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# Scaling properties of the longitudinal and transversal asymmetries of the $\vec{n}\vec{d}$ total cross section

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**Abstract**

The longitudinal and transversal asymmetries of the total $\vec{n}\vec{d}$ cross section are calculated. Four modern nucleon-nucleon interactions: AV18, CD Bonn, NijmI and NijmII, give different predictions for these observables. When the three-nucleon Hamiltonian is supplemented by the $2\pi$-exchange Tucson-Melbourne three-nucleon force (3NF), individually adjusted with each particular NN potential to reproduce the experimental triton binding energy, all predictions practically coincide. We propose to check this scaling behavior experimentally in order to get a clear signal for 3NF effects in the low energy three-nucleon continuum. Connected to that is the proposal to measure the energy at which the longitudinal asymmetry goes through zero. This energy is shifted by about 400 keV when 3NF's are acting. © 1999 Elsevier Science B.V. All rights reserved.

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Substantial progress in theoretical and numerical methods achieved recently in the study of the three-nucleon (3N) system allows now to obtain reliable theoretical predictions in the 3N continuum which are based on modern nucleon-nucleon (NN) interactions, including even three-nucleon forces (3NF) [1]. With the drastically improved quality of the present day NN potentials [2–4] shown in their unprecedented accuracy in reproducing the NN data set ($\chi^2$/datum very close to 1), it is now possible to approach the basic and interesting question concerning the significance of 3NF's in the elastic nucleon-deuteron (Nd) scattering and breakup processes. What is needed are clearcut signals coming from certain observables which deny to be explained by 3N Hamiltonians based on modern NN interactions only. The first signature for insufficient dynamics based on present day NN forces alone comes from the 3N binding energy. The modern NN interactions underbind $^3H$ by about 500-800 keV [5]. In the 3N continuum the bulk of 3N scattering observables can be described quite well in the NN force picture only [1]. Recently, however, it was found that a large discrepancy between modern NN potential predictions and data exist in the minima of the elastic Nd scattering cross sections for incoming nucleon energies greater than $\approx 60$ MeV [6]. A large part of this discrepancy can be removed when the $2\pi$-exchange


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Tucson-Melbourne (TM) 3NF [7] properly adjusted to the triton binding is included in the 3N Hamiltonian. The question arises if in the low energy 3N-continuum observables exist which also give clearcut signals for the action of 3NF's, similar to those coming from the $^3He$ and $^3H$ binding energies. The low energy vector analyzing power $A_y$ in elastic Nd scattering (as well as $iT_{11}$) could be such a case. There a dramatic discrepancies exist between the predictions based on NN forces only and both nd and pd data [8,9]. However, the present day 3NF models have insignificant effects and do not remove that discrepancy. This possibly implies that it is caused by defects in the NN force [9] or has its source in still undiscovered 3NF properties [10].

In the present study we propose additional candidates for 3NF effects, the longitudinal ($\Delta\sigma_L$) and transversal ($\Delta\sigma_T$) asymmetries of the total neutron-deuteron (nd) cross section with both neutron and deuteron polarized. They are defined as

$$\Delta\sigma_L = \sigma_{nd}^{tot}\left( p_z^n, -p_z^d \right) - \sigma_{nd}^{tot}\left( p_z^n, p_z^d \right)$$

$$\Delta\sigma_T = \sigma_{nd}^{tot}\left( p_y^n, -p_y^d \right) - \sigma_{nd}^{tot}\left( p_y^n, p_y^d \right), \tag{1}$$

where $\sigma_{nd}^{tot}$ is the nd total cross section for specified vector polarizations of the incoming neutron and deuteron (for instance $p_z^n$,($-p_z^d$) means that the neutron (deuteron) is polarized in z(-z)-direction). The z-axis is chosen parallel to the beam direction and the y-axis perpendicular to the scattering plane.

Using the optical theorem [1] it follows that the total nd cross section with the incoming polarization state described by the density matrix $\rho^{in}_{m_n,m_d;m_{n'},m_{d'}}$ is given by

$$\sigma_{tot}^{pol} = -\frac{4m}{3}\frac{1}{q_0}(2\pi)^3 \times \sum_{m_n m_d m_{n'} m_{d'}} Im\left[ \rho^{in}_{m_n m_d;m_{n'} m_{d'}} U_{m_{n'} m_{d'};m_n m_d} \right], \tag{2}$$

where $U_{m_n^{out},m_d^{out};m_n^{in},m_d^{in}}$ is the forward elastic nd scattering amplitude at relative neutron-deuteron momentum $q_0$.

