Design and Optimization of Stochastic Search Strategy for Foraging Problem

採餌問題のための確率的探索戦略の設計と最適化

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Abstract

Autonomous robot's search strategy is the set of rules that it employs while looking for targets in its environment. In this study, the stochastic movement of robots in unknown environments is statistically studied, using a Lévy walk method. Biological systems (e.g., foraging animals) provide useful models for designing optimal stochastic search algorithms. Observations of biological systems, ranging from large animals to immune cells, have inspired the design of efficient search strategies that incorporate stochastic movement. In this study, we seek to identify the optimal stochastic strategies for autonomous robots. Given the complexity of interaction between the robot and its environment, optimization must be performed in high-dimensional parameter space. The effect of the explanatory variable on the forger robot movement with the minimum required energy was also studied using experiments done by the response surface methodology (RSM). We analyzed the extent to which search efficiency requires these characteristics, using RSM. Correlation between the involved parameters via a Lévy walk process was examined through designing a setup for the experiments to determine the interaction of the involved variables and the robot movement. The extracted statistical model represents the priority influence of those variables on the robot by developing the statistical model of the mentioned unknown area. The efficiency of a simple strategy was investigated based on Lévy walk search in two-dimensional landscapes with clumped resource distributions. We show how RSM techniques can be used to identify optimal parameter values as well as to describe how sensitive efficiency reacts to the changes in these values. Here, we identified optimal parameter for designing robot by using stochastic search pattern and applying mood-switching criteria on a mixture of speed and sensor and μ to determine how many robots are needed for a solution. Fractal criterion-based robot strategies were more efficient than those based on the resource encounter criterion, and the former was found to be more robust to changes in resource distribution as well.

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1   Introduction

Many organisms, ranging from bacteria to primates, make use of stochastic movement patterns in order to find food, which is vital for their survival[1]. Such movement patterns, known as search strategies, have recently caught the attention of many ecologists interested in shedding light on the universal features of optimal foraging behavior[2]. Accordingly, three contributions to this field are discussed below. The main question :

How should robots behave in environments with a small knowledge and unknown factors to find targets?

In Section 2, A literature review is presented, with the aim of understanding the history of studies on organism's forage as a central goal of behavioral ecology. Reviewing and discussing the factors that impact levy walk and foraging behavior, this study can shed light on the interactions between organisms and their environments, predicting organisms' reactions to changing environmental conditions. A foraging organism usually has no idea of the location of food resources. Consequently, it must make use of search strategies to find them. A review of empirical studies reveals a variety of species using random movement patterns to locate food resources[2]. Some of these stochastic search strategies include the following movement patterns:

Brownian motion, Lévy walks, Straight-line (ballistic) motion.

The extent to which different stochastic movement patterns are effective in finding resources largely depends on the spatial distribution of resources. Thus, a forager's evolutionary fitness depends, by large, on the interaction between its movement strategy and the kind of landscape it is exploring. foraging is a fascinating and highly multidisciplinary field of research with implications far beyond the confines of biology. Foraging has been a canonical setting for the study of search, reward-seeking, and information processing. These and related themes have a wide impact on fields such as biology, economics, robotics, and computer science. The definition of foraging as a repeated sequence of actions: search, encounter, decide. Search can encompass a strategy of waiting in place as well as an active traversal of the environment in an effort to find resources. An encounter occurs when a food item is located, and the organism must then decide whether to attempt to appropriate the resource. Following this, a foraging strategy can be broadly considered as a strategy for searching for an environment in order to encounter and appropriate food resources. Hence, a complete strategy will cover the operationalization of a search process, encounter behaviors, and choices as to which items are considered as prey.

In Section 3,4, As a collection of statistical and mathematical techniques, design of experiment (DOE) and response surface methodology (RSM) has proved to be very useful in the development, improvement, and optimization of processes. RSM also has significant applications for the design, development, and formulation of new products. It can improve existing product designs as well. RSMs are widely used in the industrial sector, especially in situations in which multiple input variables potentially affect performance measures or quality properties of a product or process. These performance measures or quality characteristics are called the response. They are typically measured on a continuous scale, while attribute responses, ranks, and sensory responses are not unusual. The majority of real-world applications of RSM include more than one response. The input variables have come to be called independent variables as well, and they are controlled by the engineer or scientist, at least when it comes to a test or an experiment.

1.1 Biologically-inspired strategies

Biologically inspired robotics is a field of study that examines how the behaviors of living cells and organisms can be used as a basis for programming robots[3]. These behaviors are assumed to be well-honed to their purpose by natural selection. In the case of stochastic search strategies, one can look into the foraging behavior of animals, including seabirds, sharks, fruit flies, fish, bacteria, large

mammals, etc. One can even look at the behavior of cells in the human immune system, which can be thought of as "foraging" for disease cells[2].

Many of these organisms appear to move via a Lévy Walk while foraging for food, and so can be thought of as executing stochastic search strategies[4]. There are many different types of random walk and Lévy Walk and stochastic movement[5]. In a simple random walk a searcher moves a fixed distance (the step length) in a randomly chosen direction, stops, randomly chooses another direction, and moves a distance equal to the step-length in that new direction, and so on. In a Gaussian random walk, step-lengths are chosen from a Gaussian distribution (a simple random walk can be viewed as a Gaussian random walk with variance zero)[2]. At the sufficiently large time and distance scales, all Gaussian random walks converge to Brownian motion. Random walk with this property is called diffusion. When the step directions are selected from a non-uniform distribution, the result is a biased random walk. The random walk has the probability distribution of each step direction concentrated around the direction of the previous step direction. Searchers that move via correlated random walks are said to display directional persistence. A random walk can be both biased and correlated[6].

