Triton binding energies for modern NN forces and the π-π exchange three-nucleon force

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Abstract

We present high precision calculations for the triton binding energy using the most recent, phase equivalent realistic nucleon-nucleon (NN) potentials and the Tucson-Melbourne π-π three-nucleon force (3NF). That 3NF is included up to a total two-body angular momentum of $j_{max} = 6$. It is shown that the inclusion of the 3NF slows down the convergence in the partial waves and $j_{max} = 5$ is needed in order to achieve converged results within a few keV. We adjust the cut-off parameter Λ in the form factors of the Tucson-Melbourne 3NF separately for the different NN potentials to the triton binding energy. This provides a set of phenomenological 3N Hamiltonians which can be tested in three-nucleon scattering and systems with $A > 3$. Their NN correlation functions are compared to each other. © 1997 Elsevier Science B.V.

It has been possible to include a 3NF into Faddeev calculations for the 3N bound state since many years [1–4]. In configuration space calculations were performed up to a maximal two-body angular momentum of $j_{max} = 4$ [5]. The same has been achieved using an admixture of configuration and momentum space [6–8]. In momentum space no efforts beyond $j_{max} = 2$ have been reported up to now [9,10].

Recently, due to a new way of partial wave decomposition (PWD) for the 3NF in momentum space [11], it became possible to include higher partial waves of the 3NF in momentum space with $j > 2$. The old PWD [12,13] used up to now leads to untractable

numerical instabilities for partial waves with $j > 2$. Previous results based on the 3NF up to $j_{max} = 2$ are however not affected. One aim of this study is to extend the momentum space calculations for the triton binding energy to higher partial waves to demonstrate convergence within an accuracy of a few keV.

The other aim is provoked by an ambiguity in the Tucson-Melbourne (TM) π-π exchange 3NF [14,12]. In the TM 3NF the meson nucleon form factors are parametrised by a certain cut-off parameter Λ, whose value is only roughly known. That parameter Λ acts like a strength factor of that 3NF and the 3N binding energy is quite sensitive to Λ. A variation of Λ within one pion mass causes differences in the 3N binding energy of about 1.5 MeV. One can add

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additional two-meson exchange 3NFs [15,10] like the π-ρ potential, which counteracts the attraction of the π-π potential. Chiral perturbation theory suggests many more structures [16,17]. In this situation, where there is not yet a theory which predicts consistent NN and 3N forces, a phenomenological approach appears to be justified. Therefore the second aim is to adjust certain 3NFs in conjuction with given NN forces to the triton binding energy. Such an approach has been already taken by the Urbana-Argonne Collaboration [18,19]. Performing now this fit for several modern NN potentials one gets a number of 3N Hamiltonians which all give the same (correct) triton binding energy. Using these models one can then explore the 3N continuum and search for interesting 3NF effects in elastic nd scattering and in nd breakup observables. Also one might go on to $A > 3$ systems, which has already been pioneered in [20].

We fit Λ in conjuction with the most recent, phase-equivalent realistic NN potentials, the phenomenological potentials AV18 [21], Nijm I, Nijm II and Reid 93 [22], and the meson theoretical potentials Nijm 93 [22] and CD-Bonn [23]. Note that Nijm II was recently refitted in the $^1P_1$-wave in order to remove an unphysical bound state at -964 MeV [24]. The potentials Nijm II and Reid 93 are purely local, whereas Nijm 93, Nijm I and AV18 carry a small non-locality in form of p^2-terms. CD-Bonn, which is defined in momentum space, is strongly non-local and carries all the Dirac structure of the nucleon. All these potentials are fitted perfectly to the recent Nijmegen phase shift analysis [25] with a χ^2 per datum very close to one; only Nijm 93 is fitted less perfectly.

It is clear that these NN forces and the TM-3NF are inconsistent. This is a trivial statement for the phenomenological NN forces, but even for the meson theoretical ones there is no consistent scheme behind the forces. Therefore fitting Λ is just a zeroth order, purely phenomenological step, which will lay some ground to do exploratory steps in 3N scattering and for systems with $A > 3$. This might be accepted until there are predictive, generally accepted consistent NN and 3N forces. The work of the Bochum group [26,27] are steps into this direction.