Inserting the density matrix $\rho^{in}$ for the specific polarization states of Eq. (1) leads to the following expressions for the longitudinal and transversal asymmetries

$$\Delta\sigma_L = -\frac{4m}{3}\frac{1}{q_0}(2\pi)^3 p_z^n p_z^d \times Im\left[ U_{1/2,-1;1/2,-1} - U_{1/2,1;1/2,1} \right]$$

$$\Delta\sigma_T = \frac{4m}{3}\frac{1}{q_0}(2\pi)^3 \frac{1}{\sqrt{2}} p_y^n p_y^d \times Im\left[ U_{-1/2,1;1/2,0} + U_{-1/2,0;1/2,-1} \right]. \tag{3}$$

The transition amplitude for elastic Nd scattering is composed of the nucleon exchange part ($PG_0^{-1}$), the direct action of a 3NF part $V_4^{(1)}(1+P)$ and a part having its origin in the multiple interactions of 3 nucleons through 2N and 3N forces:

$$<\phi'|U|\phi> = <\phi'|PG_0^{-1} + V_4^{(1)}(1+P) + P\tilde{T} + V_4^{(1)}(1+P)G_0\tilde{T}|\phi>. \tag{4}$$

That rescattering part is expressed in terms of a $\tilde{T}$ operator which sums up all multiple scattering contributions through the integral equation [11]

$$\tilde{T}|\phi> = tP|\phi> + (1+tG_0)V_4^{(1)}(1+P)|\phi> + tPG_0\tilde{T}|\phi> + (1+tG_0)V_4^{(1)}(1+P)G_0\tilde{T}|\phi>. \tag{5}$$

Here $G_0$ is the free 3N propagator, $t$ the NN $t$-matrix, and $P$ the sum of a cyclical and anticyclical permutation of three objects. The 3NF $V_4$ is split into 3 parts

$$V_4 = \sum_{i=1}^{3} V_4^{(i)}, \tag{6}$$

where each one is symmetrical under exchange of two particles. For the $\pi - \pi$ exchange 3NF for instance [7], this corresponds to the three possible choices of the nucleon, which undergoes the (off-shell) $\pi - N$ scattering. The asymptotic state $|\phi>$ ($|\phi'>$) is a product of the deuteron wave function and the momentum eigenstate of the neutron.

We calculated $\Delta\sigma_L$ and $\Delta\sigma_T$ at a number of neutron energies ranging between 0.6 MeV and 65 MeV by solving Eq. (5) with four modern NN interactions: AV18 [3], CD Bonn [4], Nijm I and Nijm II [2]. As the 3NF we took the $2\pi$-exchange TM model [7] where the strong cut-off parameter $\Lambda$

Table 1
The longitudinal asymmetry $\Delta\sigma_L$ without and with TM 3NF included

| $E_{lab}$ (MeV) | $\Delta\sigma_L$ (mb) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | AV18 | AV18 + TM | CDBonn | CDBonn + TM | NijmI | NijmI + TM | NijmII | NijmII + TM |
| 0.6 | −2432.6 | −2577.3 | −2524.4 | −2601.6 | −2481.0 | −2598.4 | −2463.8 | −2581.6 |
| 0.8 | −2318.5 | −2463.9 | −2407.5 | −2486.6 | −2364.8 | −2484.0 | −2347.9 | −2466.9 |
| 1.0 | −2211.6 | −2356.3 | −2297.0 | −2377.1 | −2255.4 | −2375.1 | −2239.1 | −2358.1 |
| 3.0 | −1246.2 | −1355.3 | −1298.5 | −1364.1 | −1271.7 | −1365.8 | −1260.8 | −1352.7 |
| 5.0 | −698.3 | −769.6 | −726.8 | −772.3 | −712.2 | −772.8 | −697.8 | −766.8 |
| 6.88 | −366.0 | −413.8 | −382.4 | −413.6 | −372.2 | −415.1 | −369.4 | −410.9 |
| 8.92 | −154.3 | −186.1 | −163.3 | −185.1 | −156.8 | −185.9 | −155.6 | −183.9 |
| 12.0 | 7.4 | −9.9 | 4.3 | −8.6 | 7.8 | −8.6 | 7.5 | −8.5 |
| 19.0 | 113.9 | 110.6 | 115.5 | 111.6 | 116.2 | 112.2 | 114.9 | 110.9 |
| 65.0 | 27.7 | 30.8 | 28.1 | 29.7 | 28.5 | 30.9 | 27.8 | 30.0 |