Ballistic motion is the term for straight-line movement. It can be viewed as a random walk with infinite step-length. In the ballistic motion, a searcher selects a direction at random and travels in that direction indefinitely the mean-squared

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displacement of ballistic motion scales with the square of time. The movement pattern is superdiffusive if its mean-square displacement scales with time at a faster-than-linear rate; hence, ballistic motion is superdiffusive[3, 7, 8].

Lévy walks a particularly important class of superdiffusive random walks. In natural systems, food resources (targets) are often distributed in clumps. If a forager encounters a food item, likely, other food is nearby. Hence it makes sense to carefully search the nearby area, using a movement pattern such as Brownian motion. Search strategy like Brownian motion is inefficient because it involves revisiting previously explored areas[2, 9]. On the other extreme, a forager employing ballistic motion DOEs not revisit previously explored terrain but might be unlucky and move in a direction away from a clump of food resources. Lévy walk is a trade-off between these two phenomena. Lévy walk foragers are likely to take small steps (similar to Brownian motion), but will occasionally take very long steps, preventing them from wasting time intensively searching a barren region[8, 10, 11].

The Lévy foraging hypothesis has been very controversial; nonetheless, it serves as excellent motivation for programming autonomous robotic search. In this thesis, we seek to answer a fundamental question: When designing a robot for stochastic movement, what are the optimal parameter values for a Lévy Walk stochastic search strategy and what are optimal parameters for the robot itself. The answer

depends on many characteristics of the system and the robot, including the detection radius of the searcher and speed of the robot and the general spatial distribution of targets on the landscape[12, 13]. The contribution of the dissertation can be explained as follows: Unknown Environment, Stochastic Movement, Forager Robotic, Design of Robot, Statistical Analysis, Design of Experiment, Optimization of Parameter[1, 9]. Random search; optimal foraging; Brownian motion; random walk; animal movement; spatial point process; Behavior and Ethology; Numerical Analysis and Computation; Other Applied Mathematics; Other Ecology and Evolutionary Biology; Other Mathematics; Probability[7, 14].
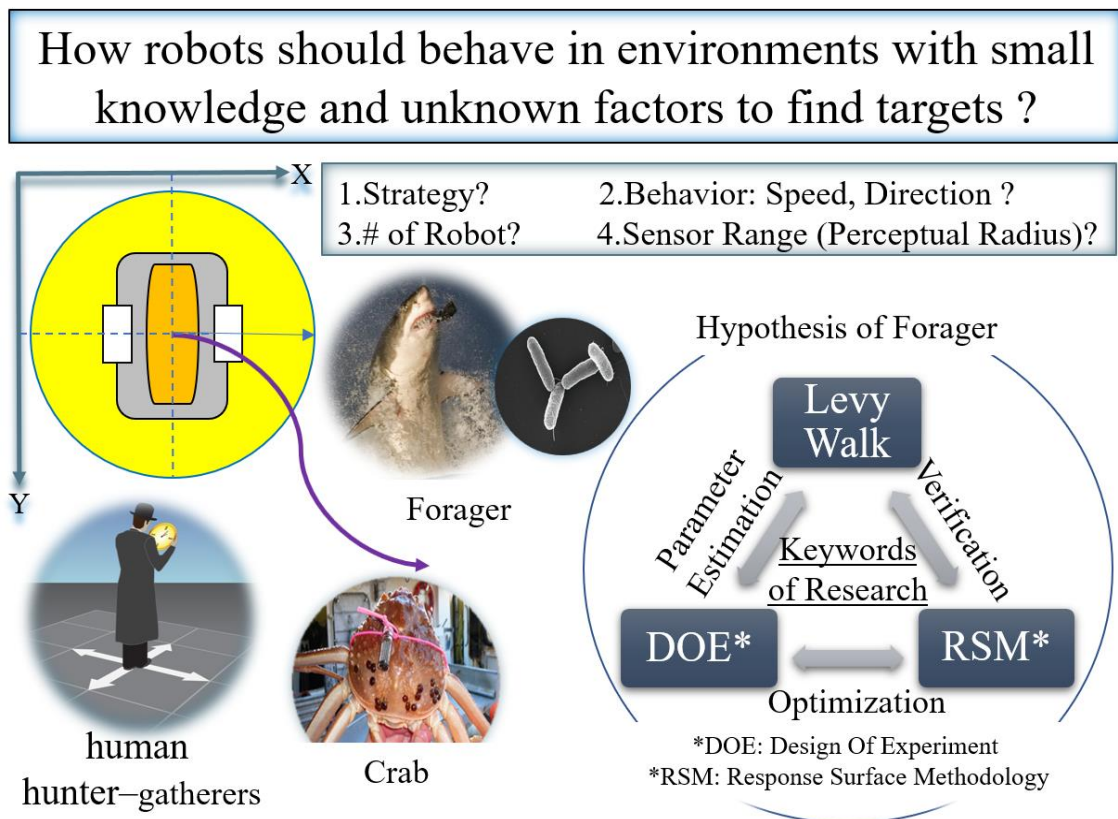
Figure 1-1. Road map

