# Systematic Study of Lattice Specific Heat of Filled Skutterudites

Kazuyuki Matsuhira<sup>\*</sup>, Chihiro Sekine<sup>1</sup>, Makoto Wakeshima<sup>2</sup>, Yukio HINATSU<sup>2</sup>, Takahiro Namiki<sup>†1,4</sup>, Keiki Takeda<sup>1</sup>, Ichimin Shirotani<sup>1</sup>, Hitoshi Sugawara<sup>3</sup>, Daisuke Kikuchi<sup>4</sup>, and Hideyuki Sato<sup>4</sup>

Graduate School of Engineering, Kyushu Institute of Technology, Kitakyushu 804-8550, Japan <sup>1</sup>Muroran Institute of Technology, Muroran, Hokkaido 050-8585, Japan

<sup>2</sup>Division of Chemistry, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan

<sup>3</sup>Faculty of the Integrated Arts and Sciences, The University of Tokushima, Tokushima 770-8502,

Japan

<sup>4</sup>Department of Physics, Tokyo Metropolitan University, Hachioji, Tokyo 192-0397, Japan (Received October 2, 2009)

The lattice specific heat  $C_{\text{lat}}$  of La-based filled skutterudites La  $T_4X_{12}$  (T = Fe, Ru and Os; X = P, As and Sb) has been systematically studied, and both the Debye temperature  $\Theta_{\text{D}}$  and the Einstein temperature  $\Theta_{\text{E}}$  of La  $T_4X_{12}$  were carefully estimated. We confirmed that a correlation exists between  $\Theta_{\text{D}}$  and the reciprocal of the square root of average atomic mass for La  $T_4\text{P}_{12}$ , La  $T_4\text{As}_{12}$ , and La  $T_4\text{Sb}_{12}$ . The  $\Theta_{\text{D}}$  of filled skutterudites was found to depend mainly on the nature of the species X forming the cage. The temperature dependence of  $C_{\text{lat}}/T^3$  for La  $T_4\text{X}_{12}$  exhibited a large broad maximum at low temperatures (10 - 30 K), which suggests a nearly dispersionless low-energy optical mode characterized by Einstein specific heat. Since no such broad maximum exists for the unfilled skutterudite RhP<sub>3</sub>, the low-energy optical modes are associated with vibration involving La ions in the  $X_{12}$  cage (the so-called "guest ion modes"). The  $\Theta_{\text{E}}$  of filled skutterudites was found to roughly correspond to the energy of low-energy guest ion optical modes. Furthermore, a good correlation was shown to exist between  $\Theta_{\text{E}}$  and  $r_{\text{R}-\text{X}} - r_{\text{R3}+}$ , where  $r_{\text{R}-\text{X}}$  is the *R*-X distance and  $r_{\text{R3}+}$  is the effective ionic radius of  $R^{3+}$ . As  $r_{\text{R}-\text{X}} - r_{\text{R3}+}$  increased,  $\Theta_{\text{E}}$  was found to decrease.

KEYWORDS: skutterudite, specific heat, Debye temperature, Einstein temperature, rattling vibration, guest ion mode

#### 1. Introduction

Recent active research on filled skutterudite compounds of the form  $RT_4X_{12}$  has revealed the wide variety of physical properties resulting from the strong *c-f* hybridization effect, the unique band structure, and the degree of freedom of the multipole moment due to *f* electrons in *R* sites of the cubically symmetric  $X_{12}$  cage.<sup>1-17</sup> In addition, it is well known that filled skutterudite compounds show a low glasslike thermal conductivity.<sup>18,19</sup> It has been presumed

 $<sup>^{*}\</sup>mbox{E-mail}$  address: matuhira@elcs.kyutech.ac.jp

<sup>&</sup>lt;sup>†</sup>Present address: Department of Physics, Tokyo Medical University, Tokyo, 160-8402

that this suppression of thermal conductivity comes from low-energy optical modes associated with the vibration of R ions in the  $X_{12}$  cage.<sup>20</sup> This characteristic vibration is called "rattling vibration" or the "guest ion mode". Furthermore, recently, some interesting phenomena such as ultrasonic absorption in  $LnOs_4Sb_{12}$  and a novel heavy-fermion state robust against magnetic fields in SmOs<sub>4</sub>Sb<sub>12</sub> have been reported.<sup>21–23</sup> It has been proposed that the electron-phonon coupling between conduction electrons and guest ion modes is responsible for these interesting phenomena.<sup>24,25</sup>

