Updating of a lumped model for an experimental web tension control system using a multivariable optimization method

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Abstract

The modelling and the control of web handling systems have been studied for a long time; correct modelling is necessary in order to design a better control system or to identify the plant parameters experimentally. On the web dynamics itself, lumped parameters expressions may be used to designate a web section between two adjacent drive rolls, and there is the necessity of incorporating the property of visco-elasticity to the web.

In this paper the lumped model of a new web tension experimental system is updated; the model is based on the conservation mass, torque balance and viscoelasticity (Voigt approach). The experimental system consists of four sections each of which is driven by a servomotor; the speed and tension feedback, by using encoders and tension sensors, drives simultaneously the four servomotors through a real time C programmed D/A board. Usually, as described in literature, these kinds of models are developed in the Laplace domain and the block scheme gives a graphical interpretation of the interaction between different sections. The transformation of the block scheme in a differential equation system in the time domain is fully described in this paper; it is not a simple step and it requires the introduction of not null initial condition for the derivative of physical variables.

Moreover, the problem of validation has been dealt with in detail in this paper, considering simultaneously 2 different combinations of input data in open loop and a multivariable optimization method in order to estimate a certain number of unknown parameters.

The results will show the accuracy of this kind of lumped parameters model for the complex experimental systems and useful information for successively designing an efficient control strategy.

*Keywords***:** Modelling, Block diagrams, Laplace transform properties, Web transport systems, optimization.

1. **Introduction**

The system handling web material such as textile, paper, plastic, polymer and metal are very common in the industry. Web is usually transported by many drive rollers and idle rollers, and it is processed in several consecutive sections such as coating, laminating and printing treatments.

The modelling and the control of web handling systems have been studied for a long time [1-8]; in order to design a better control system or to identify the plant parameters experimentally, correct modelling is necessary. On the web dynamics itself [1], lumped parameters expressions may be used to designate a web section between two adjacent drive rolls, and there is the necessity of incorporating the property of visco-elasticity to the web; at this proposal mainly two models [1] are used (Maxwell model and Voigt model). Several nonlinearities and disturbances affect the system; for this reason the tuning of a lumped and linearized model is not a simple and direct operation.

At this respect in this paper the authors studied the lumped model referred to a new experimental system [9-11] composed by four sections (an unwinder section, a leading section, a draw-roll section and a winder section) driven each by one servomotor [9-11]. In [9] the preliminary experimental system set-up was shown, while in [10] the system model validation was manually and iteratively carried out comparing the block model behaviour (Laplace domain) with the experimental data. No alternatives were given to an iterative validation procedure because the block model was developed only in the Laplace domain. In this way 5 model parameters [10] were roughly identified with a reference to a prefixed input for the system.

The possibility of transforming the block model in a differential equation system in the time domain is surely attractive; in this paper all the steps necessary to make this transformation are detailed shown. The full coincidence of the block system and of the differential equation system behaviour demonstrates that following a rigorous mathematical strategy is possible to convert the block scheme in the time domain differential equation system for describing this kind of systems.

Moreover for updating the model used in [10-11] a new validation strategy is proposed considering the differential equation system and a multivariable optimization method [12]; in this way it is possible to estimate a higher number of unknown parameters and it is possible to consider simultaneously different combinations of input data.

The details of this new validation strategy are shown in this paper, and the final comparison between experimental data and model behaviour permit to update significantly the model proposed and used in [10].

2. The system considered: theoretical model

The system that has been taken into account in the present work is schematically shown in a simplified manner in Fig. 1 and it is referred to the experimental system fully described in [10-11]. It is composed by four sections (an unwinder section, a leading section, a draw-roll section and a winder section) driven each by one servomotor and with tension sensors collocated by both the sides of the systems between the unwinder section and leading section and between draw-roll section and winder section. The system is complex (see photo in Fig. 2) and it is controlled [10] using a decentralised PID control through a real time C programmed D/A board. It consists of an unwinder section followed tension sensors, a lead section (speed controlled) with S-wrap, a draw roll section controlled by means of a couple of tension sensors and, finally, the winder section [10-11]. Every section is driven by a servo-motor, the torque controller gain of which is large enough to be assumed that the transfer function of torque control is unity.

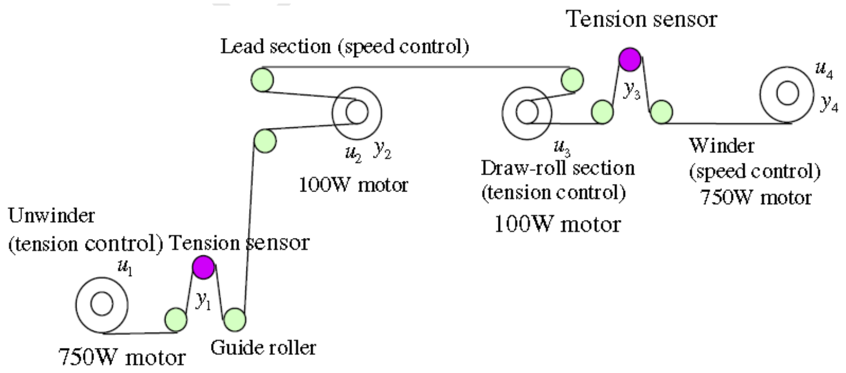

Fig. 1. Scheme of the analysed system.

The model of the web transport systems is based on three laws [10] applied at each section between two consecutive rolls:

- the law of conservation of mass for the web section for evaluating the relation between the speeds (named v_a and v_b) of two adjacent rolls and the strain ε in the web (1). For the web of the ordinary two drive rolls, the length L of the span is considered constant, because the strains in general have fairly small values compared to unity (1);

- assuming that the web does not completely slide on the roll, the web velocity is considered equal to the roll linear velocity. The velocity ω_k of the k^{th} roll can be obtained through a torque balance in function of the tension forces F_{k+1} and F_k applied to the roll from the web, (2); ω_k is the rotational speed of roll k, J_k the roll inertia, r_k the roll radius, the motor torque applied to that roll that is proportional to the motor torque control signal u_k by means of the motor constant $K_k C_k$ is the dry friction torque, K_{fk} the viscous friction coefficient;

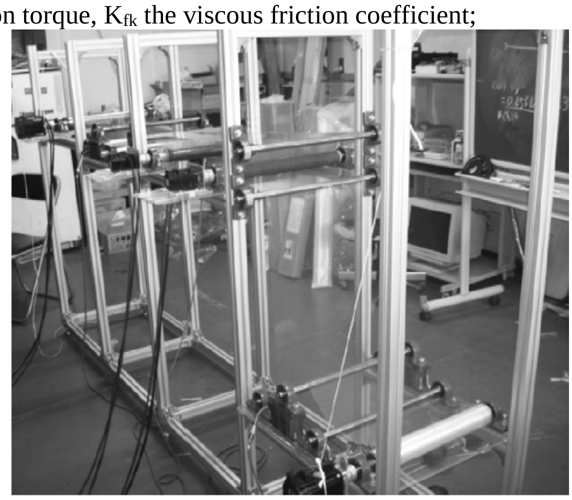

Fig. 2. Photo of the experimental system.