2   Lévy Walk

Foraging can be investigated in some biological systems. In this section, some of the biologically originated features are discussed[15].
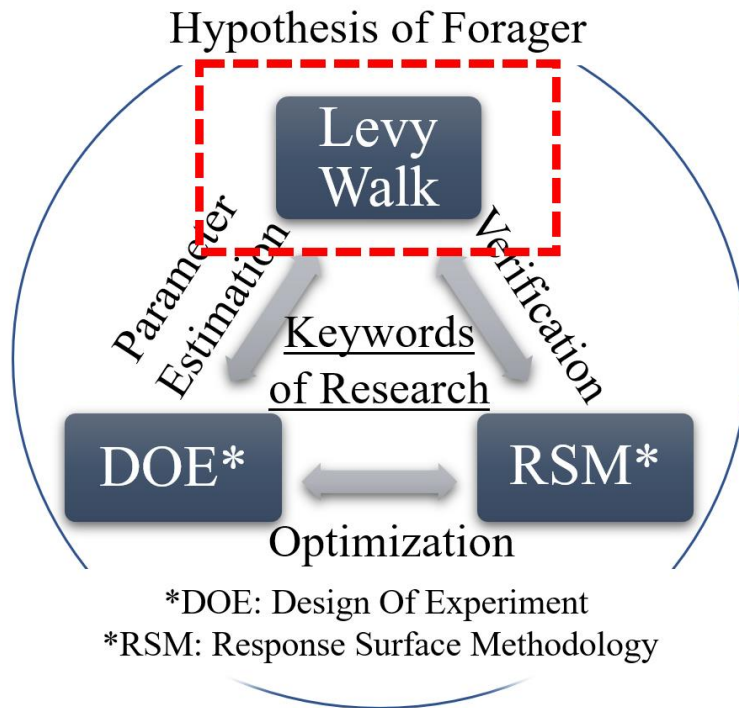
Figure 2-1Hypothesis of Forager

In a Lévy walk, step-lengths are selected from a Power-law distribution p(l) ~l $^\wedge$ μ, where l is the step-length, p(l) is the associated probability distribution, μ is a parameter, 1<μ≤3[2]. Random walks with step-lengths drawn from power-law distributions with μ>3 converge to Brownian motion as Gaussian random walks do (Figure 2-2). Lévy walks essentially represent a spectrum of random walks, with ballistic motion on one extreme (μ →1) and Brownian motion (μ ≥ 3) on the other.

Increasing μ decreases the mean-square displacement, and makes the walk "less super diffusive."

Some researchers use the name Lévy walk for the particular case μ=2, but for convenience in this thesis, we use this term to represent the entire family corresponding to $1 < \mu \leq 3$ [6]. Lévy walks differs from Lévy flights; in the former, searchers move along step-lengths at speed, while in the latter, searchers hop from the beginning of the step-length to the end[16].

Steps of a Lévy walk can be truncated if the searcher encounters a target. Many studies show that a wide variety of foraging organisms use μ=2 Lévy walks to search for food. These empirical observations, as well as the theoretical arguments that μ=2 Lévy walk is an optimally efficient stochastic search strategy, have led to the development of the Lévy Foraging Hypothesis, according to which Lévy walks are ubiquitous because they are an adaptive trait[4].

Ballistic	Levy	Brownian
$\mu \to 1^+$	$\mu = 2$	$\mu \geq 3$
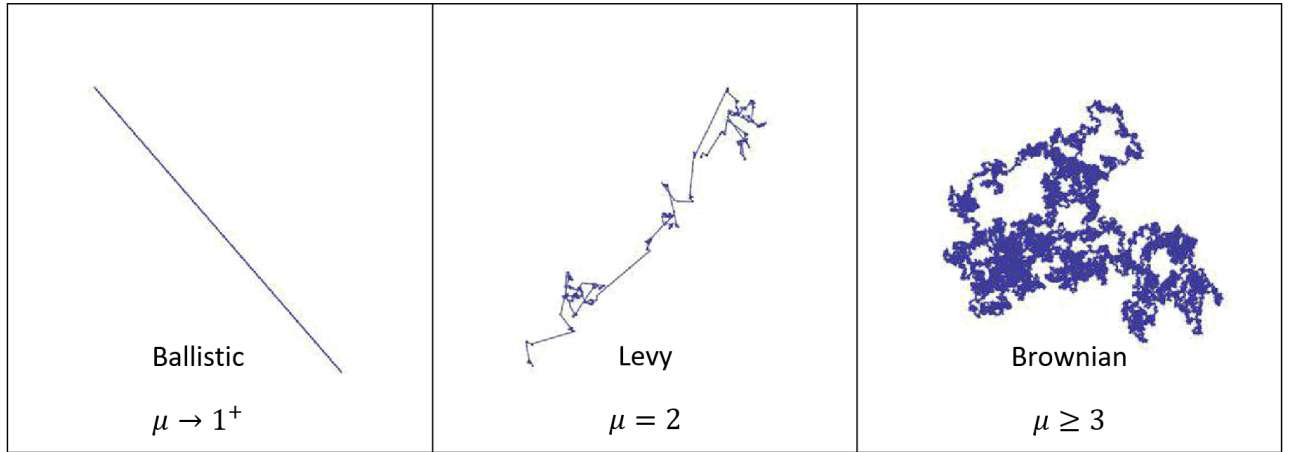
Figure 2-2 Hypothesis of Forager

Lévy flights are not to be confused with Lévy Walk. The robot moves continuously along each step length; Robot hops instantly from the start to the end of each step length[2]. Lévy Walks model cruise movement, while Lévy flights model salutatory movement. Most Lévy Walk models, including those considered in this study, are technically truncated Lévy Walks: step lengths are terminated when a resource diagnosis is reached, or when the maximum time of the simulation elapses. Fortunately, many of the essential features of Lévy Walks, including general properties of the mean-square displacement, are retained by truncated Lévy Walks[3, 5, 17].