The Faddeev equation for the 3N bound state reads [9]

$$|\psi\rangle = G_0 \, t \, P \, |\psi\rangle + G_0 \, (1 + t \, G_0) \, V_4^{(1)} \, (1 + P) \, |\psi\rangle \tag{1}$$

Here G_0 is the free 3N propagator, t the NN t-matrix and P the sum of a cyclic and an anti-cyclic permutation of the three nucleons. $|\psi\rangle$ is the Faddeev amplitude, from which one determines the wave function $|\Psi\rangle$ via

$$|\Psi\rangle = (1 + P) \, |\psi\rangle \tag{2}$$

We use the fact that all 3NFs considered up to now can be split into three parts, each of them being symmetric under exchange of two of the three particles:

$$V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)} \tag{3}$$

Therefore $V_4^{(1)}$ goes together with $t \equiv t_{23}$. We solve Eq. (1) in momentum space using a PWD. For details see [9,28].

Let us comment on the numerics. For the discretisation of the Jacobi momenta p and q (for the notation see [28]) we use 40 and 36 Gaussian points, respectively. The cut-offs of the integrals in p and q are 60 fm^{-1} and 20 fm^{-1}, respectively. For the angular integration introduced by the permutation operator P we use 16 points. The nucleon mass is chosen as $m = 938.9$ MeV. Using these sets of grid points we achieve a numerical accuracy in the binding energy of about 0.1%. A good measure for the numerical accuracy of the wave function is to evaluate the energy expectation value $\langle H \rangle \equiv \langle \Psi | H | \Psi \rangle = \langle \Psi | H_0 | \Psi \rangle + \langle \Psi | V | \Psi \rangle$ and compare it to the energy eigenvalue of Eq. (1). We find that these two numbers differ always by less than 0.05%.

First we document in Table 1 the convergence of the triton binding energy with and without 3NF. For that purpose we chose the AV14 [29] NN force together with the TM π-π 3NF ($\Lambda = 5.13 m_\pi$). For that specific model we can compare to the very recent results of the Pisa group. Let us first consider the results as a function of $j \equiv j_{max}$ without 3NF. It can be seen that in order to reach an accuracy of 0.1% one has to take into account all partial waves up to a total two-body angular momentum of $j = 4$. We also list the expectation values of kinetic energy $\langle H_0 \rangle$, NN potential energy $\langle V \rangle$, and 3N potential energy $\langle V_4 \rangle$. Table 1 also reveals that $\langle H_0 \rangle$ and $\langle V \rangle$ changes somewhat if a 3NF is included and $\langle V_4 \rangle$ is only about 2 % of $\langle V \rangle$. That ap-

Table 1
Triton binding energies E_t and energy expectation values $\langle H_0 \rangle$, $\langle V \rangle$, $\langle V_4 \rangle$ and $\langle H \rangle$ for the AV14 NN potential only and together with the TM 3NF using $\Lambda = 5.13 \, m_\pi$

j_{max}	AV14 only					AV14 + TM 3NF				
	E_t [MeV]	$\langle H_0 \rangle$ [MeV]	$\langle V \rangle$ [MeV]	$\langle V_4 \rangle$ [MeV]	$\langle H \rangle$ [MeV]	E_t [MeV]	$\langle H_0 \rangle$ [MeV]	$\langle V \rangle$ [MeV]	$\langle V_4 \rangle$ [MeV]	$\langle H \rangle$ [MeV]
2	−7.577	45.177	−52.756	–	−7.579	−8.478	49.516	−56.567	−1.430	−8.481
3	−7.659	45.596	−53.257	–	−7.662	−8.440	49.185	−56.343	−1.285	−8.443
4	−7.674	45.654	−53.330	–	−7.676	−8.490	49.370	−56.537	−1.325	−8.492
5	−7.680	45.677	−53.360	–	−7.683	−8.482	49.332	−56.509	−1.308	−8.484
6	−7.682	45.680	−53.364	–	−7.684	−8.486	49.340	−56.518	−1.310	−8.489