- for taking into account the linear viscoelasticity of web-material expressed as combination of linear springs and dashpots, the Voigt model [1] was considered. The relation between the tension force F applied to the web and the strain ε in the Laplace domain is expressed in (3); A is cross sectional area of the web, η is the viscosity modulus, $T_m = T_v = \eta / E$, E is the elastic modulus; (3) can be expressed in (4) in a simple way.

$$\varepsilon(t) = \frac{1}{L} \cdot \int [v_b(t) - v_a(t)]dt \qquad (1)$$

$$\frac{d(J_k \cdot \omega_k)}{dt} = r_k \cdot (F_{k+1} - F_k) + K_k \cdot u_k - C_k - K_{fk} \cdot \omega_k \qquad (2)$$

$$F(s) = A \cdot \eta \cdot (\frac{1 + T_v \cdot s}{T_v \cdot s}) \cdot s \cdot \varepsilon(s) \qquad (3)$$

$$F(s) = P(s) \cdot s \cdot \varepsilon(s); \qquad (4)$$

Usually the equations (1-2) are transformed in the Laplace domain for each section equations (5-6) considering starting conditions equal to 0 for the tension forces F_k and the the velocity ω_k of the k^{th} roll.

$$\varepsilon(s) = \frac{1}{L \cdot s} \cdot \left[v_b(s) - v_a(s) \right] \tag{5}$$

$$s \cdot J_k(s) \cdot \omega_k(s) = r_k \cdot \left[F_{k+1}(s) - F_k(s) \right] + K_k \cdot u_k(s) - C_k - K_{fk} \cdot \omega_k(s) \tag{6}$$

The possibility of using algebraic equations (3),(5),(6) in the Laplace domain gives the possibility of building in simple way a block model of the entire system considering the equation related to the different system sections [8,10].

In this case the unwinder section, the leading section, the draw-roll section and the winder section are respectively numbered with 1,2,3 and 4. Moreover, the input of the system are the motor torque control signals u_1,u_2, u_3 u_4 and the outputs of the system are the forces F_1 and F_3 (called y_1 and y_3 respectively), and the longitudinal speed of the rolls of the sections 2 and 4 (called y_2 and y_4).

Therefore, the block model of the system considering the relations between the sections of the system is shown in a block diagram (Fig.3); the nomenclature used in Figure 3 referred to equations (3), (5),(6) and successively used in all the paper is shown in Table 1.

The block model shown in Fig.3 was used in [10] with a commercial software to validate the experimental results and to estimate in iterative way some unknown parameters (K_{f1}, K_{f2}, K_{f3} and K_{f4} and the viscosity coefficient η).

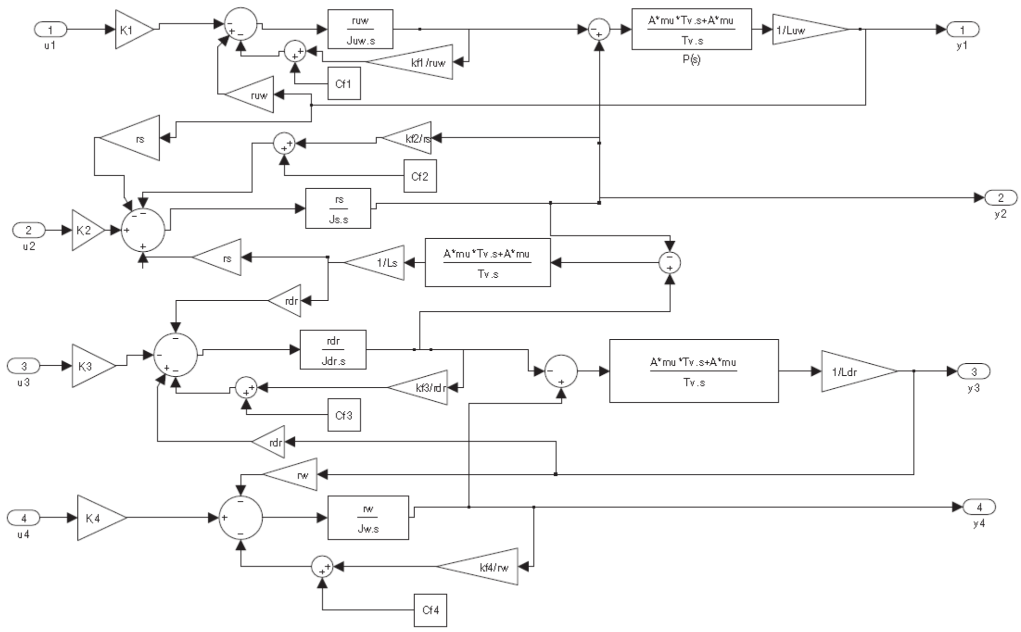

Fig. 3. System block model.

Table 1
Nomenclature of the symbols used in Fig. 3.

Parameter definition	Symbol in Eqs. (3), (5), (6)	$k = 1$ unwinder section	$k = 2$ lead section	$k = 3$ draw-roll section	$k = 4$ winder section
Moment of inertia referred to the motor shaft	J_k	J_{uw}	J_s	J_{dr}	J_w
Radius of driven rollers	r_k	r_{uw}	r_s	r_{dr}	r_w
Dry friction torques	C_k	C_{f1}	C_{f2}	C_{f3}	C_{f4}
Viscous friction coefficient	K_{fk}	k_{f1}	k_{f2}	k_{f3}	k_{f4}
Motor constant	K_k	K_1	K_2	K_3	K_4
Web length	L	L_{uw}	L_s	L_{dr}	–
Cross sectional area of the web	A	A	A	A	A
Viscosity modulus	μ	mu	mu	mu	mu
$T_v = u/E$	T_v	T_v	T_v	T_v	T_v
Motor torque control signal	u_k	u_1	u_2	u_3	u_4

The block model depicted in Fig. 3 may be easily developed in the Laplace domain using the block systems algebra in order to explicit the algebraic equation describing the system. The algebraic equations referred to the system are shown in (7) and the coefficients A_{ij} are expressed in (8).

