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New Screened Coulomb Potentials and Renormalization

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We introduce a new screening function which is useful for the few-body Coulomb scattering problem in the *screening and renormalization* scheme. We modify the Yukawa-type screened Coulomb potential more useful. In the proton-proton scattering, we demonstrate high precision calculations of the new renormalization.

1. *Introduction* When one considers a system consisting of a few charged particles, it is well known that there are serious difficulties involved in the calculation of scattering processes.[1)] This is caused by the long range nature of the Coulomb potential in coordinate space, or, equivalently, its singularity in momentum space.

Here we note two approaches for overcoming these difficulties, a modified time-dependent scattering theory,[2)] and a *screening and renormalization* method. In the former, the coherent Gaussian function is used as the initial state instead of the Coulombic plane wave. This makes the difficulties easier to treat. However, many investigations have been done in few-body systems using the latter approach. Alt et al.[3)] carried out computations for three-body scattering processes with charged particles employing the screening Coulomb potential and the renormalization scheme.

However, the screening radius R used in those calculations is typically about 600 fm, which is quite large in comparison with the short range force. At such a large radius, the screened Coulomb potential is no longer smooth, and this necessitates a careful treatment in momentum space, and great amounts of memory and cpu time in numerical computations. Therefore, it is desirable to work with a smaller radius. The purpose of this paper is to investigate a new screening function, which is different from that used by Alt et al.[3)] and to determine the precision of the results that can be obtained with results the new renormalization factor for both large and small values of R.

2. *Renormalization scheme* Before we discuss the new screening function, we would like to point out the significance of the renormalization. If one considers a bound state, numerical calculations can be performed by employing the screening method with an appropriately large radius, R, and the result will be independent of the choice of R. However, for a scattering state, the situation is completely different.

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In this case, the limit of the solutions obtained with screened Coulomb potentials as R increases will not agree with the solutions obtained using a pure Coulomb potential. This is due to the fact that different boundary conditions are used in the two cases. To solve this problem, a renormalization method, like that introduced by Taylor,[4] is necessary.

The following Yukawa-type screened Coulomb potential with screening radius R has been used:

$$V^R(r) = e^2 \frac{\exp[-(r/R)]}{r}. \tag{1}$$

In this paper we consider proton-proton scattering, and therefore, here, e represents the charge of the proton. We evaluate the phase shift for pp scattering in the state $^1S_0$. As a typical NN force, we choose the Reid Soft Core potential,[5]

$$V^{NN}(r) = -10.463\frac{\exp[-\mu r]}{\mu r} - 1650.6\frac{\exp[-4\mu r]}{\mu r} + 6484.2\frac{\exp[-7\mu r]}{\mu r} \quad [\text{MeV}], \tag{2}$$

with μ is 0.7 fm^{-1}.

The following relation,[1),4)] among various phase shifts, is obtained by considering the asymptotic behavior of the wave function:

$$\delta_0^{(R)} \xrightarrow[R\to\infty]{} \delta_0^{SC} + \sigma_0 + \phi_R. \tag{3}$$

Here, $\delta_0^{(R)}$ is the phase shift of the NN force and the screened Coulomb potential, δ_0^{SC} is that of the NN force and the pure Coulomb potential, σ_0 is the standard Coulomb phase shift obtained from $\arg \Gamma(1+i\eta)$, and ϕ_R is the renormalization phase. For higher partial waves (angular momentum l) we need to replace σ_0 by $\sigma_l = \arg \Gamma(1+l+i\eta)$. Further, $\eta = e^2 m/2p$ is the Sommerfeld parameter, with m the nucleon mass and $p(=\sqrt{mE_{cm}})$ the relative momentum. According to Taylor,[4] the renormalization phase $\phi_R(p)$ is given by

$$\phi_R(p) \equiv -\eta \int_{(2p)^{-1}}^{\infty} \frac{\exp[-(r/R)]}{r} dr. \tag{4}$$

We evaluated this by numerical integration.

The phase shifts δ_0^{SC}, $\delta_0^{(R)}$, σ_0 and ϕ_R are plotted in Fig. 1. We calculated δ_0^{SC} and $\delta_0^{(R)}$ by solving the Schrödinger equation in configuration space. The asymptotic form of the wave function for the screened Coulomb potential involves the Bessel regular/irregular functions. For the case of no screening, the pure Coulomb regular/irregular functions are used. The renormalization phase ϕ_R is not needed in the high energy region (see Fig. 1) because p in the denominator of the Sommerfeld parameter is large in that case.

