{IP}}, \quad h_\mathrm{I} = \frac{H_\mathrm{I}}{H_\mathrm{IP}}. \tag{7}$$

In the above, H_m and H_I are the maximum magnetic fiel at maximum current I_m and magnetic fiel at current I at the surface, respectively [6].

Here, the ripple loss par cycle and per unit length Q_ripple [J/m/cycle] is estimated based on Irie-Yamafuji model for DC current amplitude I_DC and superposed ripple current amplitude I_m. The results are given by

$$Q_\mathrm{ripple} = \frac{Q_0}{2} \int_{h_\mathrm{d}}^{h_\mathrm{u}} \mathrm{d}h_\mathrm{I} \left[\int_{x_1}^1 -\frac{h_\mathrm{I}^{2-\gamma}}{x} \left(h_\mathrm{I}^{2-\gamma} + 1 - x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}} \mathrm{d}x + \int_{x_1'}^1 \frac{h_\mathrm{I}^{2-\gamma}}{x} \left(h_\mathrm{I}^{2-\gamma} - 1 + x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}} \mathrm{d}x \right] \tag{8}$$

for the case of $I_\mathrm{m} < I_\mathrm{DC}$, where

$$x_1 = \left(1 - \frac{1}{2}\left(h_\mathrm{u}^{2-\gamma} - h_\mathrm{I}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}}, \quad x_1' = \left(1 - \frac{1}{2}\left(h_\mathrm{I}^{2-\gamma} - h_\mathrm{d}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}}, \tag{9}$$

and

$$
\begin{aligned}
Q_{\mathrm{ripple}} = \frac{Q_0}{2}\Bigg(\int_0^{h_{\mathrm{u}}} \mathrm{d}h_{\mathrm{I}} \quad & \int_{x_1}^{1} -\frac{h_{\mathrm{I}}^{2-\gamma}}{x}\left(h_{\mathrm{I}}^{2-\gamma}+1-x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}}\mathrm{d}x \\
+\int_0^{h_{\mathrm{d}}} \mathrm{d}h_{\mathrm{I}} \quad & \Bigg[\int_{x_2}^{x_0} \frac{h_{\mathrm{I}}^{2-\gamma}}{x}\left(-h_{\mathrm{I}}^{2-\gamma}+1-x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}}\mathrm{d}x + 2\int_{x_0}^{1} \frac{h_{\mathrm{I}}^{2-\gamma}}{x}\left(h_{\mathrm{I}}^{2-\gamma}-1+x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}}\mathrm{d}x \\
& + \int_{x_1'}^{1} -\frac{h_{\mathrm{I}}^{2-\gamma}}{x}\left(h_{\mathrm{I}}^{2-\gamma}+1-x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}}\mathrm{d}x + \int_{x_2'}^{x_0} \frac{h_{\mathrm{I}}^{2-\gamma}}{x}\left(-h_{\mathrm{I}}^{2-\gamma}+1-x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}}\mathrm{d}x\Bigg] \\
+\int_{h_{\mathrm{d}}}^{h_{\mathrm{u}}} \mathrm{d}h_{\mathrm{I}} \quad & \int_{x_2'}^{1} \frac{h_{\mathrm{I}}^{2-\gamma}}{x}\left(h_{\mathrm{I}}^{2-\gamma}-1+x^{3-\gamma}\right)^{\frac{\gamma-1}{2-\gamma}}\mathrm{d}x\Bigg) \qquad (10)
\end{aligned}
$$

for the case of $I_{\mathrm{m}} > I_{\mathrm{DC}}$, where

$$i_{\mathrm{DC}} = \frac{I_{\mathrm{DC}}}{I_{\mathrm{c}}}, \quad i_{\mathrm{m}} = \frac{I_{\mathrm{m}}}{I_{\mathrm{c}}}, \qquad (11)$$

$$h_{\mathrm{u}} = i_{\mathrm{DC}} + i_{\mathrm{m}}, \quad h_{\mathrm{d}} = |i_{\mathrm{DC}} - i_{\mathrm{m}}|\,. \qquad (12)$$

In the above, x_0, x_1, x_2, x_1' and x_2' are breaking points and given by

$$x_0 = \left(1-h_{\mathrm{I}}^{2-\gamma}\right)^{\frac{1}{3-\gamma}}, \quad x_1 = \left(1-\frac{1}{2}\left(h_{\mathrm{u}}^{2-\gamma}-h_{\mathrm{I}}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}}, \quad x_2 = \left(1-\frac{1}{2}\left(h_{\mathrm{u}}^{2-\gamma}+h_{\mathrm{I}}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}},$$

$$x_1' = \left(1-\frac{1}{2}\left(h_{\mathrm{d}}^{2-\gamma}-h_{\mathrm{I}}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}}, \quad x_2' = \left(1-\frac{1}{2}\left(h_{\mathrm{d}}^{2-\gamma}+h_{\mathrm{I}}^{2-\gamma}\right)\right)^{\frac{1}{3-\gamma}}. \qquad (13)$$

These equations, Eqs. (8) and (10) are coincident with Eq. (4) for the case of $i_{\mathrm{DC}} = 0$.

3. Results and discussion

Fig. 1 shows the ac loss par cycle and per unit length as a function of normalized maximum ripple current amplitude i_{m} for various values of γ for the case of $i_{\mathrm{DC}} = 0.01$. The chained lines represents the results for the case of $i_{\mathrm{DC}} = 0$ i.e. Q_{ac}. The value of Q_{ripple} increases with increasing i_{m} by the power of 3 to 4 depended on γ. For the case of $\gamma = 1$ which is corresponding to Bean-London model, the results of both cases, $i_{\mathrm{DC}} = 0$ and $i_{\mathrm{DC}} = 0.01$ are the same, since the magnetic fiel dependence of J_{c} is ignored.