To clarify the above-mentioned interesting phenomena, more basic information on the phonon modes is required. In particular, it is important to reveal the chemical trend on low-energy guest-ion optical modes (LGOMs). From an analysis of the specific heat of Labased filled skutterudites, we can make a rough estimate of the phonon spectrum in the low-energy region because lattice specific heat is obtained by subtracting the contribution of the electronic specific heat  $\gamma T$  from the total specific heat C, where  $\gamma$  is the electronic specific heat coefficient. We can obtain  $\gamma$  and the Debye temperature  $\Theta_{\rm D}$  by applying the Debye  $T^3$ law at low temperatures. However, for filled skutterudites, since significant deviations from the Debye  $T^3$  law are caused by LGOMs, we have to carefully estimate  $\gamma$  and  $\Theta_D$ . LGOMs characterized by Einstein specific heat lead to a broad maximum in  $(C - \gamma T)/T^3$  at  $\sim \Theta_{\rm E}/4.92$ , where  $\Theta_{\rm E}$  is the Einstein temperature; the peak in  $(C - \gamma T)/T^3$  cannot be described by Debye specific heat. Therefore, we can obtain  $\Theta_{\rm E}$  from the maximum temperature  $T_{\rm max}$  in  $(C - \gamma T)/T^3$ .  $\Theta_{\rm E}$  roughly corresponds to the energy of LGOMs. In this paper, we report the lattice specific heat of  $LaT_4X_{12}$  and discuss the  $\Theta_D$  and  $\Theta_E$  of these compounds. In addition, we examine the correlations between the energy of LGOMs ( $\Theta_{\rm E}$ ) and structural parameters in filled skutterudites.

## 2. Experimental Procedure

Polycrystalline samples of La $T_4P_{12}$ , La $T_4As_{12}$  (T=Fe, Ru, Os), and RhP<sub>3</sub> were prepared at high temperatures and high pressures. A polycrystalline sample of LaFe<sub>4</sub>Sb<sub>12</sub> was prepared as reported in ref. 26. Single crystals of La $T_4Sb_{12}$  (T=Ru, Os) were grown by the Sb-self-flux method.<sup>27</sup> Specific heat measurement was carried out by a thermal-relaxation method (PPMS, Quantum Design Inc.) from 1.8 to 300 K.

## 3. Results and Discussion

# 3.1 Analysis of specific heat

Figures 1(a) and 1(b) show the specific heat divided by the temperature C/T for LaRu<sub>4</sub>P<sub>12</sub> and LaOs<sub>4</sub>Sb<sub>12</sub>, respectively. Both compounds show a superconducting transition at  $T_{\rm SC}=7$ K and  $T_{\rm SC}=0.74$  K, respectively.<sup>28</sup> In Fig. 1(a), the anomaly in C(T) at 7 K originates from this superconducting transition. For LaRu<sub>4</sub>P<sub>12</sub>, we obtained the specific heat in the normal state by applying a magnetic field with a strength above  $H_{c2}$ . In a magnetic field of 3 T, the C(T) for LaRu<sub>4</sub>P<sub>12</sub> between 2.8 and 6 K can be well fitted by  $C/T = \gamma + \beta T^2$  (Debye  $T^3$  law);  $\Theta_{\rm D} = (12\pi^4 R_{\rm g} n/5\beta)^{1/3}$ , where  $R_{\rm g}$  is the gas constant and n=17. In this manner we obtained  $\gamma = 28.8 \text{ mJ/K}^2$  mole and  $\Theta_{\rm D}=603$  K, as previously reported.<sup>29</sup> As the temperature range in which C(T) obeys the Debye  $T^3$  law tends to be narrow in filled skutterudites, we have to carefully estimate  $\gamma$  and  $\Theta_{\rm D}$ ; significant deviations from the Debye  $T^3$  law are caused by LGOMs, as will be discussed later. As seen in Fig. 1(b), the C(T) of LaOs<sub>4</sub>Sb<sub>12</sub> obeys the Debye  $T^3$  law below 4 K, and in this region we obtained  $\gamma = 54 \text{ mJ/K}^2$  mole and  $\Theta_{\rm D}=270$ K.  $\gamma$  is consistent with previous reports.<sup>30,31</sup>



