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Calculation of critical current in DC HTS cable using longitudinal magnetic field effect

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Abstract

It is known from experimental data that the critical current for the superconducting wires in case of longitudinal magnetic field is higher than that in perpendicular magnetic field. This property may be useful for designing of DC superconducting cables. In this paper, we proposed the calculation of current carrying capacity for a new DC superconducting cable in which the transport current flowing in the outer conductors applies a longitudinal magnetic field for inner conductors. The critical current of the cable is calculated by the iteration method, and the optimal design of the multilayer cable is found. The comparing of efficiency between the force free cables and conventional superconducting cables is also discussed. It is found that the efficiency of the cable with 10 layers increases up to 13% compared with conventional cables.

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Keywords: Force-free cable; longitudinal magnetic field; carrying current capacity; cuprate high temperature superconductor DC transmission cable

1. Introduction

The effect of increasing of critical current of superconductors in the longitudinal external magnetic field was founded almost 50 years ago. Critical current density of Nb-Ti wire shows several 10 times larger value compared with transverse magnetic field [1]-[3]. The increasing of critical current causes due to a force free (FF) configuration of magnetic field (\mathbf{B}) and current (\mathbf{J}) in the superconductor in longitudinal magnetic field as [4]-[5],

$$\vec{J} \times \vec{B} = 0 . \quad (1)$$

This condition takes place when the magnetic field and the current are parallel each other. When the external magnetic field and the current apply parallel to the wide surface of the superconducting slab with thickness d along z -axis, the angle θ between z -axis and the external magnetic field linearly decreases from the surface to inner part of the superconductor as,

$$\theta = \theta_s - \alpha_f y, \quad 0 \leq y \leq \theta_s / \alpha_f, \quad (2)$$

$$\theta = 0, \quad \theta_s / \alpha_f \leq y \leq d, \quad (3)$$

where α_f is a constant, θ_s is the angle between the current and the magnetic field near the surface of the superconductor [6]. This distribution of the current in the superconductor provides the force free configuration, in which the current is parallel to the magnetic field in the superconductor. In this case, the critical current of the superconductor will be maximal.

The theoretical calculation is performed for the current carrying capacity of the DC transmission cable with the continuous distribution of the angle between the current and the axis of the cable with perfect longitudinal magnetic

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field effect on each point of the cable [7]. It is found that the current carrying capacity drastically increases compared with conventional cable. However, the present stage of cuprate superconductors show complicated characteristics in critical current density in longitudinal magnetic field. Therefore it is necessary to perform the numerical calculation of real construction of the multilayer cable. The aim of this work is the calculation of the carrying current capacity of the model of a cable with longitudinal magnetic field effect.

2. Model of DC transmission cable

The construction of multilayer cuprate superconducting cable presents on Fig. 1. Inner layer of the cable is parallel to the axis of the cable ($\theta_{\text{inn}} = 0^\circ$). Outer most layer has maximal inclination ($\theta_N = \theta_{\text{max}}$). The distribution of the tape angles between inner and outer layers is linear.

It is necessary to know the dependence of the critical current density vs. the magnetic field and the angle between the superconducting tape and the magnetic field, for the calculation of the current carrying capacity of the cable. In this work, the linear dependence of the critical current vs. the external magnetic field is assumed. It was used the same dependences of the engineering critical current densities in the longitudinal and the transverse magnetic field of the superconducting tape to compare the results of present calculation with the previous work [7]. It is chosen the linear angular dependence for intermediate angles in this work. The magnetic field and the angular dependence for the engineering critical current density J is given by

$$J(B, \theta_{B\Sigma, j}) = \alpha + \left[\beta_{\parallel} + (\beta_{\perp} - \beta_{\parallel}) \cdot \frac{|\theta_{B\Sigma, j}|}{\pi/2} \right] \cdot B, \quad (4)$$

Where $\theta_{B\Sigma, j}$ is the angle between the current and the magnetic field. $\alpha = 5 \times 10^8 \text{ A/m}^2$, $\beta_{\parallel} = 6 \times 10^8 \text{ A/m}^2$, $\beta_{\perp} = -4 \times 10^8 \text{ A/m}^2$ are parameters.

It is considered the multilayer cable containing $N+1$ layers of superconducting tapes to create the model of cable (numbering of layers, $i = 0 \dots N$). The inner radius of the superconducting cylinder is $R_0 = 20 \text{ mm}$. The thickness of the superconducting layer is $d = 0.1 \text{ mm}$. The radius of i -th layer is $R_i = R_0 + di$.