Table 2
Contributions of the NN and the 3N forces to the triton binding energy for a given j (see text)

j	AV14 only $(E_t|_j^{NN} - E_t|_{j-1}^{NN})$ [keV]	AV14 + TM 3NF $(E_t|_j^{NN+3NF} - E_t|_j^{NN})$ $-(E_t|_{j-1}^{NN+3NF} - E_t|_{j-1}^{NN})$ [keV]
3	82	$781 - 901 = -120$
4	15	$816 - 781 = 35$
5	6	$802 - 816 = -14$
6	2	$804 - 802 = 2$

proximately additional 1 MeV for $\langle V_4 \rangle$ is a big effect in relation to E_t itself, but only a small modification of the total potential energy.

Table 1 shows that the convergence of the triton binding energy with inclusion of the 3NF is slower than without 3NF. This is displayed in detail in Table 2. We denote the triton binding energy including partial waves up to a certain j by $E_t|_j$. We see that the contributions of the 3NF for a given j, $(E_t|_j^{NN+3NF} - E_t|_j^{NN}) - (E_t|_{j-1}^{NN+3NF} - E_t|_{j-1}^{NN})$, are larger than the contributions $E_t|_j^{NN} - E_t|_{j-1}^{NN}$ of the NN force (only for $j = 6$ they are both equally small). Further the contributions of the 3NF change their sign with increasing j. For even j the 3NF acts attractive and for odd j repulsive. (Of course, the step to a higher j also includes transition potentials from lower j s to that specific higher j.) This alternating sign in the contributions of the 3NF leads to the fact that for odd j the contribution of the 3NF to the triton binding energy is partially cancelled by the NN force contribution. Therefore the total change in the triton binding energy going from $j = 4$ to $j = 5$ is only 8 keV in this model.

This is about 0.1% of the binding energy and corresponds to our numerical accuracy. Therefore we chose $j = 5$ for the evaluation of the triton binding energy including a 3NF.

A comparison to the work of the Pisa group using exactly the same force model shows very good agreement. The Pisa group calculates the triton binding energy in configuration space using the pair correlated hyperspherical harmonic basis approach [30]. Thus their method is mathematically and numerically totally independent from our approach. The result given in [30] is 8.484 MeV. A recent increase in accuracy leads to 8.486 MeV [31]. Both numbers are in excellent agreement to our most advanced result of 8.486 MeV given in Table 1.

The results shown in the following refer to the choice, that NN and 3N force components are kept up to $j = 5$. The NN potentials under discussion include charge independence breaking (CIB) and on top for AV18 and CD-Bonn also charge symmetry breaking (CSB). In other words, all these potentials are given in an np, pp and two also in an nn version. The np and pp versions are obtained by fitting to the corresponding sets of NN phase shifts, whereas the choices of the nn versions are nonuniform [32] and lead to different behaviours, as will be shown below for AV18 and CD-Bonn. Because of that and since only little is known about the CSB in the NN force, we replace the nn by the pp force, except for AV18 and CD-Bonn, where we present both results. On the other hand the CIB in the NN force is known rather well and we have to take it into account. We do this according to [33,34] by choosing an effective t-matrix

$$t_{\text{eff}} = 1/3 \, t_{np} + 2/3 \, t_{pp} \tag{4}$$

Table 3

Triton binding energies E_t for various realistic NN potentials in charge dependent calculations without $T = 3/2$. The numbers in parenthesis refer to np and nn forces. The adjusted cut-off parameters Λ in the 3NF leading to the resulting triton binding energies (last column)

Potential	E_t [MeV] $t = \frac{1}{3}\ t_{np} + \frac{2}{3}\ t_{pp}$	Λ/m_π	E_t [MeV] $t = \frac{1}{3}\ t_{np} + \frac{2}{3}\ t_{pp}$
CD-Bonn	7.953 (8.014)	4.856 (4.79)	8.483
Nijm II	7.709	4.990	8.477
Reid 93	7.648	5.096	8.480
Nijm I	7.731	5.147	8.480
Nijm 93	7.664	5.207	8.480
AV18	7.576 (7.685)	5.215 (5.107)	8.479
Ruhrpot	7.644	5.306	8.459

Fig. 1. Triton binding energies versus Λ according to Table 3.