Our model deals with stochastic movement robots. The correlated random walk provides another approach to modeling movement on the Brownian to ballistic

spectrum. The Lévy Walk and correlated random walk approaches are compatible and mathematically linked. We draw on Lévy Walks to develop our models[2].

2.1 Movement

Naturally, animals need to move to eat and they need to keep away from their predators, with their movement depending on various factors including climate, temperature, concentrations of other organisms in a local area (including humans)[3]. Although such factors may affect the sinuosity, velocity, or specific trajectory taken, they do not change the primary reasons underlying the movement: the biological necessity of interactions or "encounters" with other organisms. Given the ubiquity of moving organisms, some essential questions arise naturally[3]. For instance, as of now, the priorities order of driving factors motivating the animal movement is not yet well understood[18, 19]. It may be the case that such a movement is driven by the specific activity an organism performs at a given time[20, 21]. However, some new insights have been gained on how organisms move, that is, what patterns the trajectories follow. Another relevant question is, " What factor or factors determine(s) the shape and the statistical properties of such trajectories?" Knowing the answers to these questions, we are able to go beyond the phenomenological descriptions and contemplation about causation: As for a specific species of organism, one question may be posed as

follows: "why do the organisms move as they do?" that is, what advantages or benefits do a specific species reap from such behavior?

Lengths with power-law distribution and angle with uniform distribution reconstruct Lévy Walk behavior[22, 23].
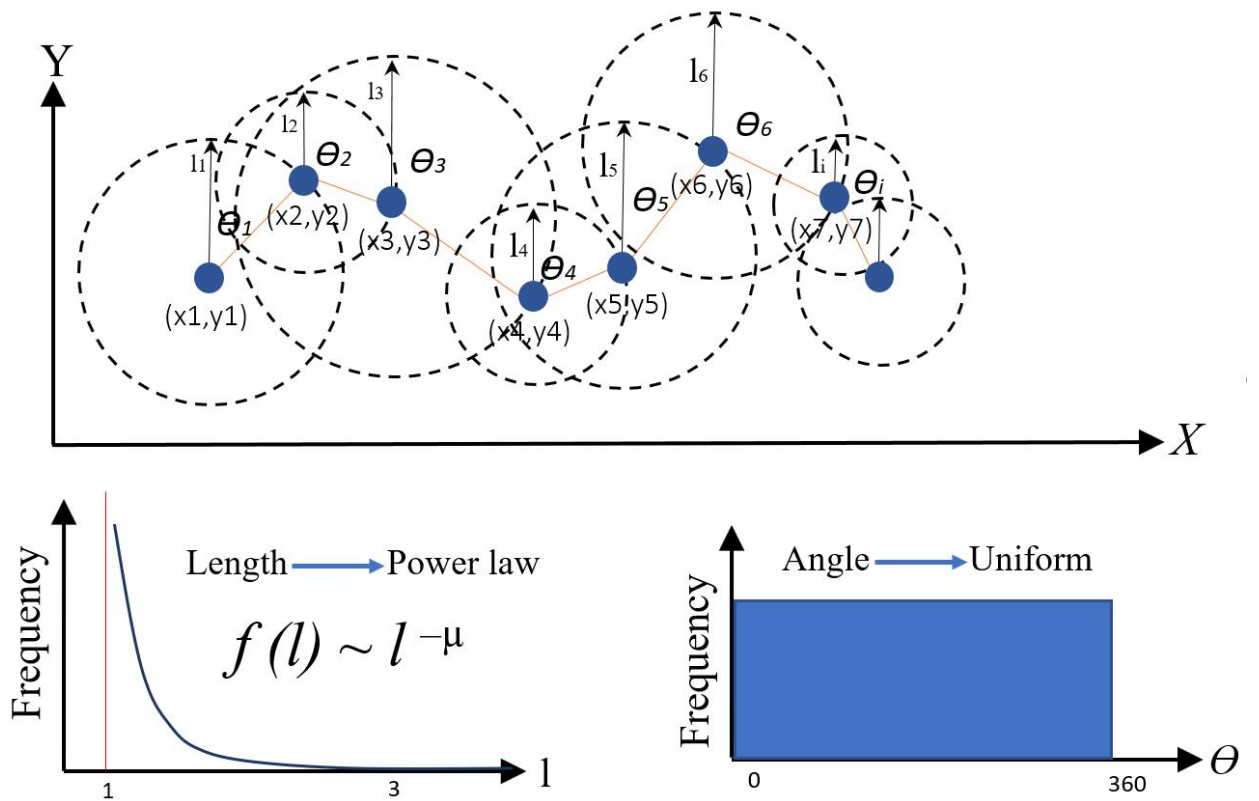
Figure 2-3 Reconstruction of Lévy Walk behavior with lengths and angle

Furthermore, another question can be asked as to "how did the specific biological mechanisms used for generating the behavior evolve?" These questions have led to studies on the new interdisciplinary subfield, which has come to be known as movement ecology[14, 24, 25]. Given that these questions have to do with such

research areas as random walk theory, stochastic processes, and anomalous diffusion, they have also been the focus of the attention of physicists[25-27].