has been adjusted individually together with each NN force to the experimental triton binding [5]. In the calculations we included all partial wave states with total angular momenta in the two-nucleon subsystem up to $j_{max} = 3$. For more details on the underlying theoretical formalism and the numerical performance we refer to [1,8]. The resulting values for $\Delta\sigma_L$ and $\Delta\sigma_T$ are shown in Tables 1 and 2, respectively.

We found that the predictions for $\Delta\sigma_L$ and $\Delta\sigma_T$ obtained with NN interactions only scatter within a range of $\approx 5$–10% depending on the energy. However, including the 3NF drastically reduces the spread of $\Delta\sigma_L$ and $\Delta\sigma_T$ values to within a range of $\approx 1$–2% at incoming neutron energies smaller than $\approx 20$ MeV. This scaling behavior is especially pronounced for $\Delta\sigma_L$, where it extents up to $\approx 40$ MeV. The scaling behavior diminishes and is finally lost for both $\Delta\sigma_L$ and $\Delta\sigma_T$ at higher energies.

Even more interesting is that these scaled values of $\Delta\sigma_L$ and $\Delta\sigma_T$ differ by about 5% from the averaged predictions of NN potentials only. This make both asymmetries good candidates for providing a clear 3NF signal detectable by appropriate measurements. In Figs. 1 and 2 we show the ratios of $\Delta\sigma_L/p_z^n p_z^d$ and $\Delta\sigma_T/p_y^n p_y^d$ to their average value (averaged over four potential results when 2N and 3NF's are acting). Both cases are shown: pure NN force predictions and NN + 3NF predictions. In Fig. 1 it is seen that for $\Delta\sigma_L$ the influence of the 3NF is especially large around $\approx 12$ MeV where $\Delta\sigma_L$ changes its sign. At 12 MeV the pure 2N force predictions for $\Delta\sigma_L^{2NF}/ < \Delta\sigma_L^{2NF+3NF} >$ (not shown in Fig. 1), are negative; (the corresponding

Table 2
The transversal asymmetry $\Delta\sigma_T$ without and with TM 3NF included

| $E_{lab}$ (MeV) | $\Delta\sigma_T$ (mb) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | AV18 | AV18 + TM | CDBonn | CDBonn + TM | NijmI | NijmI + TM | NijmII | NijmII + TM |
| 0.6 | −2464.2 | −2608.2 | −2556.4 | −2632.0 | −2512.2 | −2628.1 | −2494.0 | −2610.4 |
| 0.8 | −2355.0 | −2499.2 | −2444.3 | −2520.8 | −2400.4 | −2517.2 | −2382.3 | −2499.0 |
| 1.0 | −2258.6 | −2401.4 | −2344.2 | −2420.7 | −2301.1 | −2417.5 | −2283.4 | −2399.0 |
| 3.0 | −1478.1 | −1586.2 | −1531.5 | −1591.9 | −1501.3 | −1591.8 | −1487.1 | −1575.0 |
| 5.0 | −1034.0 | −1107.8 | −1065.0 | −1106.8 | −1046.8 | −1107.6 | −1025.6 | −1095.6 |
| 6.88 | −737.0 | −791.7 | −756.1 | −788.0 | −742.5 | −789.7 | −735.0 | −779.7 |
| 8.92 | −529.7 | −569.8 | −541.7 | −565.5 | −532.1 | −567.2 | −526.4 | −559.3 |
| 12.0 | −345.3 | −371.6 | −351.7 | −367.6 | −345.6 | −369.1 | −341.6 | −363.3 |
| 19.0 | −162.9 | −174.2 | −164.2 | −171.4 | −161.8 | −172.3 | −159.7 | −169.1 |
| 65.0 | −18.8 | −18.6 | −17.6 | −17.6 | −17.7 | −18.0 | −17.3 | −17.2 |