We neglect the total isospin $T = 3/2$ admixture, which is justified for the triton (and many but not all observables in nd scattering [35]).

In Table 3 we show the triton binding energies calculated for the the different NN potentials using the effective NN t-matrix (4). We see the well known gap to the experimental number of 8.48 MeV. Only CD-Bonn with its strong non-locality sticks out. Just for the sake of information, we also present the results if we replace t_{pp} by t_{nn} in Eq. (4), which is possible for AV18 and CD-Bonn. The results are shown in parenthesis and we find an effect of CSB of about 100 and 60 keV, respectively. This shift in energy is about what is needed to "understand" the mass difference between ^{3}H and ^{3}He on top of electromagnetic effects and n-p mass differences [6-8]. The larger effect for the AV18 potential is due to the fact that changes have been introduced on an operator level in going from the pp to the nn system [32], whereas in CD-Bonn only the $^1S_0$ component of the NN force has been changed [36]. The theoretical binding energy for Nijm II differs from the result shown in [37,38] because there the old potential version has been used; also we increased now the accuracy in going to $j = 5$. The difference to AV18 (cd) shown in [37,38] is again due to $j = 5$, but more due to the fact that in [37,38] t_{nn} instead of t_{pp} has been used in Eq. (4). Also our numbers differ from the ones given in [39], since we allow for CIB. We also included the Ruhrpot NN interaction [26,27] for reasons explained below.

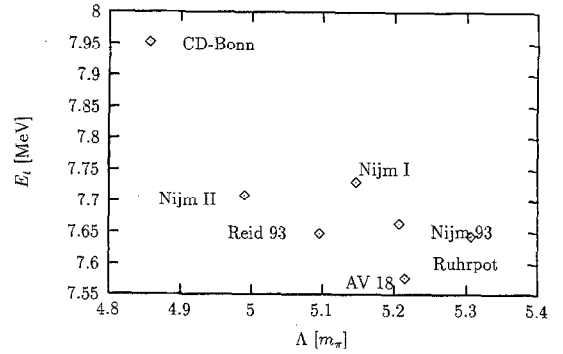Now let us come to the main point, the adjustment of the triton binding energy by choosing the appropri-ate cut-off parameter Λ in the strong form factors of the TM π-π 3NF. They are shown in Table 3 together with the resulting binding energies. The less accurate adjustment for the Ruhrpot interaction is sufficient for the purposes discussed below. Inspection of Table 3 reveals that the connection between Λ and the triton binding energy without 3NF is not linear as one might expect naively: for example the two NN forces Reid 93 and Nijm 93 give nearly the same value for the triton binding energy without 3NF, but their Λ s are quite different. This fact demonstrates the subtle interplay of the 3NF with the various NN forces which can lead to unexpected results. Since the NN forces are phe-nomenological and therefore no internal consistency exists to that 3NF, this is not necessarily surprising and has been noticed before [40,41]. We illustrate our findings in Fig. 1, were we plot the fitted Λ s against the triton binding energy without 3NF. One feature sticks out: For Nijm II, Reid 93 and AV18 the con-nection between Λ and E_t is very well linear. Is there anything special about these three potentials?

A guess might be that the probability to find the three nucleons in the triton at a certain distance might be important. A hint in that direction is the NN corre-lation function in the triton, which is defined as

$$C(r) \equiv \frac{1}{3}\frac{1}{4\pi} \int d\Omega_r \langle \Psi | \sum_{i<j} \delta(r - \hat{r}_{ij}) | \Psi \rangle \quad (5)$$