In the Laplace domain the block model of Fig. 3 is a system of 4 polynomial algebraic equation with U_1, U_2 U_3 U_4 the Laplace transforms of the inputs u_1,u_2, u_3 u_4 and the variables Y_1,Y_2 Y_3, Y_4 the Laplace transforms of the output y_1,y_2, y_3 y_4 in (7).

$$\begin{cases} A_{10}\cdot s^2\cdot Y_1 + A_{11}\cdot s\cdot Y_1 + A_{12}\cdot Y_1 + A_{13}\cdot s^2\cdot Y_2 + A_{14}\cdot s\cdot Y_2 + A_{15}\cdot Y_2 = C_1\cdot A\cdot\mu\cdot r_{uw} + K_1\cdot U_1\cdot A\cdot\mu\cdot r_{uw} + C_1\cdot A\cdot\mu\cdot r_{uw}\cdot T_v\cdot s + K_1\cdot A\cdot\mu\cdot T_v\cdot s \\ A_{21}\cdot s\cdot Y_1 + A_{22}\cdot s^2\cdot Y_2 + A_{23}\cdot s\cdot Y_2 + A_{24}\cdot Y_2 + A_{25}\cdot s\cdot Y_3 + A_{26}\cdot s\cdot Y_4 + A_{27}\cdot Y_4 = r_s\cdot C_2\cdot L_s\cdot T_v\cdot s - r_s\cdot L_s\cdot T_v\cdot K_2\cdot U_2\cdot s \\ A_{30}\cdot s^2\cdot Y_2 + A_{31}\cdot s\cdot Y_2 + A_{32}\cdot Y_2 + A_{33}\cdot s^3\cdot Y_3 + A_{34}\cdot s^2\cdot Y_3 + A_{35}\cdot s\cdot Y_3 + A_{39}\cdot s^3\cdot Y_4 + A_{36}\cdot s^2\cdot Y_4 + A_{37}\cdot s\cdot Y_4 + A_{38}\cdot Y_4 = \\ = A\cdot\mu\cdot L_s\cdot T_v\cdot s\cdot K_3\cdot U_3 + A\cdot\mu\cdot T_v^2\cdot L_s\cdot K_3\cdot U_3\cdot s^2 + A\cdot\mu\cdot L_s\cdot T_v\cdot s\cdot C_3 + A\cdot\mu\cdot L_s\cdot T_v^2\cdot C_3\cdot s^2 \\ A_{41}\cdot s\cdot Y_4 + A_{42}\cdot Y_4 + A_{43}\cdot Y_3 = K_4\cdot U_4 - C_4 \end{cases} \tag{7}$$

The coefficients A_{ij} are defined in (8).

$$\begin{cases} A_{10} = J_{uw}\cdot L_{uw}\cdot T_v \\ A_{11} = (r_{uw}{}^2\cdot A\cdot\mu\cdot T_v + K_{f1}\cdot L_{uw}\cdot T_v) \\ A_{12} = (r_{uw}{}^2\cdot A\cdot\mu) \\ A_{13} = (-J_{uw}\cdot A\cdot\mu\cdot T_v), \\ A_{14} = -J_{uw}\cdot A\cdot\mu - K_{f1}\cdot A\cdot\mu\cdot T_v \\ A_{15} = -(K_{f1}\cdot A\cdot\mu) \\ A_{21} = -(r_s{}^2\cdot L_s\cdot T_v) \\ A_{22} = -(J_s\cdot L_s\cdot T_v) \\ A_{23} = -(K_{f2}\cdot L_s\cdot T_v + r_s^2\cdot A\cdot\mu\cdot T_v) \\ A_{24} = -(r_s^2\cdot A\cdot\mu) \\ A_{25} = -(r_s^2\cdot L_{dr}\cdot T_v) \\ A_{26} = (r_s^2\cdot A\cdot\mu\cdot T_v) \\ A_{27} = r_s^2\cdot A\cdot\mu \\ A_{30} = (r_{dr}\cdot A^2\cdot\mu^2\cdot T_v^2) \\ A_{31} = (2\cdot r_{dr}\cdot A^2\cdot\mu^2\cdot T_v) \\ A_{32} = (r_{dr}\cdot A^2\cdot\mu^2) \\ A_{33} = (L_s\cdot T_v^2\cdot L_{dr}\cdot \frac{J_{dr}}{r_{dr}}) \\ A_{34} = (r_{dr}\cdot A\cdot\mu\cdot L_s\cdot T_v{}^2 + r_{dr}\cdot A\cdot\mu\cdot L_{dr}\cdot T_v^2 + K_{f3}\cdot L_{dr}\cdot T_v^2\cdot \frac{L_s}{r_{dr}}) \\ A_{35} = (r_{dr}\cdot A\cdot\mu\cdot L_{dr}\cdot T_v + A\cdot\mu\cdot L_s\cdot T_v\cdot r_{dr}) \\ A_{36} = (-r_{dr}\cdot A^2\cdot\mu^2\cdot T_v{}^2 - (J_{dr}\cdot L_s\cdot T_v\cdot A\cdot\frac{\mu}{r_{dr}}) - K_{f3}\cdot A\cdot\mu\cdot T_v^2\cdot\frac{L_s}{r_{dr}}) \\ A_{37} = (-2\cdot r_{dr}\cdot A^2\cdot\mu^2\cdot T_v - K_{f3}\cdot A\cdot\mu\cdot L_s\cdot\frac{T_v}{r_{dr}}) \\ A_{38} = (-r_{dr}\cdot A^2\cdot\mu^2) \\ A_{39} = (-J_{dr}\cdot A\cdot\mu\cdot L_s\cdot\frac{T_v{}^2}{r_{dr}}) \\ A_{41} = \frac{J_w}{r_w} \\ A_{42} = \frac{K_{f4}}{r_w} \\ A_{43} = r_w, \end{cases} \tag{8}$$

3. Transformation of the Laplace block model in the differential equation system

The block scheme shown in Fig. 3 is graphically significant about the interaction of the different subsystems that form the full system model; the research [3,6,8] was concentrated on the way for dividing the subsystems in such a way to minimize the interaction effects. But the knowledge of a system model only in the Laplace domain doesn't give a complete knowledge of the system dynamics; moreover some important operation like as example the parameters estimation of unknown parameters using optimization techniques could not be realized in the Laplace domain.

The main difficulty of inverse Laplace transform operation referred to (7) lies in the presence of the variable s multiplying the constant terms (second member) of first, second and third equation of (7). These terms are difficult to consider because the multiplication for the s variable in the Laplace domain, correspond, roughly, to a derivative in the time domain. For this reason it is important to choose a prefixed behaviour for the input variables u_1, u_2, u_3 and u_4; in this analysis a step input type was chosen for all the 4 input variables.