Fig. 1. (a) (Color online) Specific heat divided by temperature C/T for LaRu<sub>4</sub>P<sub>12</sub> of magnetic fields of 0 and 3 T. The dash-dotted line represents a fit using the Debye  $T^3$  law. (b) (Color online) Specific heat divided by temperature C/T for LaOs<sub>4</sub>Sb<sub>12</sub>. The dash-dotted line shows C/T = $\gamma + C_D(T, \Theta_D)/T$  with  $\gamma = 54 \text{ mJ/K}^2$  mole,  $\Theta_D = 270 \text{ K}$ . The dotted line represents  $C_E(T, \Theta_E)/T$ whith  $\Theta_E = 60.5 \text{ K}$ . The solid line shows the sum of these contributions.

Figure 2 shows the temperature dependence of  $(C - \gamma T)/T^3$  for La  $T_4 X_{12}$ . Below 10 K,  $(C - \gamma T)/T^3$ 

 $\gamma T)/T^3$  tends to have a finite value ( $\beta$ ). This behavior is characterized by the Debye specific heat  $C_{\rm D}(T, \Theta_{\rm D})$  at low temperatures;  $C_{\rm D}(T, \Theta_{\rm D} = 670 \text{ K})$  is shown in Fig. 2. Furthermore, we can easily see the contribution of LGOMs in the  $C/T^3$  plot. It should be noted that all of the compounds exhibit a large broad maximum at around 10 - 30 K, which is a commonly observed feature in filled skutterudites. The broad maximum is characterized by the Einstein specific heat  $C_{\rm E}(T, \Theta_{\rm E})$ ;  $C_{\rm D}(T, \Theta_{\rm D})$  has no maximum in the  $C/T^3$  plot. The maximum in the  $C_{\rm E}/T^3$  plot is located at  $T_{\rm max} \cong \Theta_{\rm E}/4.92$ ;  $C_{\rm E}(T, \Theta_{\rm E} = 140 \text{ K})$  is also shown in Fig. 2. Therefore, we can estimate  $\Theta_{\rm E}$  from  $T_{\rm max}$  in  $(C - \gamma T)/T^3$ . Recent Raman and inelastic X-ray scattering (IXS) studies have observed LGOMs. The values of  $\gamma$ ,  $\Theta_{\rm D}$ , and  $\Theta_{\rm E}$  of La  $T_4 X_{12}$  are shown in Table I. The LGOMs of La  $T_4 X_{12}$  are centered at around 60 - 140 K.

It has been reported that the specific heat of  $AT_4Sb_{12}$  (A=Ca, Sr, Ba, La; T=Fe, Ru, Os) is roughly the sum of  $C_E(T, \Theta_E)$  and  $C_D(T, \Theta_D)$ .<sup>35</sup> In the same way, we found that the specific heat of La  $T_4X_{12}$  is roughly equal to the sum of  $C_E(T, \Theta_E)$  and  $C_D(T, \Theta_D)$  (not shown). This result suggests that the dispersion of LGOMs is weak in filled skutterudites. For example, we obtained a good fit (solid line) for the specific heat of LaOs<sub>4</sub>Sb<sub>12</sub> below 9 K (Fig. 1 (b)). The fitting curve is described by  $C/T = \gamma + C_D(T, \Theta_D = 270 \text{ K})/T + C_E(T, \Theta_E = 60.5 \text{ K})/T$ , where  $\gamma$  is 54 mJ/K<sup>2</sup> mole.