2.2 Robot Movement

In the proposed model, a robot starts moving by choosing a heading and a step length, with the heading randomly selected based on a uniform distribution on [0,360]. The step length is selected from a Power-law distribution with parameter μ (for a non-composite robot). The method for simulating ballistic motion was an exception. In the case of non-ballistic motion, the selected heading and step length in combination, determine a random walk step. The robot moves along a random walk step at a speed of between 1 to 10 per time [2, 5]. The robot's speed determines how finely its movement is discretized, and 1 was the lowest speed for functional simulation. It takes a robot many time steps to complete a typical random walk step[5]. When the robot comes up with a resource while moving along a random walk step, it first truncates the random walk step, moves to the resource, and consumes the resource[2, 6]. Consumed pollutions are not replaced; hence, our simulations represent a destructive robot (resource pollution depletion). If a robot reaches a landscape boundary before completing a random walk step, it truncates the random walk [2, 4]. Ending a random walk step (whether truncated or not), the robot randomly selects another heading and step length, and the same procedure is repeated[4]. Here, Power-law distributions are not used by

simulations of ballistic motion ($\mu \rightarrow 1$) to generate step lengths. A robot using ballistic motion selects a heading and moves in that direction until it encounters a resource or landscape boundary[28-30].
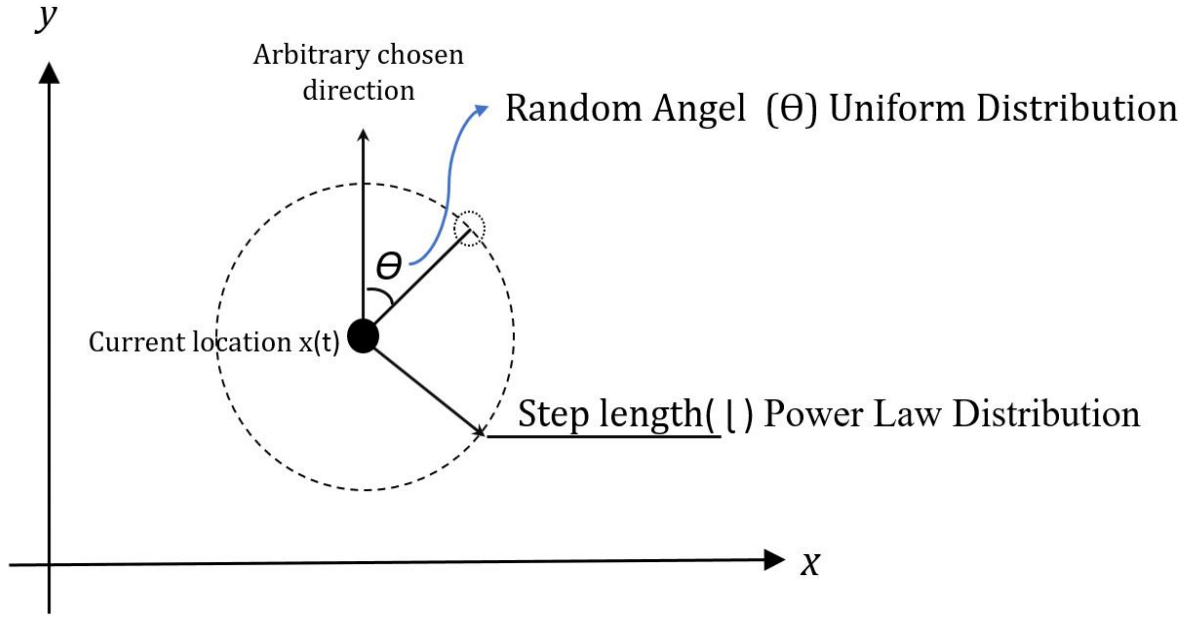
Figure 2-4 Robot movement

The robot moves at cruise speed but changes parameter for another test[30]. When a resource falls within its detection radius, the robot moves in a straight line to the resource and detects and saves it; otherwise, the robot performs a random search strategy[30, 31]. Random search strategies are comprised of a set of probabilistic movement rules. Although the resulting movement patterns are stochastic, the probability distributions that yield the movement offers a search structure. Like many theoretical studies conducted on optimal random search behavior, the

proposed model in this study is very general, with parameters not being specific to any particular species. The distance and time units in our simulation set the distance and time scales of the system[31]. These units could be quantified in terms of meters and seconds to represent a specific system. Our simulations use a square landscape of 100 units in length and width, and robots have a detection radius of between 1 to 10 [31].

2.3 Landscape Characteristics

The source was distributed across landscapes. We selected this clan of point processes because it allowed us to adjust both the intensity and aggregation of the distributions. The source distributions were specified by two parameters: the radius of the clusters of sources and the total initial number of resources. We used 100 out of 1000 as our premier resource levels, and cluster radius of 4 out of 64[2]. The algorithm started with the DOE method by drawing the number of source aggregations, or clusters from a Poisson distribution with an expected value of 15 (**Table 1**). This was followed by randomly assigning a point in the landscape to the center of each cluster (i.e., parent point). Then sources were sequentially assigned to a random parent and randomly placed within a specified radius (i.e., cluster radius) of the parent point. This continued until all resources were distributed among the parents[2]. Therefore, in each simulation, the algorithm randomly determined the number of clusters and the number of resources per cluster, though

the premier total resource density and the cluster radius were fixed. Changing a single parameter (i.e., cluster radius), we could vary the degree of aggregation of resources, ranging from tightly clumped (cluster radius = 4) to dispersed (cluster radius = 64)[2, 5].