where r is the distance of two of the three nucleons and $\hat{r}_{ij}$ the corresponding operator. C provides the proba-bility to find two nucleons at a distance r. It is shown in Fig. 2 for the various NN potentials. We see that there is indeed a significant difference between the po-tentials of the two groups mentioned above. For the
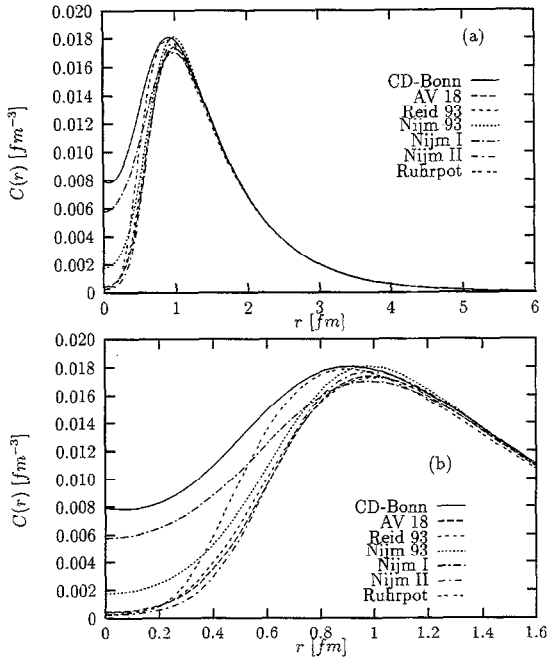
Fig. 2. Two-body correlation functions for the triton using various NN potentials. Subfigure (b) shows an enlargement for small r of subfigure (a).

potentials Nijm II, Reid 93 and AV18 C is essentially zero at $r = 0$ and their C s are very similar, whereas the probabilities $C(r)$ are much less suppressed at short distances for the potentials CD-Bonn, Nijm I, and Nijm 93. The potentials, which are strongly repulsive at short distances require a smaller strength factor Λ in the 3NF to achieve the triton binding energy than the weaker repulsive ones. For instance Reid 93 and Nijm 93 give nearly the same triton binding energy (without 3NF) but require different strength factors Λ. The one which allows two nucleons to come closer to each other, Nijm 93, needs a larger Λ. The corresponding remark applies to the pair Nijm II and Nijm I. CD-Bonn has no local, strongly repulsive partner to compare with, but the nearly linear correlation with Λ for the three potentials Nijm 93, Nijm I, and CD-Bonn shown in Fig. 1 has obviously to do with the increasingly weaker suppression of $C(r)$ at $r = 0$. We also included a 7th potential, the Ruhrpot [26,27], which is a meson theoretical interaction, but there the χ^2 is not pushed to that accuracy as for the other potentials and it is fitted to a different set of NN phase

shift parameters, namely the Arndt phases [42]. Also the Ruhrpot model is provided only in a np version. As seen from Fig. 2 $C(r)$ for that potential is also strongly suppressed near $r = 0$ but nevertheless the Λ is quite large. Its $C(r)$ behaves however differently from the others, since it rises very quickly to its maximum. We included that potential as an example for a qualitatively different behaviour of $C(r)$. Apparently that quick rise of $C(r)$ is more important than the strong suppression of $C(r)$ at very small $r \lesssim 0.2$ fm^{-1} and causes the large Λ. This demonstrates the subtlety of the interplay of properties of the NN forces and that 3NF. The detailed behaviour of the forces at short distances below about 1 fm is important.

It is obvious that this observations can be further illustrated and possibly understood by investigating the configuration space properties of that 3NF and the NN forces together with the behaviour of the 3N wavefunction (also with respect to spin-isospin properties). Since we work in momentum space this is not directly accessible to us and we leave that as a suggestion.

The various short range behaviours of NN correlation functions have been emphasised before in [43]. They play a crucial role for instance in inclusive electron scattering at intermediate energies [44,45] or in (e,e',p) processes, where nucleon momentum distributions enter, which are again influenced by NN correlations [46].

In that context it is also of interest to see how $C(r)$ changes, once that 3NF has been included. Our results shown in Fig. 3 tell that the C s do not change qualitatively. The C s increase in the maximum at $r \approx 1$ fm including that 3NF. This is connected with the stronger decrease at larger r s. The change at very short distances is nearly zero for the very repulsive NN potentials and increases with decreasing repulsion. We also determined the probability to find one nucleon at a certain distance from the centre of mass for the various NN forces, with and without 3NF. The effect of that 3NF was to increase the probability slightly for $r \lesssim 1$ fm. Especially around 0.5 fm, where the density without 3NF starts to flatten towards $r = 0$ the density aquires a small hump due to the 3NF. Our results are very similar to the one already found in [47].