As mentioned in introduction, the screening radius R must have a large value when we use the screening function in Eq. (1), in order to obtain a good numerical result by using $\delta_0^{(R)} - \sigma_0 - \phi_R$ instead of δ_0^{SC} [see Eq. (3)]. Therefore we should search for a new screening function that gives good numerical results for smaller values of R.

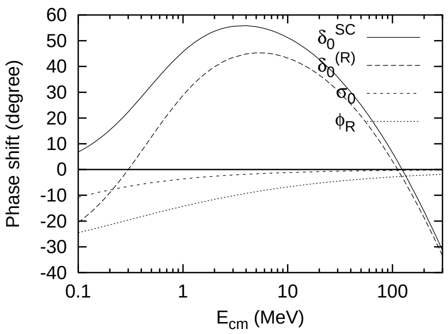

Fig. 1. Comparison of the phase shifts δ_0^{SC}, $\delta_0^{(R)}$, σ_0 and ϕ_R for $R = 50$ fm.

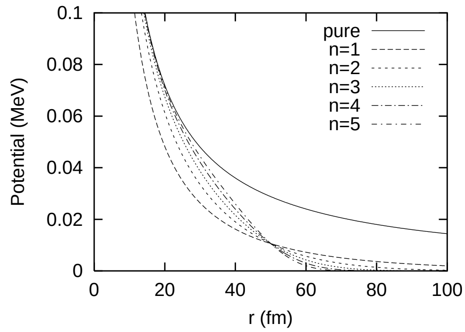

Fig. 2. Pure and screened Coulomb potentials for $n = 1 - 5$.

3. *New screening functions* We now introduce a new screened Coulomb potentials of the form

$$V_n^R(r) = e^2 \frac{\exp[-(r/R)^n]}{r}. \tag{5}$$

Note that Eq. (5) reduces to the Yukawa-type potential for $n = 1$ [see Eq. (1)] and that it converges to a sharply truncated potential as $n \to \infty$,

$$\lim_{n\to\infty} V_n^R(r) = \frac{e^2}{r} \cdot \theta(R - r), \tag{6}$$

where $\theta(r)$ is the step function. The pure and screened Coulomb potentials for $n =$ 1 to 5 at $R = 50$ fm are displayed in Fig. 2. This figure reveals that as n increases, the cutoff becomes sharper, and the pure Coulomb potential is better represented in the inner region.

As a measure of the quality of the new screening function, we introduce the quantity

$$\varDelta\delta \equiv \ \delta_0^{SC} + \phi_n^R + \sigma_0 - \delta_0^{(R)}. \tag{7}$$

We next study the dependences of (7) on the c.m. energy E_{cm}, the screening radius R and the power n. The renormalization phase $\phi_n^R(p)$ is calculated as

$$\phi_n^R(p) = -\eta \int_{(2p)^{-1}}^{\infty} \frac{\exp[-(r/R)^n]}{r} dr. \tag{8}$$

This quantity generally depends on the power n. Although we integrate (8) numerically in this paper, an analytic expression is given with any value of n:

$$\phi_n^R(p) = -\eta \left[\ln(2pR) - \frac{\gamma}{n}\right] + \eta \sum_{k=1}^{\infty} \frac{(-1)^k}{knk!(2pR)^{kn}}, \tag{9}$$

with the Euler constant γ $(= 0.5772\cdots)$. The last term has an order of $0(1/R^n)$, and the first term is derived from

$$\int_0^R \frac{\exp[-(r/R)^n] - 1}{r} dr + \int_R^{\infty} \frac{\exp[-(r/R)^n]}{r} dr = -\frac{\gamma}{n}, \tag{10}$$

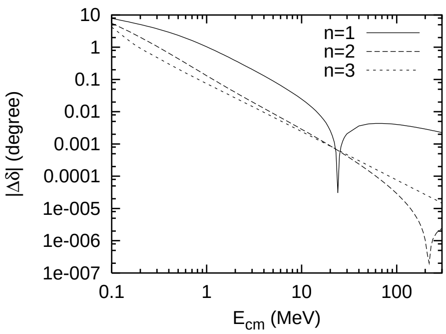

Fig. 3. $|\Delta\delta|$ for $R = 50$ fm as a function of E_{cm} for various values of n.