So, considering, a step input type, a correct use of the Laplace transform properties applied to derivatives for the first equation of (7) gives (9).

$$A_{10}\cdot\frac{d^2y_1(t)}{dt^2}+A_{10}\cdot\frac{dy_1(0)}{dt}+A_{10}\cdot y_1(0)+A_{11}\cdot\frac{dy_1(t)}{dt}+A_{11}\cdot y_1(0^+)+A_{12}\cdot y_1(t)+A_{13}\cdot\frac{d^2y_2(t)}{dt^2}+A_{13}\cdot\frac{dy_2(0)}{dt}+A_{13}\cdot y_2(0)+ \\ +A_{14}\cdot\frac{dy_2(t)}{dt}+A_{14}\cdot y_2(0)+A_{15}\cdot y_2(t)=C_1\cdot A\cdot\mu\cdot r_{uw}+K_1\cdot U_1\cdot A\cdot\mu\cdot r_{uw}+C_1\cdot A\cdot\mu\cdot r_{uw}\cdot T_v\big|_{t=0}+K_1\cdot A\cdot\mu\cdot T_v\big|_{t=0} \tag{9}$$

The second member terms multiplying s variable, influence only the starting conditions for t=0 of the derivatives of y_1, y_2, y_3 and y_4; moreover evaluating (9) in t=0, it is possible to obtain (11) the equation of starting conditions; so the equation (9) may be decomposed in (10) with starting conditions (11).

$$A_{10}\cdot\frac{d^2y_1(t)}{dt^2}+A_{11}\cdot\frac{dy_1(t)}{dt}+A_{12}\cdot y_1(t)+A_{13}\cdot\frac{d^2y_2(t)}{dt^2}+A_{14}\cdot\frac{dy_2(t)}{dt}+A_{15}\cdot y_2(t)=C_1\cdot A\cdot\mu\cdot r_{uw}+K_1\cdot U_1\cdot A\cdot\mu\cdot r_{uw} \tag{10}$$

$$A_{10}\cdot\frac{dy_1(0)}{dt}+A_{10}\cdot y_1(0)+A_{11}\cdot y_1(0)+A_{13}\cdot\frac{dy_2(0)}{dt}+A_{13}\cdot y_2(0)+A_{14}\cdot y_2(0)=C_1\cdot A\cdot\mu\cdot r_{uw}\cdot T_v+K_1\cdot A\cdot\mu\cdot T_v \tag{11}$$

Moreover remembering that the variables y_1, y_2, y_3 and y_4 are equal to 0 at t=0, (11) may be simplified in (12).

$$A_{10}\cdot\frac{dy_1(0)}{dt}+A_{13}\cdot\frac{dy_2(0)}{dt}=C_1\cdot A\cdot\mu\cdot r_{uw}\cdot T_v+K_1\cdot U_1\cdot A\cdot\mu\cdot T_v \tag{12}$$

Following the same way the second equation of (7) may be expressed in (13) with starting conditions (14), the third equation of (7) in (15) with starting conditions (16) and (17) and, finally, the fourth equation of (7) in (19) with starting conditions (20) and (21).

$$A_{21}\cdot\frac{dy_1(t)}{dt}+A_{22}\cdot\frac{d^2y_2(t)}{dt^2}+A_{23}\cdot\frac{dy_2(t)}{dt}+A_{24}\cdot y_2(t)+A_{25}\cdot\frac{dy_3(t)}{dt}+A_{26}\cdot\frac{dy_4(t)}{dt}+A_{27}\cdot y_4(t)=0 \tag{13}$$

$$A_{22}\cdot\frac{dy_2(0)}{dt}=C_2\cdot L_s\cdot T_v-r_s\cdot L_s\cdot T_v\cdot K_2\cdot U_2 \tag{14}$$

$$A_{30}\cdot\frac{d^2y_2(t)}{dt^2}+A_{31}\cdot\frac{dy_2(t)}{dt}+A_{32}\cdot y_2(t)+A_{33}\cdot\frac{d^3y_3(t)}{dt^3}+A_{34}\cdot\frac{d^2y_3(t)}{dt^2}+A_{35}\cdot\frac{dy_3(t)}{dt}+A_{39}\cdot\frac{d^3y_4(t)}{dt^3}+A_{36}\cdot\frac{d^2y_4(t)}{dt^2}+A_{37}\cdot\frac{dy_4(t)}{dt}+A_{38}\cdot y_4(t)=0 \tag{15}$$

$$A_{30}\cdot\frac{dy_2(0)}{dt}+A_{33}\cdot\frac{d^2y_3(0)}{dt^2}+A_{34}\cdot\frac{dy_3(0)}{dt}+A_{39}\cdot\frac{d^2y_4(0)}{dt^2}+A_{36}\cdot\frac{dy_4(t)}{dy}=A\cdot\mu\cdot L_s\cdot T_v\cdot K_3\cdot U_3+A\cdot\mu\cdot L_s\cdot T_v\cdot C_3 \tag{16}$$

$$A_{33}\cdot\frac{dy_3(0)}{dt}+A_{39}\cdot\frac{dy_4(0)}{dt}=A\cdot\mu\cdot T_v^2\cdot L_s\cdot K_3\cdot U_3+A\cdot\mu\cdot L_s\cdot T_v^2\cdot C_3 \tag{17}$$

The fourth equation of (7) may be re-written in (18) multiplying both members for the term s^2.

$$A_{41}\cdot s^3\cdot Y_4+A_{42}\cdot s^2\cdot Y_4+A_{43}\cdot s^2\cdot Y_3=K_4\cdot U_4\cdot s^2-C_4\cdot s^2 \tag{18}$$

Using the same procedure of third equation, the (18) may be transformed in (19) with starting conditions (20) and (21).

$$A_{41}\cdot\frac{d^3y_4(t)}{dt^3}+A_{42}\cdot\frac{d^2y_4(t)}{dt^2}+A_{43}\cdot\frac{d^2y_3(t)}{dt^2}=0 \tag{19}$$

$$A_{41}\cdot\frac{d^2y_4(0)}{dt^2}+A_{42}\cdot\frac{dy_4(0)}{dt}+A_{43}\cdot\frac{dy_3(0)}{dt}=0 \tag{20}$$

$$A_{41}\cdot\frac{dy_4(0)}{dt}=K_4\cdot U_4-C_4 \tag{21}$$

So the block model in (7) may be transformed in the time domain in the equations (10), (13), (15), (19) with starting conditions (12), (14), (16), (17), (20), (21).

4. Validation of the differential equation system

The system, previously introduced, has been analyzed in the classical form of (22) with Y= [y_1, y_2 y_3 y_4 dy_1/dt dy_2/dt dy_3/dt d^2y_3/dt^2 dy_4/dt d^2y_4/dt^2] and A, B, C calculated expressing (10), (13), (15), (19) in a vector form and explicated in (23), (24), (25).

$$A\cdot Y=B\cdot Y+C \tag{22}$$

$$A=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0\\0&0&0&0&A_{10}&A_{13}&0&0&0&0\\0&0&0&0&0&A_{22}&0&0&0&0\\0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&A_{30}&0&A_{33}&0&A_{39}\\0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&A_{41}\end{bmatrix} \tag{23}$$

$$B=\begin{bmatrix}0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&1&0\\-A_{12}&-A_{15}&0&0&-A_{11}&-A_{14}&0&0&0&0\\0&-A_{24}&0&-A_{27}&-A_{21}&-A_{23}&-A_{25}&0&-A_{26}&0\\0&0&0&0&0&0&0&1&0&0\\0&-A_{32}&0&-A_{38}&0&-A_{31}&-A_{35}&A_{33}&0&A_{39}\\0&0&0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&-A_{43}&0&-A_{42}\end{bmatrix} \tag{24}$$

$$C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ C_1 \cdot A \cdot \mu \cdot r_{uw} \cdot T_v + K_1 \cdot U_1 \cdot A \cdot \mu \cdot T_v \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

The 10 starting conditions are reported in the equations (12), (14), (16), (17), (20), (21) together with starting condition equal to 0 for y_1, y_2, y_3 and y_4.