According to a common misunderstanding, the negative binomial distribution is the best tool for modeling clusters, describing the probability of finding a specific number of points within a sample area; yet, it DOEs not directly identify the positions of points. No fixed spatial point process yields a negative binomial distribution of points in all possible sample areas. The selection of boundary conditions for the landscape was aimed at minimizing the effect of boundary artifacts [2]. The aim was to ensure that no resources were too close to the landscape boundary (which would protect them from approaching from one or more sides). Reaching a boundary, the forager was relocated to a random position in the landscape, and it starts over its search (starting by drawing a new step length). The schematic of the environment the cluster Radius should be larger than the detection radius[2, 16, 31].

Table 1 Independent variables and their coded and actual values

Parameter for Robot		
Robot Number	1 to 10	Robots to the landscape preliminary tests indicate that, for example, one forager searching for 1000-time steps is the same as ten robots searching for 100-time steps.
Source	100 to 1000	Type of resource parent or offspring; used in distributing resources according to the Neyman-Scott process (constrained by the total number of resources).
Cluster Number	1 to 20	The max extent of the area occupied by parent resources.
Radius of Cluster	4 to 64	Creates a local variable with the distance from the forager to the nearest resource within its perceptual radius.
μ	$1 < \mu \leq 3$	Limit of μ to 1, the Power-law distribution approaches an infinite uniform distribution; In the limit of μ to 3, the Power-law distribution approaches a normal distribution.
Speed Robot	1 to 10	Speed is set to a value that is a fraction of the perceptual radius to ensure that the forager never steps over any resources (i.e., cruise forager) Perceptual radius where forager knows the exact location of the resource.
Radius for Search	1 to 10	Several of these variables are state variables for the forager but were treated as global because they are the same for all foragers and static throughout the simulation (should be changed for increased flexibility).

Recently, some algorithms have been developed for a probabilistic search for static and moving targets, including the approach (based on the foraging theory that hypothesizes optimality of the search by animals and mimicking such behavior)[2, 32].Also, the report presents a brief account of the history of mobile robots and multi-robot systems, stressing their essential properties and the problems associated with the search by mobile robot teams. It presents recently developed algorithms of universal search[33, 34].

The probabilistic algorithms of the search were developed by Prof. Irad Ben-Gal, and the methods of foraging and agent-based techniques have been studied[16].

The problem of search for a hidden object, chasing prey and catching a target is one of the oldest mathematical problems. It requires knowing how best to search for an object when the amount of searching effort is limited, and only probabilities of the object's possible positions are given[34].

A general overview of the main existing methods of probabilistic search for static and moving targets yields recently developed algorithms of search by autonomous mobile agents. The algorithms implement a probabilistic version of local search with estimated global distances, resulting in the agents' paths over a domain. It requires developing autonomous mobile agents, which demonstrate the same behavior. The report overviews the main algorithms and models of search applied

in the foraging theory and presents some recently developed methods of control of autonomous mobile agents, which follow the ants' foraging activity. The presented probabilistic algorithms of the search were developed. The methods of foraging and agent-based techniques are studied[16].

Lévy Walk movement pattern as a type of search strategies has caught the attention of ecologists who are eager to identify universal properties of optimal foraging behavior. The robot contribution to this field is discussed. First, a way is proposed to extend the Lévy Walk used for robotics Value Theorem to the spatially explicit framework of stochastic search strategies[29, 30, 35]. Next, simulations are described, with a focus on comparing the efficiencies of the design of robotics sensor and speed search strategies. Different parameters are used in making robotics. Finally, the design of the experiment is analyzed to identify the factors that contribute to foraging[16].Experimentation plays an essential role in the industry, robotics, engineering, and science.

2.4 Biological Encounters as Reaction-Diffusion Process

Based on the research findings, biological encounters naturally include two main components: diffusive, transport, and reactive, i.e., interaction, such as eating or mating. Therefore, they serve as a particular case of reaction-diffusion processes. Normally, the diffusion processes are linear in that the probability density functions of the random walkers follow the superposition principle[35].

The superposition principle guarantees that the probability of finding one of many random walkers at a specified position will equal the sum of the probabilities of finding each of them individually at that position[17, 28]. In more technical terms, the superposition principle guarantees the existence of random walk propagators[3]. However, should the superposition principle hold, the random walkers must avoid engaging in interaction with each other since such interactions will usually result in nonlinear effects[28]. Noninteracting random walkers should constantly follow linear Fokker-Planck equations associated with the probability density function for the walkers[4, 28]. Research findings show that such a one-dimensional approach to diffusion is of great use. For example, the study carried out by Sparrevohn et al. has found that thousands of fish released at a single point diffuse as random walkers given the movement of the water (i.e., advection). In contrast, the reaction process necessarily involves one "particle" interacting with another, resulting in the emergence of nonlinear phenomena[4]. Take, for instance, the "reactions" represented by a predator intent on eating its prey. Though two meals of prey are likely, in principle, to be approximately twice as beneficial as a single meal, 100 meals do not necessarily mean that it is 100 times more beneficial[21, 36]. Therefore, it follows that the reactions between predator and prey inherently deviate from linear behavior. Eating, mating, and pollination are distinct reactions[26]. By large, such biological interactions are divided into two main general categories. The first category includes interspecific interactions,