It will be interesting to repeat this sort of study for other 3NFs and to pin down possible effects in inclusive and exclusive electron scattering.
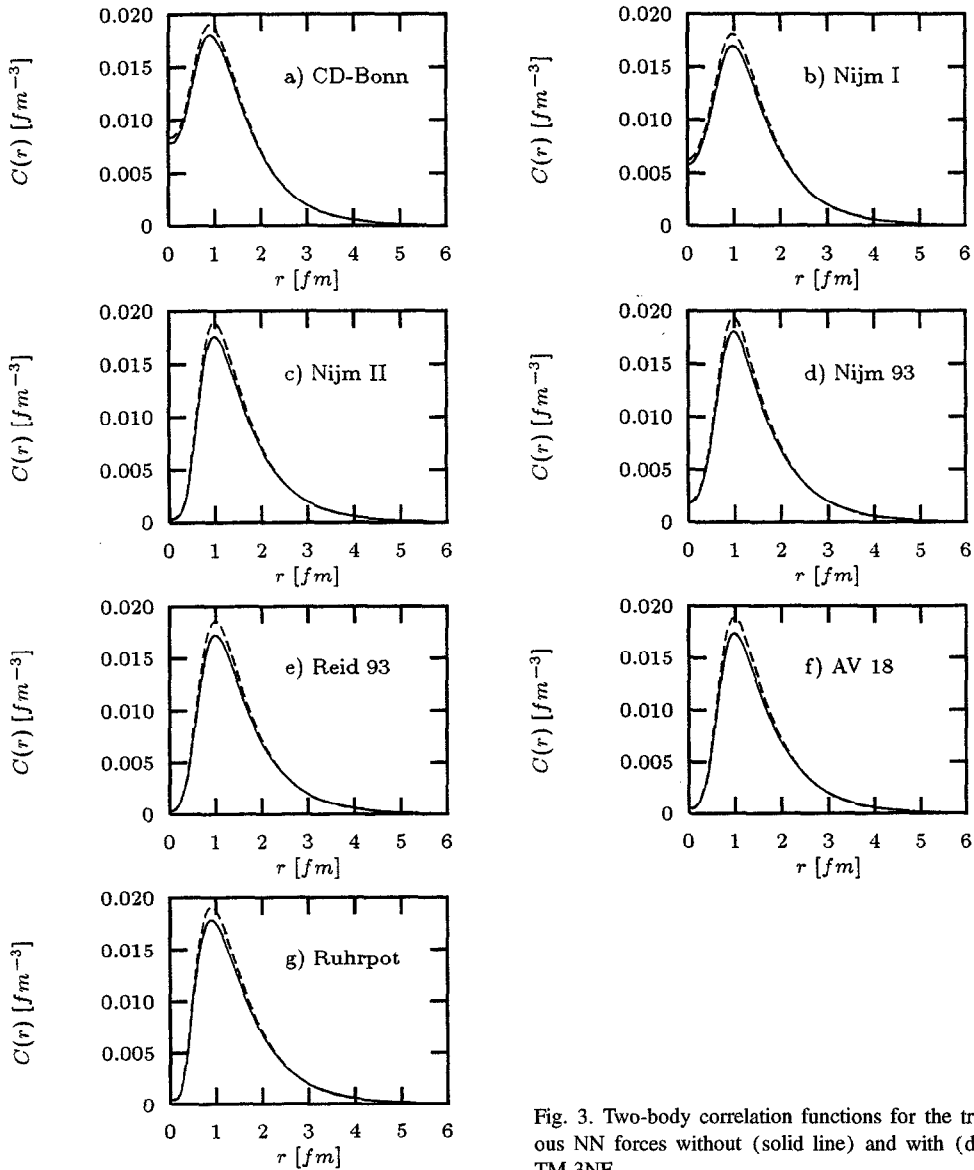
Fig. 3. Two-body correlation functions for the triton using various NN forces without (solid line) and with (dashed line) the TM-3NF.

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