The validation of the model expressed in (22) has been done using an iterative technique based on the explicit Runge-Kutta method [13]. The perfect overlapping between the block model output (named model 2) and the new model in time domain (named model 1) is shown in Fig.4; in this figure, in order to clearly distinguish the results plots, a constant value (2% of the mean value) is added to the data of the model 1. A sampling time of 0.001 s was used in the comparison in Fig. 4, with a time range of 10 seconds.

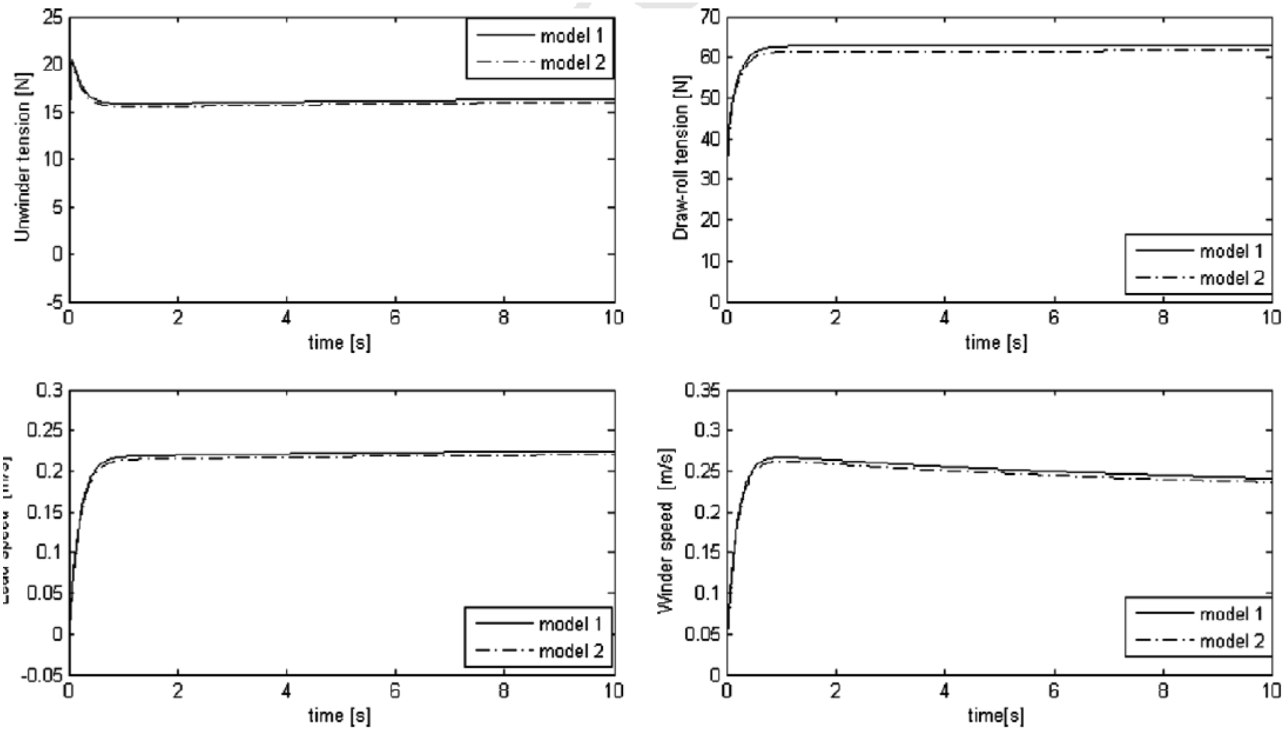

Fig. 4. Comparison of the differential equation system with the block model data.

Many comparisons carried out have shown in all the cases the perfect equivalence between the 2 models.

The physical values of parameters in the comparison in Fig.4 were chosen close to the values used in [10]; they are resumed in Table II. In order to demonstrate the perfect equivalence of the models 1 and 2 all the coefficients C_i and K_{fi} and the inputs u_i were chosen different from 0 (differently from successive experimental validation), in such a way to consider the contributes of all the terms of the block model.

5. Validation strategy and experimental repeatability

In [10] the identification strategy was manually and iteratively carried out comparing the block model behaviour with the system output Y= [y_1 y_2 y_3 y_4]. In this way 5 parameters (the viscous friction coefficients K_{f1}, K_{f2}, K_{f3}, K_{f4} and the viscosity coefficient η) were roughly identified with a reference to a single combination of input U =[u_1, u_2, u_3, u_4].

Table 2
Data assumed for testing the system models.

Name	Symbol	Value
Total length between winder and unwinder	L (m)	3.73
Unwinder section length	L_{uw} (m)	1.33
Lead section length	L_l (m)	1.18
Draw-roll section length	L_{dr} (m)	1.22
Unwinder roller radius	r_{uw} (m)	0.029
Lead section roller radius	r_l (m)	0.02
Draw-roll section roller radius	r_{dr} (m)	0.02
Winder roller radius	r_w (m)	0.025
Web thickness	T_h (mm)	0.04
Young modulus of the web	E (N/m^2)	10^9
Web width	W (m)	0.35
Moment of inertia for unwinder	J_{uw} (kg m^2)	0.0126
Moment of inertia for lead section roller	J_l (kg m^2)	0.0022
Moment of inertia for draw-roll section roller	J_{dr} (kg m^2)	0.0022
Moment of inertia for unwinder	J_w (kg m^2)	0.0126
Viscosity coefficient (Voigt model)	η (Ns/m^3)	10^9
Viscous friction coefficient for Section 1	K_{f1}	0.04
Viscous friction coefficient for Section 2	K_{f2}	0.027
Viscous friction coefficient for Section 3	K_{f3}	0.05
Viscous friction coefficient for Section 4	K_{f4}	0.005
Dry friction forces for Section 1	C_1	0.004
Dry friction forces for Section 2	C_2	0.045
Dry friction forces for Section 3	C_3	0.005
Dry friction forces for Section 4	C_4	0.005
Motor constant for Section 1	K_1	0.7967
Motor constant for Section 2	K_2	0.1060
Motor constant for Section 3	K_3	0.1060
Motor constant for Section 4	K_4	0.7967
Torque control signal for Section 1	u_1 (V)	0.2
Torque control signal for Section 2	u_2 (V)	0.5
Torque control signal for Section 3	u_3 (V)	0.5
Torque control signal for Section 4	u_4 (V)	2

Considering the nonlinearities and disturbances that affect this kind of systems and considering the complexity of the experimental analysed system more extensive and generalized analysis is considered necessary for improving the model affordability and for automating the tuning of unknown parameters by means of optimization techniques. At this proposal the block model of Fig.3, referred to the Laplace domain is now fully replaced by the differential equation system (22) with starting conditions (12), (14), (16),(17),(20),(21) together with starting condition equal to 0 for y_1, y_2, y_3 and y_4.