Fig. 2. (Color online) Temperature dependence of  $(C - \gamma T)/T^3$  for La $T_4X_{12}$ .

| Compound                            | $\gamma \ (mJ/K^2 \ mole)$ | $\Theta_{\rm D}~({\rm K})$ | $\Theta_{\rm E}~(K)$ |
|-------------------------------------|----------------------------|----------------------------|----------------------|
| $LaFe_4P_{12}$                      | 52                         | 670                        | 138                  |
| $LaRu_4P_{12} \\$                   | 29                         | 603                        | 128                  |
| ${\rm LaOs_4P_{12}}$                | 20                         | 482                        | 131                  |
| $\rm LaFe_4As_{12}$                 | 135                        | 421                        | 113                  |
| $\mathrm{LaRu}_{4}\mathrm{As}_{12}$ | 58                         | 355                        | 98.4                 |
| ${\rm LaOs_4As_{12}}$               | 49                         | 360                        | 99.4                 |
| $\rm LaFe_4Sb_{12}$                 | 122                        | 314                        | 87.6                 |
| $\rm LaRu_4Sb_{12}$                 | 47                         | 275                        | 72.8                 |
| $LaOs_4Sb_{12}$                     | 54                         | 270                        | 60.5                 |

Table I. Electronic specific heat coefficients  $\gamma$ , Debye temperatures  $\Theta_{\rm D}$ , and Einstein temperatures  $\Theta_{\rm E}$  for La  $T_4 X_{12}$ .

## 3.2 Specific heat of unfilled skutterudite RhP<sub>3</sub>

Figure 3 shows the temperature dependences of  $(C - \gamma T)/T^3$  for LaRu<sub>4</sub>P<sub>12</sub> and RhP<sub>3</sub>. The skutterudite RhP<sub>3</sub> has no La ions in the P<sub>12</sub> cage, and is therefore a good reference compound for LaRu<sub>4</sub>P<sub>12</sub> to clarify the effect of LGOM. For comparison with the lattice specific heat of LaRu<sub>4</sub>P<sub>12</sub>, the specific heat  $(C - \gamma T)/T^3$  of RhP<sub>3</sub> is multiplied by 4 in Fig. 3. Although RhP<sub>3</sub> is thought of as a semiconductor,<sup>36</sup> a small contribution to the specific heat from a T-linear term was observed. The C(T) curve for RhP<sub>3</sub> between 5 and 30 K can be well fitted by  $C/T = \gamma + \beta T^2$ , from which we obtained  $\gamma = 3.79 \text{ mJ/K}^2$  mole and  $\Theta_D = 498 \text{ K}$  using n=4; the origin of the small T-linear term is not yet clear. Although the  $(C - \gamma T)/T^3$  curve for LaRu<sub>4</sub>P<sub>12</sub> exhibits a large broad maximum at around 26 K, the curve for RhP<sub>3</sub> has no such feature. This is good evidence that LGOMs are due to optical modes related to the presence of La ions that fill in the P<sub>12</sub> cage. Therefore, we may conclude that the  $\Theta_E$  of filled skutterudites roughly corresponds to the energy of LGOMs in the  $X_{12}$  cage.

#### 3.3 Debye temperature

We next discuss the Debye temperature of La  $T_4X_{12}$ . According to the Debye model,  $\Theta_D$  is proportional to the velocity of sound in a solid. This in turn is proportional to the reciprocal of the square root of the density of the solid. Since density roughly corresponds to the average atomic mass  $M_{av}$ ,  $\Theta_D$  is ultimately linearly proportional to the reciprocal of the square root of  $M_{av}$ . The gradient of this linear relationship is roughly proportional to the square root of a force constant. Figure 4 shows the dependence of  $\Theta_D$  on average atomic mass for La  $T_4X_{12}$ . We found correlations between  $\Theta_D$  and the reciprocal of the square root of  $M_{av}$  for La  $T_4P_{12}$ , La  $T_4As_{12}$ , and La  $T_4Sb_{12}$ . The gradient for La  $T_4P_{12}$  is 1.2 times larger than that for La  $T_4As_{12}$ and 1.4 times larger than that for La  $T_4Sb_{12}$ . Therefore, La  $T_4P_{12}$  is expected to be roughly



Fig. 3. (Color online) Temperature dependences of  $(C - \gamma T)/T^3$  for LaRu<sub>4</sub>P<sub>12</sub> and RhP<sub>3</sub>. The  $(C - \gamma T)/T^3$  curve for RhP<sub>3</sub> is multiplied by a factor of 4 for easier comparison.

