

DETERMINATION OF STRESS INTENSITY FACTOR FOR INTERFACE CRACK UNDER UNIFORM HEAT FLOW BY CRACK TIP STRESS METHOD

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Summary This paper deals with the analysis of the thermal stress intensity factor for interfacial crack in dissimilar materials under uniform heat flow by using the finite element method. This method is based on the fact that the singular stress field near the interface crack tip is controlled by the stress values at the crack tip node calculated by FEM. The calculation shows that the present method has the sufficient accuracy in the interface crack problems under thermal stress.

INTRODUCTION

The determination of stress intensity factors (SIFs) for interface cracks under thermal boundary conditions is a subject of practical importance. Many methods have been developed to calculate the SIFs of an interface crack by using the finite element method. However, it is still not necessarily easy to analyze SIFs of interface crack under thermal stress by FEM because of the oscillatory stress singularity.

In this paper, an interface crack in a dissimilar plate under uniform heat flow is considered as shown in Fig.1(a). To determine the SIFs under thermal stress, the stresses at the interface crack tip calculated by FEM are used and are compared with the results of reference problem (Fig.1b) using the same mesh pattern near the crack tip. Here, a central interface crack will be considered with varying crack length and material combination. Then, the effect of material combination on the SIFs will be discussed.

METHOD OF ANALYSIS

Recently, the effective method was proposed for calculating the stress intensity factor of interface crack by using FEM [1]. The method utilizes the stress values at the crack tip by FEM. From the stresses σ_y , τ_{xy} along the interface crack tip, stress intensity factors are defined as shown in Eq.1.

$$\sigma_y + i\tau_{xy} = \frac{K_1 + iK_2}{\sqrt{2\pi r}} \left(\frac{r}{2a} \right)^{i\varepsilon}, \quad \varepsilon = \frac{1}{2\pi} \ln \left[\frac{\left(\frac{\kappa_1 + 1}{G_1} \right) / \left(\frac{\kappa_2 + 1}{G_2} \right)}{\left(\frac{\kappa_2 + 1}{G_2} \right) / \left(\frac{\kappa_1 + 1}{G_1} \right)} \right] \quad (1)$$

Here, $\kappa_j = 3 - 4\nu_j$ ($j=1, 2$) for plane strain condition. From Eq.(1), SIFs may be separated as

$$K_1 = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_y \{ \cos Q + (\tau_{xy} / \sigma_y) \sin Q \}, \quad K_2 = \lim_{r \rightarrow 0} \sqrt{2\pi r} \tau_{xy} \{ \cos Q - (\sigma_y / \tau_{xy}) \sin Q \} \quad (2)$$

$$Q = \varepsilon \ln \{ r / (2a) \} \quad (3)$$

Here, r and Q can be chosen as constant values since the reference and unknown problems have the same crack length and the same material combination. Therefore if Eq.4 is satisfied, Eq.5 may be derived from Eq.3.

$$\tau_{xy}^* / \sigma_y^* = \tau_{xy} / \sigma_y \quad (4)$$

$$K_1^* / \sigma_y^* = K_1 / \sigma_y, \quad K_2^* / \tau_{xy}^* = K_2 / \tau_{xy} \quad (5)$$

Here, σ_y^* , τ_{xy}^* are stresses of reference problem near the crack tip, and σ_y , τ_{xy} are stresses of given problem. By using the stress values at the interface crack tip calculated by FEM, the SIFs of the given problem can be obtained by

$$K_1 = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}^*} K_1^*, \quad K_2 = \frac{\tau_{xy0,FEM}}{\tau_{xy0,FEM}^*} K_2^* \quad (6)$$

SIFs of the reference problem are defined by Eq.7.

$$K_1^* + iK_2^* = (T + iS) \sqrt{\pi a} (1 + 2i\varepsilon) \quad (7)$$

In order to determine the loading stresses T and S , the reference problem is expressed by superposing the tension and shear problems. The stresses at the interface crack tip of the reference problem are expressed by

$$\sigma_{y0,FEM}^* = T \cdot \sigma_{y0,FEM}^{T=1} + S \cdot \sigma_{y0,FEM}^{S=1}, \quad \tau_{xy0,FEM}^* = T \cdot \tau_{xy0,FEM}^{T=1} + S \cdot \tau_{xy0,FEM}^{S=1} \quad (8)$$

where $\sigma_{y0,FEM}^{T=1}$ stands for the stress σ_y at the crack-tip node calculated by FEM in the condition of $T=1$ and $S=0$. From the condition that the stresses at the crack tip between the given and the reference problems are the same, we obtain the loading stresses T and S as follows,

$$T = \frac{\sigma_{y0,FEM} \cdot \tau_{xy0,FEM}^{S=1} - \sigma_{y0,FEM}^{S=1} \cdot \tau_{xy0,FEM}}{\sigma_{y0,FEM}^{T=1} \cdot \tau_{xy0,FEM}^{S=1} - \sigma_{y0,FEM}^{S=1} \cdot \tau_{xy0,FEM}^{T=1}}, \quad S = \frac{\sigma_{y0,FEM}^{T=1} \cdot \tau_{xy0,FEM} - \sigma_{y0,FEM} \cdot \tau_{xy0,FEM}^{T=1}}{\sigma_{y0,FEM}^{T=1} \cdot \tau_{xy0,FEM}^{S=1} - \sigma_{y0,FEM}^{S=1} \cdot \tau_{xy0,FEM}^{T=1}} \quad (9)$$