The problem of the model validation has been analysed comparing the experimental values in different operating conditions. In particular two different experimental input combinations have been considered (see Table III); in both the combinations, 4 system servomotors are powered by constant electrical tension values. In detail in the combination 1 only the biggest motors, unwinder motor (u_1) and winder motor (u_4) are powered, while in combination 2 all four motors are simultaneously powered.

Table 3
Input combination.

	u_1 (V)	u_2 (V)	u_3 (V)	u_4 (V)
Combination 1	0.2	0	0	2.0
Combination 2	0.3	0.5	0.5	2.5

Moreover a repeatability test was preliminarily done for the two combinations; in Fig. 5, 3 repetition tests for the combination 1 and in Fig. 6, 2 repetition tests for the combination 2.

It is evident that the sensor feedback is in some manner influenced by some kind of electrical noise, for this reason the operation of model validation and parameters estimation may not be carried out with extreme accuracy and only the average values of the combinations 1 and 2 will be considered as experimental data for the model validation.

Moreover it may be noted by comparing Figs 5 and 6 that the input combination 2 produces higher values of tension on the web (y_1 y_3) and higher values of speed (y_2 y_4). This is the reason because the experimental acquisition of combination 2 is shorter (less than 15 seconds) considering the same web length moving on the experimental system.

5. 1 Tuning multistep strategy

In the present work, all the parameters not directly measurable used in (22) including dry friction coefficient (C_1, C_2, C_3, C_4), viscous friction coefficients K_{f1}, K_{f2}, K_{f3} and K_{f4}, motor inertia (J_w, J_l, J_{uw}, J_{dr}) and viscosity coefficient (η), have been estimated.

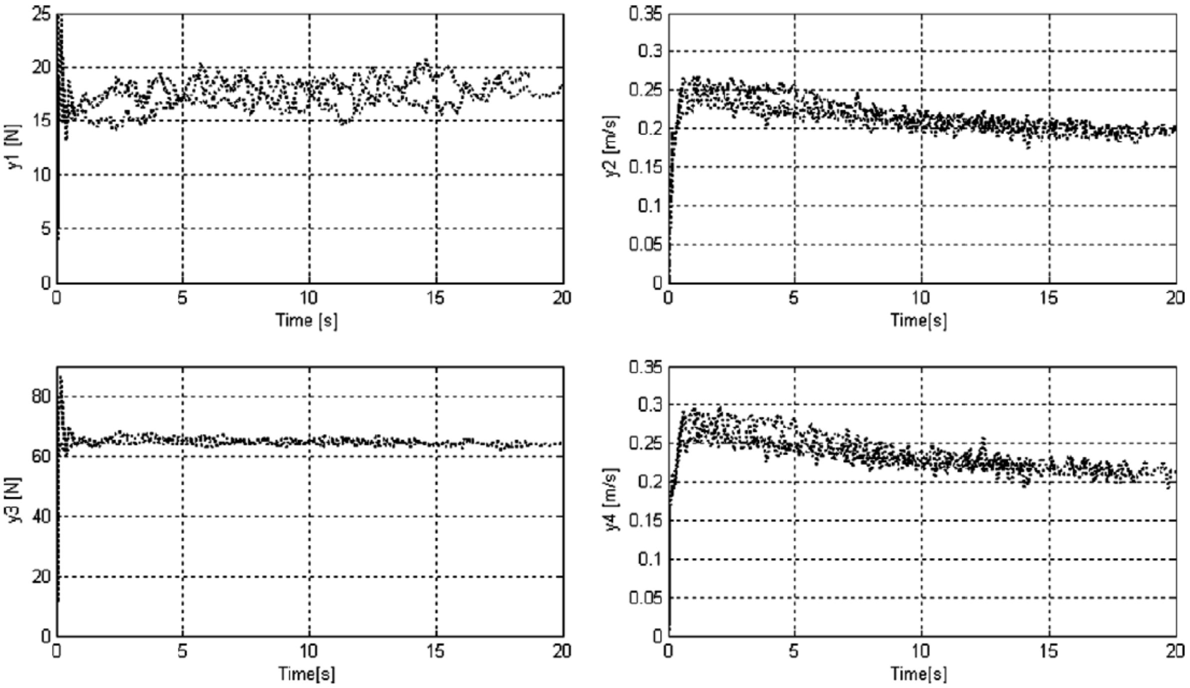

Fig. 5. Experimental repeatability for combination 1.