2 times harder than  $\text{La}T_4\text{Sb}_{12}$ . The bulk moduli of  $\text{La}\text{Ru}_4\text{P}_{12}$  and  $\text{La}\text{Ru}_4\text{Sb}_{12}$  are reported to be 172 and 98 GPa, respectively,<sup>37</sup> and the above results are consistent with these values. The  $X_{12}$  cage in filled skutterudites consists of  $X_4$  rings formed by strong covalent bonds. The present result indicates that the  $\Theta_{\rm D}$  of filled skutterudites depends mainly on the nature of X.



Fig. 4. (Color online) Dependence of  $\Theta_D$  on average atomic mass.

# 3.4 Specific heats of $GdOs_4P_{12}$ and $GdRu_4P_{12}$

We next discuss the lattice specific heats of  $GdOs_4P_{12}$  and  $GdRu_4P_{12}$ .  $GdOs_4P_{12}$  exhibits a ferromagnetic transition at  $T_C=5$  K,<sup>38,39</sup> whereas  $GdRu_4P_{12}$  shows an antiferromagnetic transition at  $T_N=22$  K.<sup>7,40</sup> Figure 5 shows the specific heats ( $\Delta C$ ) and entropies ( $\Delta S$ ) of  $GdOs_4P_{12}$  and  $GdRu_4P_{12}$ . To obtain a 4f magnetic contribution, we estimated the  $\Delta C$  values of  $GdOs_4P_{12}$  and  $GdRu_4P_{12}$  by subtracting the specific heats of  $LaOs_4P_{12}$  and  $LaRu_4P_{12}$ , respectively, as nonmagnetic components. We now focus on the magnetic contribution of  $GdOs_4P_{12}$ . A broad peak in  $\Delta C$  for  $GdOs_4P_{12}$  appears at around 30 K in Fig. 5(a). The  $\Delta S$  value of  $GdOs_4P_{12}$  estimated using  $\Delta C$  data is beyond Rln8 (14.9 J/K<sup>2</sup> mole), which is expected to be in the ground state S=7/2 multiplet of  $Gd^{3+}$ . However, in general, there is no splitting of CEF levels in the ground state of  $Gd^{3+}$ , and therefore the broad peak does not come from the Schottky anomaly.

The broad peak is best explained by a lattice contribution resulting from a significant shift of LGOMs towards a low energy. We have already reported on a similar phenomenon in SmOs<sub>4</sub>Sb<sub>12</sub>.<sup>31</sup> The effect of the energy shift of LGOMs on lattice specific heat is evaluated using  $\Theta_{\rm E}$  as a fitting parameter. We estimated the change in  $\Theta_{\rm E}$  for GdOs<sub>4</sub>P<sub>12</sub> from  $\Theta_{\rm E}$ =131 K for LaOs<sub>4</sub>P<sub>12</sub>. The resulting  $\Delta C_{\rm E}$  curve is shown in Fig. 5(a) where  $\Delta C_{\rm E}(T) = C_{\rm E}(T,$  $\Theta_{\rm E}) - C_{\rm E}(T, \Theta_{\rm E} = 131 \text{ K})$ . In this manner, we obtained  $\Theta_{\rm E}$ =87 K for GdOs<sub>4</sub>P<sub>12</sub>. The abovementioned Schottky-like anomaly at 80 K is well reproduced by the curve. The *true* magnetic contribution appears below 20 K. In addition,  $\Delta S$  after correcting for the contribution from  $\Delta C_{\rm E}$  is close to *R*ln8 at 80 K. In this analysis, excess entropy can also be explained by the low-energy shift of LGOMs. In a similar manner, we estimated  $\Theta_{\rm E}$ =98 K for GdRu<sub>4</sub>P<sub>12</sub>. The results for  $\Delta C_{\rm E}$  are shown in Fig. 5(b). The long tail in  $\Delta C$  above  $T_{\rm N}$ =22 K is mainly due to a significant energy shift of LGOMs towards a low energy. The *true* magnetic contribution appears below 30 K. A short-range ordering in this antiferromagnetic transition develops below this temperature.