typically, a trophic interaction between a consumer and consumable, which can take on the form of predation, parasites fiction, or mutual rewarding (e.g., flowers and pollinators)[2]. The second category represents the interactions between members of the same species, that is, mating or territorial competition. Therefore, one can use two-species reaction-diffusion models, i.e. those with two reacting species to describe various ecological systems[3]. Most importantly, the diffusion, i.e., movement emerging out of such diverse reactions remains the same, at least in a first approximation[26]. To be more specific, the randomness in the movements is not expected to rely strongly on the organism's foraging for food or searching for a mate (for something else) as long as relevant search cues including the density of organisms are comparable[2]. Being valid, this premise justifies the examination of the diffusive properties of biological encounter processes regardless of the nature of the reactive processes[16]. This study focuses largely on the "not encounter" rates between organisms, i.e., the diffusive aspects of the underlying reaction-diffusion process is only considered. Such an approach can be tailored to consider new kinds of behaviors as search for food may not necessarily be dominant. Avoiding predators may also be important[15, 20]. A predating organism may benefit from increased encounters with its prey, while simultaneously is net benefitting from lower encounter rates with its predators. It is claimed that conditioning encounter rates between organisms have an important role in the ecological constraints, contributing greatly to the life evolution[27]. Multiple

potential factors, as well as many ecological adaptive pathways, are involved in such interactions. The importance of movement is indisputable. For instance, there is coordination between locomotion and its detection. Therefore, it is hypothesized that the sudden spike in spatial complex, as well as the patchiness of the marine odor landscape during the Ediacaran-Cambrian interval about five hundred million years ago, resulted in the gradual evolution of external bilateral sensory organs (e.g., nose and ears). Foraging and search strategies are considered as one of the crucial factors influencing encounter rates[15, 21]. Consequently, a question can be posed as to whether they might have contributed to the evolution of the sensory apparatus indirectly. In this study, encounter rates are examined in a framework that makes a distinction between two types of interacting organisms[22, 23]. The organism is categorized either as a searcher, e.g., forager, predator, parasite, pollinator, or the actual gender in the search activity engaged the mating process, or it is a target, e.g., prey, food, or the passive gender in the mating activity[14]. Statistical models of foraging do not need to take into consideration the "microscopic" details of the process they are essentially irrelevant to the averages. recognizing the limitations and applicability of such models is important[3]. Despite this "coarse-graining" perspective, these models yield statistically robust results since they do not rely on a specific type of biological implementation of the search mechanisms[24, 25]. They have a long tradition in statistical physics in which simple models lead to a remarkably good agreement with experiment (e.g.,

the Is in model ferromagnetic phase transitions). The framework chosen in this study makes it possible to have considerable variation, easily generalizing to new cases. For example, the search can be guided almost completely via external cues, using either the searcher's cognitive (memory) skill or its detection (olfaction, vision, etc.) skill[2]. Alternatively, the searches might not be oriented, hence effectively stochastic processes. Even when the actual process is thoroughly deterministic, a statistical approach can be of great use, or perhaps even necessary when the environment is considered as a disordered medium[5]. Deterministic walks (e.g., the traveling seller problem and the traveling tourist problem) in the context of random environments can be clear made distinct from (genuinely stochastic) random walk[23, 37].

2.5   Group Testing

Two classic versions of the problem in the form of a search for a hidden object were developed during World War II. The first one was formulated in 1942 as a problem of search for all fault units in a given pool. Initially, it required finding an optimal procedure for testing blood samples for the presence of an antigen[14]. A set of units were tested simultaneously, and if the test indicated a presence of antigen, then the set was partitioned into subsets, and each subset was tested separately. The procedure of partitioning and testing continued up to finding a unit or units with the antigen. Sterrett (1957) extended the Dorfman procedure to the

search with multiple targets, with the number of targets being unknown. Later, the online procedure of multiple-target search for a known number of targets was suggested by Hwang[14]. A group-testing approach addresses mainly the problem of statistical decisions, which include the selection of the best action under uncertainty conditions. It involves certain payoffs and a determination of the size of the test samples concerning the results of the previous tests. An implementation of this approach to the search problem results in the following procedure. The searcher acts on a set of possible locations of the target[14, 37]. At each step, the searcher chooses a subset of the locations and checks whether the target is somewhere in this subset or not. The procedure continues recursively on the subsets where the target is detected. The search terminates when the searcher detects the target in a single-point set. In this procedure, the main problem is concerned with the determination of the size and the location of the subsets, based on a given constant or varying detection function[24, 37]. An optimal solution to this problem with perfect detection was developed by Zimmerman in 1959. Later, it was found that the Zimmerman procedure is equivalent to the Huffman's optimal coding procedure (Huffman, 1952), and the length of the testing procedure up to the identification of the faulty unit is analogous to the length of the binary code. Abrahams (1994) generalized this procedure to the search by multiple searchers. In 2005, this procedure was distributed on the group-testing search with coalitional and no coalitional decision making, and an online algorithm of the search was

suggested; A detailed description of this model and an overview of the other group testing search algorithms are presented by Kagan and Ben-Gal (2013b).

2.6   Search and Screening

This problem was named by Koopman as "search and screening problem" and was widely accepted. Nowadays, this problem is integrated into the theory of search and screening, which according to Frost and Stone is the study of how to employ limited resources most effectively while trying to find a target whose location is not precisely known. The goal is to use the search assets, intending to maximize the probability of locating the search object given the resources available[25, 26]. Sometimes this target is stated in terms of minimizing the time to find the search object.