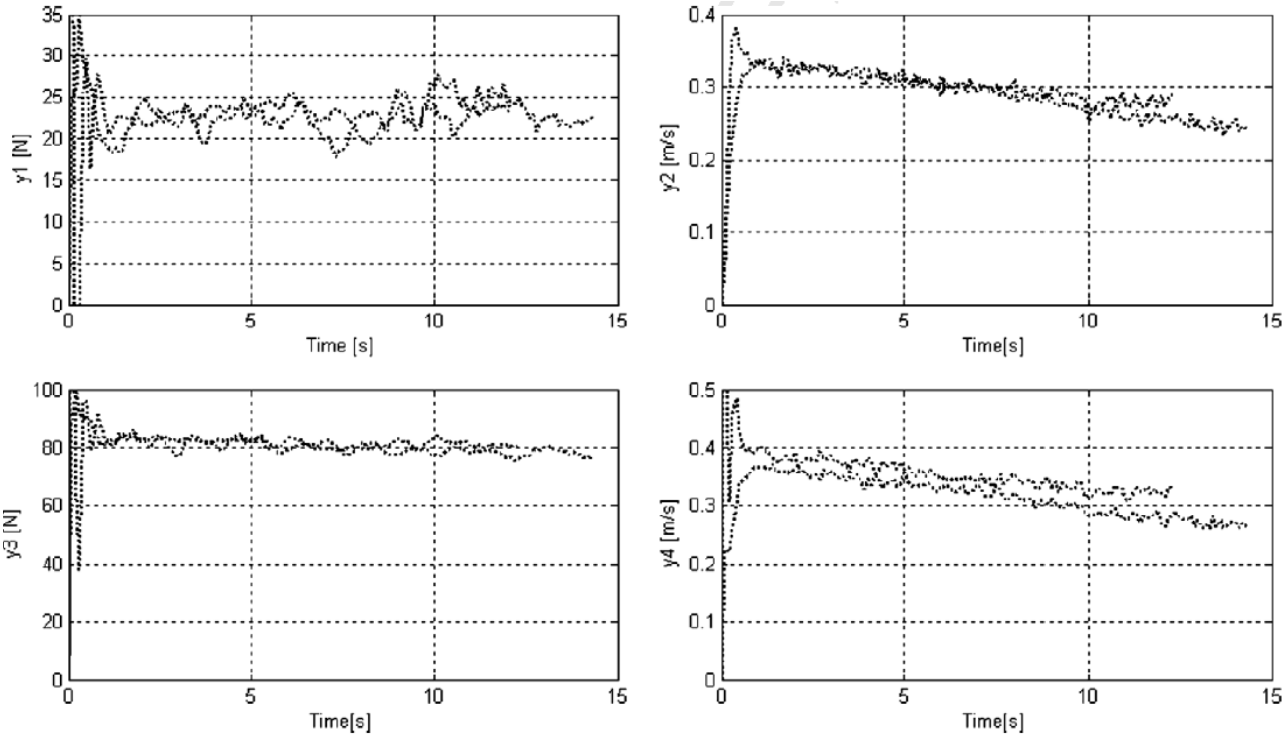

Fig. 6. Experimental repeatability for combination 2.

So, globally, 11 unknown parameters are the objects of estimation; the vectors X of unknown parameters which is defined as $X=[k_{f1}, k_{f2}, k_{f3}, k_{f4}, J_{uw}, J_{dr}, C_1, C_2, C_3, C_4, \eta]$; it is important to note that the inertia of the winder motor (J_w) and of the lead section (J_l) are considered equal respectively to J_{uw} and J_{dr} for the correspondence of motor type and roll size of these sections and so only 2 inertia parameters have to be estimated for the 4 motors .

The use of 2 different combinations of input gives the possibility of choosing a tuning multistep strategy and of trying to estimate the unknown parameters vector X.

In detail the parameters estimation has been carried out like a minimum research problem as follows (26);

$$\min(\frac{y_i^{\exp} - y_i^{th}}{y_i^{\exp}})^2 \tag{26}$$

where y_i^{exp} $_{i=1...4}$ are the experimental values for the 4 output variables, while y_i^{th} $_{i=1...4}$ are solutions of the system model (22), with the same system input U_i. The solutions y_i^{th} $_{i=1...4}$ were calculated solving (22) through Runge-Kutta method [13]. In particular, the classical fourth-order Runge-Kutta method was found to be a useful compromise between computational efforts and accuracy of the solution. The mentioned fourth-order Runge-Kutta method was implemented [13] through a fixed integration step of 10 ms after testing the numerical integration with several steps.

The minimum research problem was solved using a sequential quadratic programming (SQP) method [12] solving (26) in the time interval [0-10] seconds with sampling time 100 ms (sampling time of experimental acquisition) and considering reasonable constraints for all the parameters to estimate.

It means that (26) becomes a minimum research problem of 11 variables (vector X) with a system of 400 equations (100 for each y_i). The choice of using an observation period of 10 seconds is linked to the necessity of considering a time period long enough for reducing the importance of the transitory phase.

But the particularity of the proposed tuning strategy lies in the possibility of using 2 combinations of input; at this proposal a preliminary parameters estimation was carried out using only the combination 1; the results $\underline{X}$ of this optimization problem surely gives a minimum point for the difference between the experimental data and the model for combination 1.

At this point a new optimization problem was considered, where the combination 2 and its corresponding experimental data were introduces. Two differential equation systems (22) with different vectors input U (Table III) are simultaneously solved and used for the optimization process in the time interval [0-10 seconds] with sampling time 100 ms. The starting point of the new optimization problem is $\underline{X}$.

Moreover it has been noted that a different weight between different y_i could give back a better optimization process; in particular, considering that the speed values (variables y_2 and y_4 expressed in [m/s]) are remarkably lower than the tension force values (variables y_1 and y_3 expressed in [N]), a weight constant multiplying the error contribute for speed variables y_2 and y_4 has been attributed.

The analysis of the efficiency of the optimization was carried out using 2 global indicators (global mean of quadratic error) defined in (27) for combination 1 and analogously for combination 2:

$$e_1 = \frac{1}{4} \cdot \left[(\frac{y_1^{\exp} - y_1^{th}}{y_1^{\exp}})^2 + (\frac{y_2^{\exp} - y_2^{th}}{y_2^{\exp}})^2 + (\frac{y_3^{\exp} - y_3^{th}}{y_3^{\exp}})^2 + (\frac{y_4^{\exp} - y_4^{th}}{y_4^{\exp}})^2 \right] \tag{27}$$

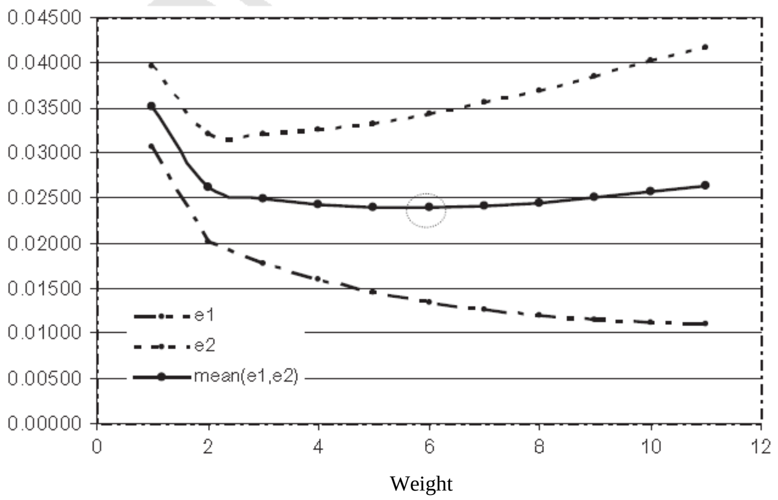

Fig. 7. Error indicators increasing the weight of variables y_2 and y_4.

In Fig 7 the global error indicators e_1 and e_2 obtained with increasing the weight coefficient from the value 1 (all variables weighted at same manner) to the value 11 are shown. For the last value (weight=11), the final result of optimization is exactly equal to the starting point $\underline{X}$.

A further increase of the weight coefficient doesn't change anymore the optimization which is always equal to $\underline{X}$.

Moreover in Fig 7 the mean value of global indicators is shown; it is evident that the minimum of the mean of global indicators e_1 and e_2 corresponds to a solution that minimizes the quadratic error in respect to both the combinations. And this value X_f is the final result of the multistep tuning strategy.

6. RESULTS AND COMMENTS

The weight value that minimizes the mean of e_1 and e_2 is equal to 6 (Fig. 7). For this value the global optimization converges to the solution X_f as follows (28):

$$X_f = \begin{bmatrix} 0.03869 \\ 0.02817 \\ 0.05119 \\ 0.00089 \\ 0.00887 \\ 0.00788 \\ 0.00372 \\ 0.04513 \\ 0.00012 \\ 0.00010 \\ 1.50 \cdot 10^9 \end{bmatrix} \qquad (28)$$

The comparison of experimental data and model results using parameters estimated in (5) is shown in Fig. 8 (combination 1) and in Fig. 9 (combination 2).