# 3.5 Correlation between structure parameters and $\Theta_E$

We will now discuss the correlations between structure parameters and  $\Theta_E$  for La $T_4X_{12}$ , Gd  $T_4P_{12}$ , and Sm  $T_4X_{12}$ . First, we define the guest free distance  $r_{\text{GFD}}$  as

$$r_{\rm GFD} = r_{\rm R-X} - r_{\rm R3+} - r_{\rm X},\tag{1}$$

where  $r_{\rm R-X}$  is the distance between R and X,  $r_{\rm R3+}$  is the effective ionic radius of  $R^{3+}$  for a 12-coordination-number site, and  $r_{\rm X}$  is the covalent radius of X.<sup>41</sup> If the energy of LGOMs depends only on the structure parameters, we would expect to see a good correlation between  $r_{\rm GFD}$  and  $\Theta_{\rm E}$ . In this estimation of  $r_{\rm GFD}$ , we used the lattice parameters shown in Table II. In considering the effective space in the cage, we should take into account the effective ionic radius of rare-earth ions. The effective ionic radius of Sm<sup>3+</sup> (1.24 Å) is smaller than



Fig. 5. (Color online) Specific heats ( $\Delta C$ ) and entropies ( $\Delta S$ ) of (a) GdOs<sub>4</sub>P<sub>12</sub> and (b) GdRu<sub>4</sub>P<sub>12</sub>. The broken lines show  $\Delta C_{\rm E}(T)$  (see text for details). The solid lines show the true magnetic entropy after correcting for the contribution of  $\Delta C_{\rm E}$ .

that of La<sup>3+</sup> (1.36 Å). The effective ionic radius of  $\mathrm{Gd}^{3+}$  for a 12-coordination-number site has not been reported so far. Therefore, we estimated it to be 1.21 Å by extrapolation from those of other rare-earth elements. To derive  $r_{\mathrm{R-X}}$ , we need the lattice parameters u and v. Unfortunately, these parameters, which determine the position of X atoms, have not been reported so far for LaRu<sub>4</sub>As<sub>12</sub> or LaOs<sub>4</sub>As<sub>12</sub>. However, the relationship  $u + v \sim 0.50$  is wellknown for filled skutterudites. In fact, from Table II, we can see that  $u + v = 0.500 \pm 0.005$ for the other compounds. Therefore, for the estimation of the  $r_{\mathrm{R-X}}$  values of LaRu<sub>4</sub>As<sub>12</sub> and LaOs<sub>4</sub>As<sub>12</sub>, we used the lattice parameters  $u = 0.350 \pm 0.005$  and  $v = 0.150 \mp 0.005$ . Next, since the lattice parameters u and v for  $\mathrm{Sm} T_4 X_{12}$  and  $\mathrm{Gd} T_4 P_{12}$  have also not been reported except in the case of the compound  $\mathrm{SmOs}_4 \mathrm{Sb}_{12}$ , we substituted the parameters of the La analogue for those of  $\mathrm{Sm} T_4 X_{12}$  and  $\mathrm{Gd} T_4 P_{12}$ . Although ambiguities in u and v lead to an error of roughly 1% in the estimation of  $r_{\mathrm{R-X}}$ , this is not significant in the present discussion.

| Compound                          | a       | (u,v)               | Ref. |
|-----------------------------------|---------|---------------------|------|
| $LaFe_4P_{12}$                    | 7.83160 | (0.3539, 0.1504)    | 42   |
| $\rm SmFe_4P_{12}$                | 7.8029  | nd                  | 42   |
| $\mathrm{LaRu}_4\mathrm{P}_{12}$  | 8.0610  | (0.3591,  0.1428)   | 43   |
| $\mathrm{SmRu}_4\mathrm{P}_{12}$  | 8.0397  | nd                  | 8    |
| $GdRu_4P_{12}$                    | 8.0375  | nd                  | 7    |
| $\mathrm{LaOs}_4\mathrm{P}_{12}$  | 8.08197 | (0.35700, 0.14002)  | 44   |
| $\mathrm{GdOs}_4\mathrm{P}_{12}$  | 8.0657  | nd                  | 38   |
| $LaFe_4As_{12}$                   | 8.3252  | (0.34556, 0.15474)  | 45   |
| $\rm SmFe_4As_{12}$               | 8.3003  | nd                  | 46   |
| $LaRu_4As_{12}$                   | 8.50810 | nd                  | 45   |
| $LaOs_4As_{12}$                   | 8.54370 | nd                  | 45   |
| $LaFe_4Sb_{12}$                   | 9.1395  | (0.33696, 0.16042)  | 47   |
| $\rm SmFe_4Sb_{12}$               | 9.130   | nd                  | 48   |
| $LaRu_4Sb_{12}$                   | 9.2732  | (0.34174,  0.1581)  | 49   |
| $LaOs_4Sb_{12}$                   | 9.30799 | (0.34118,  0.1565)  | 49   |
| $\mathrm{SmOs}_4\mathrm{Sb}_{12}$ | 9.3085  | (0.34009,  0.15589) | 50   |