Because the layers act as the hollow cylinders with the axial and the azimuthal components of the current, it means that the current in k -th internal layer induces the azimuthal magnetic field to outer layers, and is given by

$$B_{i,k,\text{in}} = \frac{\mu_0 \cdot I_k \cdot \cos(\theta_k)}{2 \cdot \pi \cdot R_i}, \quad (5)$$

where I_k is the summation of all currents in k -th layer tapes, R_i is the radius of i -th layer, θ_k is the angle between the direction of the current flow in the tapes of k -th layer and the axis of cable.

On the other hand, the azimuthal component of the current in k -th layer induces axial magnetic field to i -th layer, and is given by

$$B_{i,k,\text{out}} = \frac{\mu_0 \cdot I_k \cdot \sin(\theta_k)}{2 \cdot \pi \cdot R_k} \cdot \tan(\theta_k), \quad (6)$$

where R_k is the radius of k -th layer.

The magnetic field in the i -th layer $B_{i,\Sigma}$ is the combination of the magnetic field from inner layer $B_{i,\text{in},\Sigma}$ and the magnetic field from outer layer $B_{i,\text{out},\Sigma}$ and is given by

$$B_{i,\text{in},\Sigma} = \sum_{k=0}^{i-1} \frac{\mu_0 \cdot I_k \cdot \cos(\theta_k)}{2 \cdot \pi \cdot R_i}, \quad (7)$$

$$B_{i,\text{out},\Sigma} = \sum_{k=i+1}^N \frac{\mu_0 \cdot I_k \cdot \sin(\theta_k)}{2 \cdot \pi \cdot R_k} \cdot \tan(\theta_k), \quad (8)$$

$$B_{i,\Sigma} = \sqrt{(B_{i,\text{in},\Sigma})^2 + (B_{i,\text{out},\Sigma})^2}, \quad (9)$$

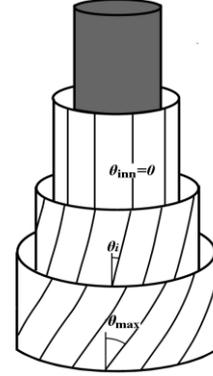


Fig. 1. Structure of multilayer cable designed for force free configuration of current and magnetic field.

The angle between the current in the i -th layer and the magnetic field acting to the i -th layer is given by

$$\theta_{B_{i,\Sigma},i} = \theta_i - \arctan\left(\frac{B_{i,in,\Sigma}}{B_{i,out,\Sigma}}\right) = \theta_i - \arccos\left(\frac{B_{i,out,\Sigma}}{B_{i,\Sigma}}\right). \quad (10)$$

The critical current in i -th layer can be estimated from the critical current density in i -th layer J_i given by Eq. (4) and magnetic fields given by Eqs. (5)—(8) as

$$I_i = J_i \cdot 2 \cdot \pi \cdot R_i \cdot d \cdot \cos(\theta_i). \quad (11)$$

The current carrying capacity I_t is the summation of the critical current at each layer given by Eq. (11). Each critical current can be numerically calculated by the iteration method. The real amount of iteration during the calculation was up to 100, and the value of total inaccuracy was less than 10^{-6} A/m² for critical current density.

3. Results and discussion

The aim of this work is finding the optimal configuration of the cable. Fig. 2(a) shows the angle of the maximal inclination of the outer layer, θ_{\max} , dependence of the current carrying capacity for the cable with 7-layers without external magnetic field, $B_{\text{ext}} = 0$ T. Top curve in Fig. 2(a) shows the result that angles between layers and cable axis are linearly increased as a function of the number of layer from inner part to outer part of the cable. It is found that the current carrying capacity has the optimal construction of the cable when θ_{\max} is near 55° . It should be noted that the each angle between current of each layer and local magnetic field is not 0° except of $\theta_{\max} = 90^\circ$, while the transport current of outer layer becomes small.