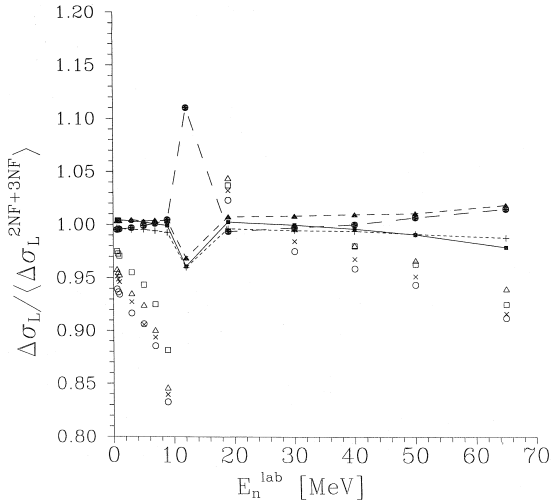


Fig. 1. The ratios of $\Delta\sigma_L$ to their average value (see text) as a function of incoming neutron lab. energy. The open circles, open squares, open triangles, and crosses are the AV18, CDBonn, NijmI, and NijmII potential predictions, respectively. The full circles connected with a long dashed line, the full squares connected with a solid line, the full triangles connected with a dashed line, and the pluses connected with a short dashed line are the corresponding predictions when the 3NF is included.

values are $-0.84$, $-0.48$, $-0.87$, and $-0.84$ for AV18, CDBonn, NijmI and NijmII, respectively) while adding the 3NF positive predictions result. However, even in this energy region the scaling behaviour is not destroyed. At 12 MeV only the AV18 + 3NF prediction deviates by about 15% from a practically overlapping results for the other three potentials. This deviation, however, is small compared to the changes caused by adding the 3NF to the pure 2N force predictions at this energy.

Another interesting point is that the pure 2N force predictions lead to the energy $E_n^{cross}(2N) \approx 11.8$ MeV at which $\Delta\sigma_L$ changes sign. This value is different by about 400 keV when the 3NF is acting. Then $\Delta\sigma_L$ crosses zero at $E_n^{cross}(2N+3NF) \approx 12.2$ MeV. The magnitude of that energy shift is large enough to be detected by the measurement of $\Delta\sigma_L$.

The situation is similar for $\Delta\sigma_T$ as shown in Fig. 2. There is, however, no zero crossing.

Summarizing, we have shown that the 3NF changes the magnitudes of the longitudinal and transversal asymmetries of the total $\vec{n}\vec{d}$ cross sections. These changes show at low energies scaling character bringing different predictions based on modern 2N potentials together when the proper 3NF is added, which together with the particular NN interaction reproduces the experimental triton binding energy. The magnitude of the effect is of the order of 5%-10%, which is large enough to be measured. In addition, the zero-crossing energy of the longitudinal asymmetry is shifted by about 400 keV when the 3NF is acting. This shift is also sufficiently large to be checked experimentally. The results of such measurements would form a data basis to test the present day 3NF models in the low energy region of the 3N continuum.

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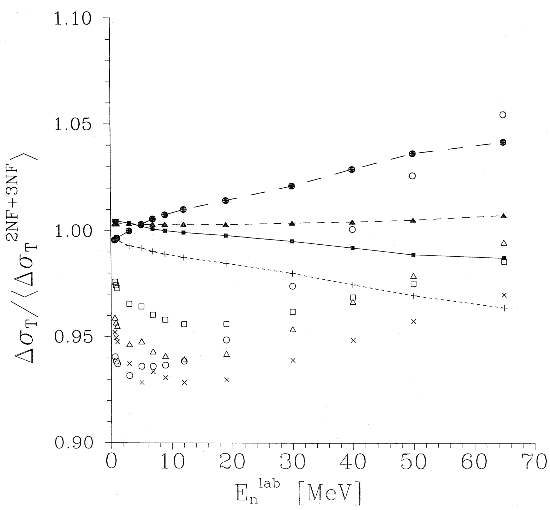


Fig. 2. The same as in Fig. 1 but for $\Delta\sigma_T$.

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