Table II. Lattice parameters (a (Å) and (u, v)) for  $RT_4X_{12}$ . nd: not determined.

The  $r_X$  values of P, As, and Sb are 1.06, 1.19 and 1.38 Å, respectively. Actually, the average X-X distance in the  $X_4$  ring is close to  $2r_X$  and slightly larger than  $2r_X$ .

Figures 6(a) and 6(b) show the dependences of  $\Theta_{\rm E}$  on  $r_{\rm GFD}$  and  $r_{\rm R-X}-r_{\rm R3+}$ , respectively. For Sm  $T_4X_{12}$ , the energy of LGOMs obtained by Raman scattering and IXS is shown in the figures.<sup>32–34</sup> We found that the correlation between  $r_{\rm GFD}$  and  $\Theta_{\rm E}$  is not strong at first sight. Instead, there appears to be a better linear correlation between  $r_{\rm R-X} - r_{\rm R3+}$  and  $\Theta_{\rm E}$ , which suggests that  $r_{\rm X}$  does not affect  $\Theta_{\rm E}$ . One reason for the deviation may be the rigidity of the  $X_{12}$  cage. The  $X_4$  ring has a strong covalency as mentioned above, and the  $X_{12}$  cage is formed by six such rings, each of which connects between two  $X_{12}$  cages. Associated with each  $X_{12}$ cage, there are two different X-X distances. The X-X distance in the  $X_4$  ring is close to  $2r_{\rm X}$ . However, the second X-X distance is 1.4 - 1.7 times longer than  $2r_{\rm X}$ . Since the degree of covalency in the  $X_{12}$  cage is not so strong, the cage is not very rigid. As noted previously in the discussion on the  $\Theta_{\rm D}$  of La  $T_4X_{12}$ , La  $T_4{\rm P}_{12}$  is roughly 2 times harder than La  $T_4{\rm Sb}_{12}$ ; thus, we can expect that the Sb<sub>12</sub> cage is less rigid. For a nonrigid cage, the concept of guest free distance collapses. Therefore, the deformation of the  $X_{12}$  cage may cause a significant deviation from the linear correlation between  $r_{\rm GFD}$  and  $\Theta_E$ . Another possibility is that, since all of these compounds are metallic, both the  $X_{12}$  cage and the simple cubic T-lattice are conductive. Since bondings in the  $X_{12}$  cage, in the simple cubic *T*-lattice, and between *X* and *T* sites are the most important contributions to metallicity, the covalent radius of the  $X_{12}$  cage is not important for guest ions. Therefore,  $\Theta_{\rm E}$  is unaffected by  $r_{\rm X}$ .

Next, it should be noted that the deviation from the linear relationship between  $r_{R-X} - r_{R3+}$  and  $\Theta_E$  for Os skutterudites with f electrons can be seen in Fig. 6(b). This suggests that some other factors affect LGOMs. One such factor is a strong electron-lattice interaction, which can lead to anharmonic vibration.<sup>51</sup> When LGOM exhibits strong anharmonic vibration, the energy of LGOM decreases upon cooling.<sup>52</sup> In fact, the energy of LGOMs for LaOs<sub>4</sub>Sb<sub>12</sub> is decreased by 5% from 300 down to 4 K; it is suggested that LGOMs for LaOs<sub>4</sub>Sb<sub>12</sub> are anharmonic.<sup>33</sup> Although in these calculations we used  $\Theta_E$  estimated at low temperatures, the structure parameters used were determined at room temperature. At low temperatures, these differences are expected to become larger. The present result suggests a strong anharmonic vibration of LGOMs in Os skutterudites. Another possible origin is a strong c-f hybridization effect, which may lead to a strong interaction between the R-filler with 4f electrons and the  $X_{12}$  cage and the T lattice with conduction electrons. Actually, novel types of behavior resulting from a strong c-f hybridization have been reported in many Os skutterudites with f electrons.<sup>9, 22, 23, 53-57</sup> Further study is needed to clarify how the interaction between the guest ions and the host cage due to c-f hybridization affects LGOM.