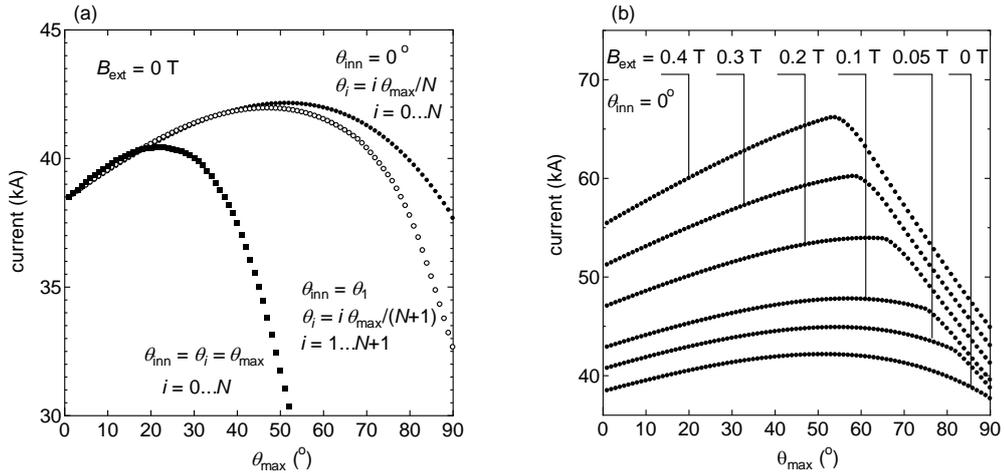


Fig. 2. (a) The dependences of current carrying capacity for 7-layers cable vs. maximal inclination of outer layer θ_{\max} . Top curve for the cable with linear distribution of tape angles from $\theta = 0^\circ$ for inner layer and linear distribution up to θ_{\max} for outer layer. Second curve for the cable with linear distribution of tape angles from first step of inclination θ_1 up to θ_{\max} (also 7 layers). Lower curve represents for the cable in which the inclinations for all layers are θ_{\max} . (b) The dependence of critical current for 7-layers cable under different longitudinal magnetic field.

The practical realization of the cable with inner layer parallel to the axis is difficult, because this cable is not possible to bend. Second curve in Fig. 2(a) shows the result that the first layer has angle of θ_1 to solve this problem. The value of maximal current carrying capacity in this case decreases about 0.5% comparing with top curve. It means that it is possible to omit the first inner layer which is parallel to the axis of the cable without large decrease of total current carrying capacity.

Next task of this work, the characteristics of force free cable is comparing with characteristics of conventional superconducting cables. The current carrying capacity I_t of the same 7-layer cable with the same inclination of layers θ_{\max} for the all layers is calculated as shown in lower curve in Fig. 2(a). The maximal current carrying capacity of this cable is less than that of the same force free cable about 4.2%.

The increasing of current carrying capacity is expected by adding of the external longitudinal magnetic field. The influence of external longitudinal magnetic field on the 7-layer FF cable is shown on the Fig. 2(b). Table 1 shows the characteristics of maximum value of current carrying capacity for cables with different amount of layers. It is found that external magnetic field B_{ext} leads to the drastically increasing of I_{tmax} of the cable. The value of the field B_{ext} was chosen to have the maximal longitudinal magnetic field effect under $\theta_{\text{max}} \sim 55^\circ$.

Table 1. The efficiency of cables with different number of layers.

| No. of layers | B_{ext} [T] external magnetic field | I_t [kA] (max. current carrying capacity under $\theta_{\text{max}} \sim 55^\circ$) | I_0 [kA] (total current carrying capacity under $\theta_{\text{max}} = 0^\circ$) | $I_{\text{tmax}}/I_0(\theta_{\text{max}} = 0^\circ)$ (this work) | $I_{\text{tmax}}/I_0(\theta_{\text{max}} = 0^\circ)$ [7] | I_{tmax} of FF/ I_{tmax} of conventional cable |
|---------------|--|--|---|--|--|---|
| 4 | 0.164 | 28.5 | 23.5 | 1.21 | 1.24 | 1.008 |
| 5 | 0.226 | 38.3 | 28.7 | 1.33 | 1.40 | 1.017 |
| 6 | 0.301 | 50.6 | 33.6 | 1.51 | 1.59 | 1.026 |
| 7 | 0.396 | 67 | 38.5 | 1.74 | 1.82 | 1.042 |
| 8 | 0.517 | 85 | 43 | 1.98 | 2.13 | 1.064 |
| 9 | 0.680 | 111 | 47.5 | 2.34 | 2.53 | 1.100 |
| 10 | 0.909 | 146 | 51.5 | 2.83 | 3.11 | 1.139 |

The coefficients of increasing of current carrying capacity for cables with different amount of layers between cables with longitudinal magnetic field effect and external magnetic field (B_{ext}) and cables without external magnetic field where all superconducting tapes is parallel to the axis of the cable ($I_{\text{tmax}}/I_0(\theta_{\text{max}} = 0^\circ)$) present on the Table 1. In the paper [7], it is assumed that θ_i changes continuously and all parts of cable are in under perfect longitudinal magnetic field. It is possible to see that calculated coefficient ($I_{\text{tmax}}/I_0(\theta_{\text{max}} = 0^\circ)$) is close to the coefficient from previous work [7]. In the last column of the Table 1 represents the efficiency coefficient of the force free cable comparing with convenient cable in which all layers have the same inclination. It is found that the force free cable is more effective than usual cable.

4. Conclusion

In this work, the modeling of multilayer HTS cable was performed. It is found that longitudinal magnetic field effect allows drastically improvement in the current carrying capacity of the superconducting DC cables. The proposed method for modeling multilayer cables and calculating current carrying capacity may be useful to design cables with real dependences of critical current as functions of field and angle between field and current.

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