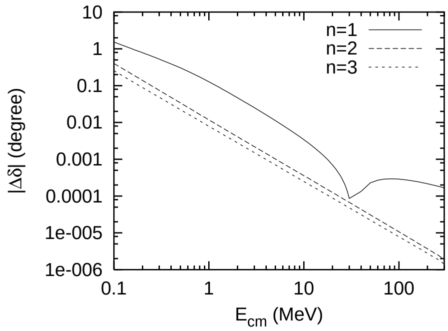

Fig. 4. The same as Fig. 3 for $R = 500$ fm.

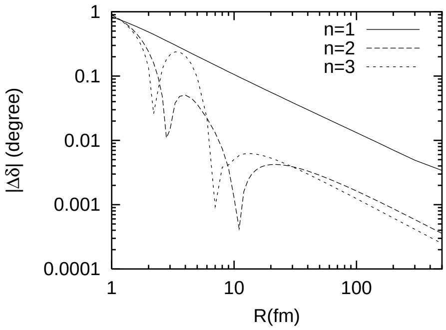

Fig. 5. $|\Delta\delta|$ for $E_{cm} = 10$ MeV as a function of R for various values of n.

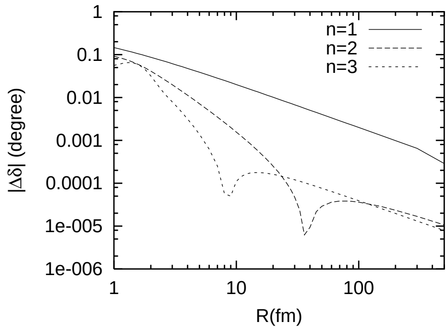

Fig. 6. The same as Fig. 5 for $E_{cm} = 100$ MeV.

where we use the replacement $s = (r/R)^n$.

The quantity $|\Delta\delta|$ is plotted in Figs. 3–8. In Figs. 3 and 4 we plot the dependence of $|\Delta\delta|$ on E_{cm} for $R = 50$ fm and 500 fm, respectively. The value of $|\Delta\delta|$ decreases rapidly with E_{cm}. These figures clearly show that the calculations for $n \geq 2$ are more precise than that for n=1. In consideration of Fig. 1, we are led to conjecture that this is due to the fact that the screened Coulomb potential is closer to the pure Coulomb potential in the inner region when n is larger than 1.

In Figs. 5 and 6, we illustrate the R dependence of $|\Delta\delta|$ for two fixed values of E_{cm}, 10 and 100 MeV. We find that $|\Delta\delta|$ does indeed decrease as R increases, but again, the results are better behaved for $n > 1$ than for $n = 1$. Figures 7 and 8 plot the n dependence of $|\Delta\delta|$ for fixed E_{cm} and various values of R at E_{cm}= 10 and 100 MeV, respectively. It is seen that $R = 50$ fm and n=2 is sufficient to obtain precise results.

4. *Summary* We generalized the Yukawa-type screened potential that was adopted by Alt et al.[3)] to the form given in Eq. (5). We find that already for n=2, the screened Coulomb potential given in Eq. (5) is closer to the pure Coulomb potential in the inner region. Our numerical results reveal that high precision calculations can be carried out with the new screened Coulomb potentials for $n \geq 2$ and a screening radius as small as R=50 fm.

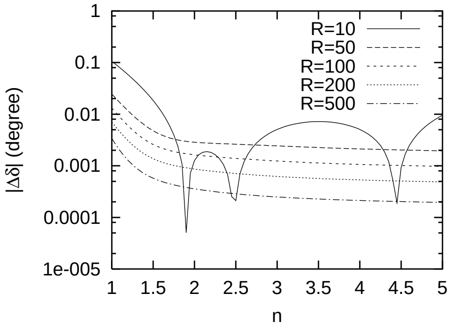

Fig. 7. $|\Delta\delta|$ for $E_{cm} = 10$ MeV as a function of n for various values of R.

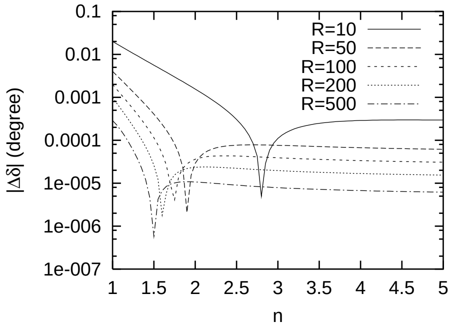

Fig. 8. The same as Fig. 7 for $E_{cm} = 100$ MeV.

Looking to further works, we note that the new screened Coulomb potential allows us to calculate values for 3-body scattering systems more accurately than the former potential with which Alt used the large radius R=600 fm.[1),3)]

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