Now, we discuss the significant deviation from the linear relationship between  $r_{\rm R-X} - r_{\rm R3+}$ and  $\Theta_{\rm E}$  for SmOs<sub>4</sub>Sb<sub>12</sub> in Fig. 6(b). This compound is known to be an intermediate-valence heavy-fermion compound; the average valence of Sm ions is 2.83 at room temperature.<sup>54</sup> From a recent result on the temperature dependence of structure parameters for SmOs<sub>4</sub>Sb<sub>12</sub>, the Sm-Sb distance has been found to decrease by 0.016 Å as the temperature changes from 300 to 20 K.<sup>56</sup> Thus, the guest free distance of SmOs<sub>4</sub>Sb<sub>12</sub> decreases with a decrease in temperature. The deviation from the linear correlation between  $r_{\rm R-X} - r_{\rm R3+}$  and  $\Theta_{\rm E}$  becomes much greater at low temperatures. Furthermore, the average effective ionic radius of Sm ions in SmOs<sub>4</sub>Sb<sub>12</sub> is larger than  $r_{\rm R3+}$  for Sm<sup>3+</sup> because of the existence of Sm<sup>2+</sup> with a larger effective ionic radius. Since the difference between the Sm-Sb distance and the average effective ionic radius of Sm is much smaller than  $r_{\rm R-X} - r_{\rm R3+}$ , the deviation seen in Fig. 6(b) becomes clear. This means that the energy of LGOMs in SmOs<sub>4</sub>Sb<sub>12</sub> is not determined by only the structure parameters. The above results strongly suggest that the guest-host interaction due to c-fhybridization plays an important role in determining the LGOMs of SmOs<sub>4</sub>Sb<sub>12</sub>.

# 4. Conclusions

We have systematically studied the lattice specific heat of La-based filled skutterudites  $\text{La}T_4X_{12}$  (*T*=Fe, Ru and Os; *X*=P, As and Sb), and their  $\Theta_{\rm D}$  and  $\Theta_{\rm E}$  were carefully determined. The  $\Theta_{\rm D}$  of filled skutterudites was found to depend mainly on the nature of the *X* species forming the cage;  $\text{La}T_4P_{12}$  was shown to be roughly 2 times harder than  $\text{La}T_4\text{Sb}_{12}$ .



Fig. 6. (Color online) Dependences of  $\Theta_{\rm E}$  on (a) guest free distance  $r_{\rm GFD}$  and (b)  $r_{\rm R-X} - r_{\rm R3+}$  for  $RT_4P_{12}$  (open circles: R=La, closed circles: R=Sm and Gd),  $RT_4As_{12}$  (open squares: R= La, closed squares: R=Sm), and  $RT_4Sb_{12}$  (open triangles: R=La, closed triangles: R=Sm and Gd).

The  $\Theta_{\rm E}$  of filled skutterudites was found to correspond roughly to the energy of LGOMs. We found a good linear correlation between  $r_{\rm R-X} - r_{\rm R3+}$  and  $\Theta_{\rm E}$ , with  $\Theta_{\rm E}$  decreasing as  $r_{\rm R-X} - r_{\rm R3+}$  increases. However, we found a deviation from the linear relationship between  $r_{\rm R-X} - r_{\rm R3+}$  and  $\Theta_{\rm E}$  for Os compounds with f electrons. This was discussed in terms of the effects of both anharmonic vibration due to a strong electron-lattice interaction and guest-host interaction due to c-f hybridization in these compounds. In particular, for SmOs<sub>4</sub>Sb<sub>12</sub>, the present results suggest that the guest-host interaction due to c-f hybridization has the most critical effect on LGOMs.

Full Paper

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