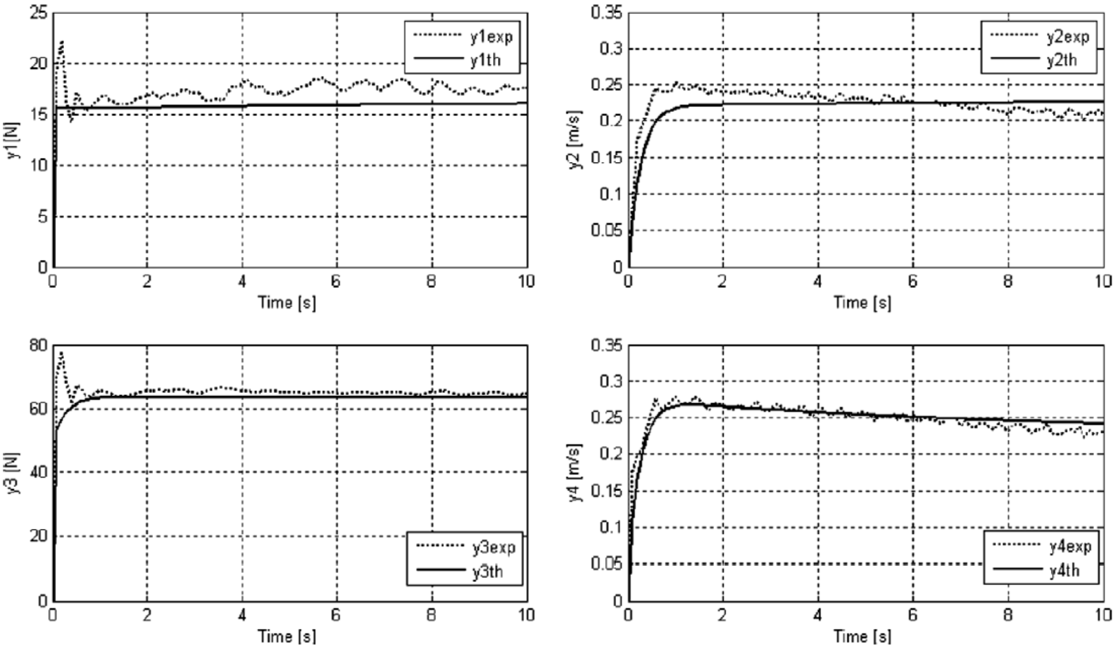

Fig. 8. Model validation for combination 1.

It is evident that the validation is very good and it is much closer than the validation results iteratively obtained in [10]. This result is equally satisfying considering the experimental dispersion (Figs 5 and 6); the model data are widely comprised in the experimental uncertainty for both the two different input combinations.

It is to underline that the model considered for this parameters tuning is a lumped linear model that assumes that the web does not completely slide on the roll, the web velocity is considered equal to the roll's linear velocity; this condition may be partially dissatisfied especially considering higher speeds (case of combination 2). Moreover in the model equation the inertia and radius of unwinder and winder sections are considered constant, and it may be another reason of the light discrepancy between the model and the experimental data.

7. CONCLUSIONS

The lumped model of an experimental web transport system recently developed by the authors [9-11], characterized by four different sections, high number of rollers positioned at different levels, long way of the web (polypropylene as example) in the different sections has been updated in this study.

This paper describes at first the mathematical technique to transform the block scheme usually used in web system lumped model (Voigt) in a differential equation system in the time domain. This transformation is not a simple step and it requires the introduction of not null initial conditions for the derivatives of the variables. Otherwise the possibility of having a mathematical model in the time domain is very attractive and useful for managing automatic methods for parameters estimation and/or for control design.

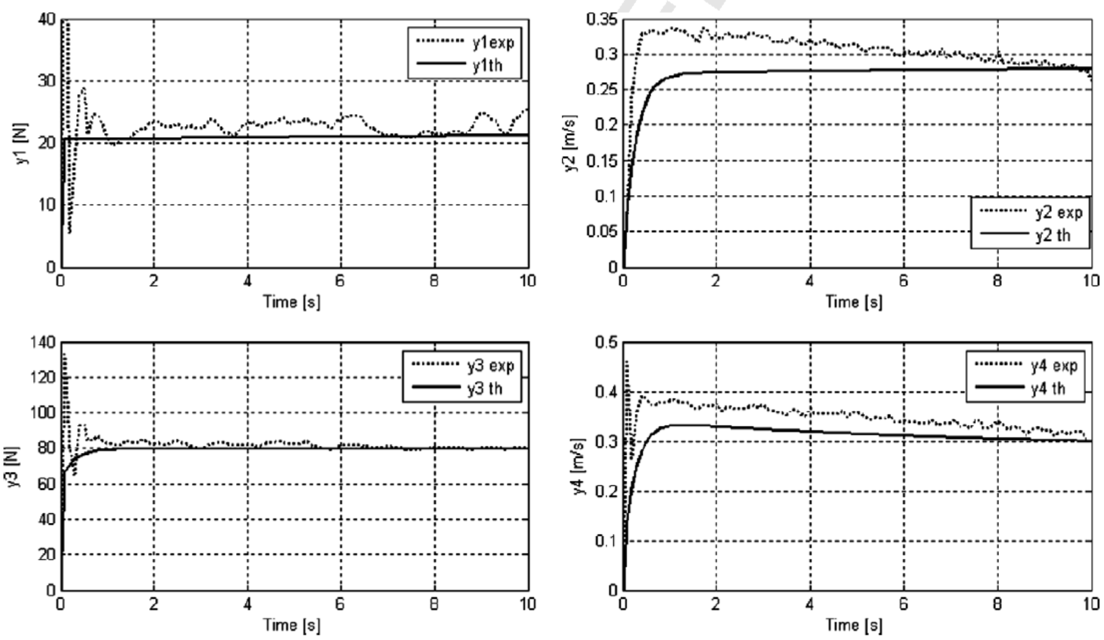

Fig. 9. Model validation for combination 2.

The full coincidence shown in this paper between the block system and the differential equation system demonstrates that, following a rigorous mathematical strategy, it is possible to convert this type of overlapped block scheme in the time domain differential equation system.

Moreover a multistep strategy for the estimation of several parameters for the time domain model of the experimental web system is introduced. The particularity of this strategy lies in the use of an SQP method on the differential equation system with two different combinations of step inputs and in the iterative refinement of weight coefficients used in the optimization process through the minimization of an opportune error indicator.

The good results obtained through this validation technique may be considered very attractive for analyzing the system behaviour using a lumped model; considering the improvements that the time domain model gives, the experience described in this paper could be a useful instrument for researcher involved in modelling problems related to similar systems.

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