On the other hand, it is found that both results of with and without DC current are different when γ is not unity. The difference between Q_{ac} and Q_{ripple} becomes large with decreasing γ, when the magnet fiel dependence of J_{c} is large. Hence the magnetic fiel dependence of J_{c} largely affects Q_{ripple}. The AC ripple loss Q_{ripple} for $i_{\mathrm{DC}} = 0.01$ is larger than the pure AC loss Q_{ac} for $i_{\mathrm{DC}} = 0$ at $i_{\mathrm{m}} < i_{\mathrm{DC}}$. This is due to the large penetration fiel resulting in large movement of magnetic fiel by small J_{c} in the magnetic fiel by DC current. However, opposite result that the AC ripple loss Q_{ripple} for $i_{\mathrm{DC}} = 0.01$ is smaller than the pure AC loss Q_{ac} for $i_{\mathrm{DC}} = 0$ is obtained at $i_{\mathrm{m}} > i_{\mathrm{DC}}$, since i_{m} is larger than i_{DC} and large J_{c} at zero magnetic fiel is obtained at the surface of the cylindrical superconductor.

Fig. 2 shows Q_{ripple} and Q_{ac} with various value of i_{DC}. The results of Norris ellipse and strip are also plotted for comparison. It is also found that the crossing point of Q_{ripple} and Q_{ac} is obtained at $i_{\mathrm{m}} \simeq i_{\mathrm{DC}}$. The difference between Q_{ripple} and Q_{ac} becomes saturated at low values of i_{m}.

In practical DC transmission cable, it is considered that the cross section of cable is layered structure made by thin superconducting tapes, and it is not whole cylindrical superconductor. In this case, the theoretical result can be obtained by the case that the current is limited at the surface area of the cylindrical superconductor. Therefore I_{c} is large and corresponding i_{DC} and i_{m} are quite smaller than I_{c}. In addition, $i_{\mathrm{DC}} \gg i_{\mathrm{m}}$ is satisfie in the practical case. Therefore, Q_{ripple} is several to ten times larger than Q_{ac} at low i_{m} when γ is not unity. However the value of Q_{ripple} is quite smaller than Q_0 as shown in Fig. 2. In sammry, Q_{ripple}/Q_0 becomes very small in practical DC transmission cable, since I_{c} is very large and the corresponding i_{m} and i_{DC} become very small.

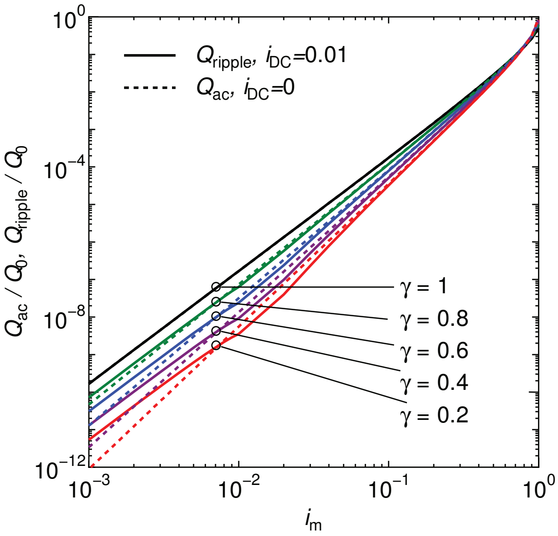

Fig. 1. AC ripple current amplitude dependence of reduced AC loss of ripple current for cylindrical superconductor with various value of γ. Solid and Dotted lines represent the cases of $Q_{ripple}(i_{DC} = 0.01)$ and $Q_{ac}(i_{DC} = 0)$, respectively.

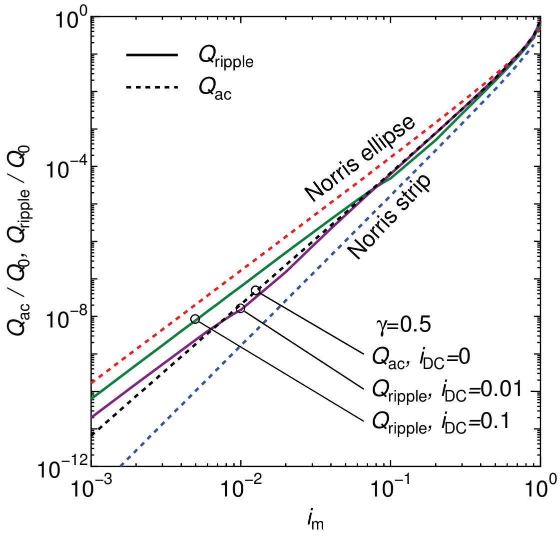

Fig. 2. AC ripple current amplitude dependence of reduced AC loss of ripple current with various i_{DC} for $\gamma = 0.5$.

4. Conclusions

In present study, the AC loss of ripple current of cylindrical superconductor is theoretically investigated, since AC ripple current is remained in superconducting DC transmission cable system for AC-DC conversion. It is predicted that the AC losses with and without DC are the same when the magnetic fiel dependence of J_c is ignored based on Bean-London model. On the other hand, the AC loss of ripple current is larger than pure AC loss at low ripple current $i_m < i_{DC}$, and the result is opposite for the case of $i_m > i_{DC}$ based on Irie-Yamafuji model in which the magnetic fiel dependence of J_c is taken into account. In practical case of DC transmission cable, since $i_m < i_{DC}$ is satisfied the ripple loss is larger than pure AC loss. However, corresponding I_m/I_c is very small in layered structure of DC transmission cable made by thin tape superconductor. Therefore, the value of Q_{ripple} is quite smaller than Q_0 which is the AC loss at $I_m = I_c$.

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