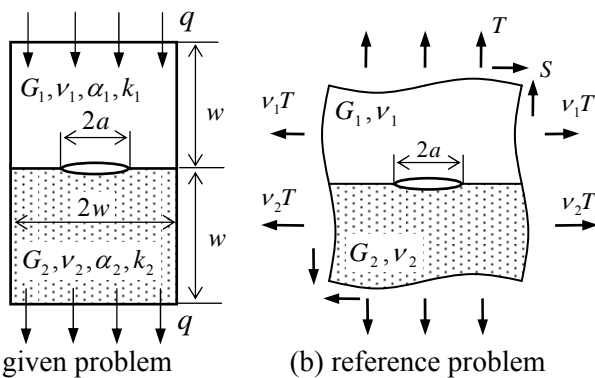
NUMERICAL RESULTS AND DISCUSSION

Crack in a homogeneous plate

In order to examine the accuracy of the present method, a center crack in a homogeneous plate ($G_1/G_2=1$) subjected to uniform heat flow is analyzed. Table 1 shows the comparison between the present results and results computed by the body force method [2]. In Table 1, K_{II}^∞ is the exact solution of a crack in an infinite plate subjected to uniform heat flow [3]. Both results are in good agreement with each other. From Table 1, it is found that the present method gives the numerical results with good accuracy for the thermal crack problem.

Centre interface crack in a bonded plate

Figures 2 and 3 show the normalized stress intensity factors of the center interface crack in bonded dissimilar plate subjected to uniform heat flow (Fig.1). To observe the relation between the material combinations G_1/G_2 and SIFs, the shear modulus $G_1=3.85 \times 10^5$ ($E_1=10^6$) [MPa], Poisson's ratio $\nu_1=\nu_2=0.3$, thermal expansion $\alpha_1=\alpha_2=10^{-7}$ [1/K], thermal conductivity $k_1=k_2=100$ [W/mK], uniform heat flow $q=10^5$ [W/m²] and plane strain state are assumed. In Figs 2 and 3, As G_1/G_2 increases, the normalized values F_1 and F_2 increase.



(a) given problem (b) reference problem
Fig.1 Interface crack (a) in a bonded finite plate under uniform heat flow, (b) in a bonded dissimilar semi-infinite plate subjected to mechanical loads.

Table 1 Normalized SIFs of center crack in a plate under uniform heat flow ($G_1/G_2=1$ in Fig.1a).

a/w	K_{II} / K_{II}^∞		Error [%]
	Present	BFM [2]	
0.1	0.999	0.999	0.00
0.2	0.990	0.997	-0.70
0.3	0.999	0.999	0.00
0.4	1.013	1.013	0.00
0.5	1.049	1.049	0.00
0.6	1.122	1.121	0.09

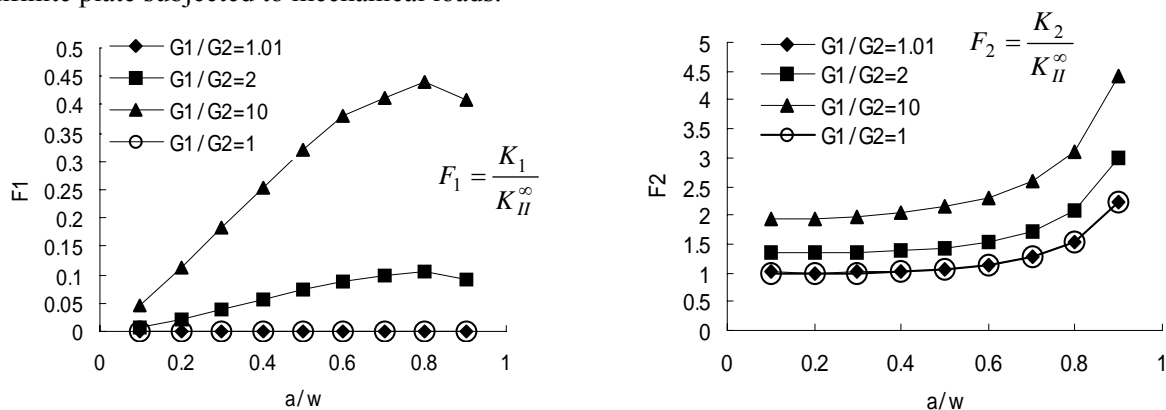


Fig.2. Variation of F_1 of center interface crack in a bonded dissimilar plate under uniform heat flow.

References

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