It is assumed that the searcher acts under uncertain conditions, accumulating information about the target location during the search. The amount of available information is specified by a detection function, which defines the probability of detecting the target given the search efforts made. The most popular detection function is a Koopman function that has an exponential form related to search efforts and is concave in time. Originally, the theory of search dealt with offline search planning, and the solution of the problem was specified in the form of the optimal distribution of search efforts[26, 38]. This solution assumes that a group of search agents conducts the search, and an overall search effort is large enough as it

starts from the initial task. The primary cause of the search planning problem for the static target was solved for different distributions of the target, using the Koopman detection function[27, 33]. Detailed consideration of analytical results and algorithms was published in 1975 by Stone (1975) and then in 1992 by Iida (1992). Recent results obtained in the theory and military applications were presented in the reports by Frost and Stone (2001) and by Cooper, Frost, and Robe (2003), and by Washburn and Kress (2009). Stone, in particular, presented the algorithm of building an optimal search plan for the search in discrete space and time. Drawing on the Stone's algorithm and using Koopman detection function, Brown (1980), Washburn (1980, 1983), and Eagle (1984) developed algorithms of optimal search planning for a Markovian target moving in the discrete domain at the beginning of the 1980s. Recently, Singh and Krishnamurthy (2003) generalized this approach and reported about the algorithm, which is applicable both for non-Koopman detection function as well as for a search planning in the case of an infinite horizon[33, 39].

In parallel to the search for a moving target in discrete space and time, the search planning problem was considered in continuous time and space. In particular, Hellman (1972) formulated a general equation of the target's movement and found necessary conditions for the search optimality given a given finite period of search. Lukka (1977) mitigated this problem by drawing on certain assumptions regarding

the motion abilities of the searcher and the target. Using these assumptions, he derived necessary conditions for the optimality of search paths. Later, in 1981, more general models were studied by Mangel (1981). A detailed description of the models and results in search planning in continuous space and time was published in 1985; An introductory presentation with examples of applications was presented by Washburn (1989). Later, Ohsumi (1991) performed a search for a target moving according to a diffusion process and found optimal search paths, using smoothness and concavity of the Koopman detection function. The optimal paths provide a maximum probability of detecting the target in a finite fixed period for some individual cases of search[39].

The ideas of informational group-testing search and the search in the nonfixed period with different termination time were studied, resulting in the development of heuristic near-optimal algorithms. In particular, Kagan (2010), Goren, and Ben-gal (2010) suggested an online algorithm of the search for static and moving targets in discrete time and space. The algorithm required perfect detection function without Koopman function. Two years later, this algorithm was modified for the search with imperfect detection, including the application of the Koopman detection function. In the same year, Israel, Khmelnitsky, and Kagan (2012) applied a discrete variant of the ohsumi model for the search over terrain with shadowing and suggested an online search algorithm for a static target and the

target governed by not necessarily fair diffusion process. All these algorithms act online and yield a near-optimal path of the searcher[34, 39].

2.7 Foraging Robotics

Foraging robotics is a benchmark problem, especially for the multi-robot system. It is the primary benchmark problem for several reasons[24]:

1. Complex foraging involved in many social animal and fish and insects provides both inspiration and system-level models artificial systems, provided the processes are well understood.

2. Forager robots perform a collection task involving the coordination of each of multiple problematic tasks, including useful detection and identification (searching) of target or food. Physical collection (harvesting) of target or food almost certainly requires physical manipulation, transporting the food or target, homing or navigating while carrying the prey or food back to a home site and saving the food item at home before returning to the forager.

3. Foraging movement requires cooperation between individuals involving either communication to signal to others where food or prey may be found (e.g., pheromone trails or direction, giving) or cooperative transport of food items to home and saving items for a single individual to transport[24].

Some types of foraging robots have been successfully used in practical applications. Most foraging robots are found in research laboratories and simulation method in a computer. If these robots are intended for practical applications, they are at the stage of proof of concept or prototype. Forager robotics is a complex field which requires a range of competencies tightly integrated within the practical robotics[31, 37]. Although the principles of robotics forager are now becoming recognized, many of the sub-system technologies necessary for forager robots remain very challenging. In particular, situational awareness and sense; energy and power actuation, locomotion, autonomy; and safe navigation in unknown environments and proof of dependability and safety all remain difficult problems in robotics[24].

Therefore, it is important to describe and define the principles of the robotic forager. The majority of samples will necessarily be laboratory samples and simulations by computer systems, which are not aimed at solving real-world applications. They are designed to be used in the simulation model, illuminating and demonstrating those principles[37]. Then, it is necessary to develop a classification of robotic forager, encompassing important design features.Such a classification is a requirement for any forager robotic, whether operating singly or in a multi-robot team, and some technologies are currently available to implement those features; single robot foraging, including commercially available

robotics[37]. Elaborates on the recent developments in multi-robot (collective) foraging; strategies employed for cooperation, such as cooperative transport, information sharing, and labor division (task allocation), approaches to the mathematical modeling of multi-robot foraging[37].In forager robotics, self-determination conventionally discusses the degree to which a robot can make its own decisions based on which next actions will take place. Thus, a full autonomous forger robot would be capable of carrying out its entire mission or purpose without human control or intervention. Semiautonomous forger robot would have a degree of autonomy but needs some human supervision[37, 40].

Behavior-based control describes a class of forger robot control systems characterized by a set of conceptually independent task achieving modules, or behaviors. All tasks achieving modules can access the robot's sensors, and when a particular module becomes active, it can temporarily take control of the robot's actuators. Braitenberg vehicle: In robotics, a Braitenberg vehicle is a theoretical autonomous robot in which easy sensors are connected directly to wheels. Therefore if, for instance, a front right side sensor is connected to the left side drive wheels and vice-versa; if the sensors are light-sensitive, the robot will automatically steer towards a light source[37].
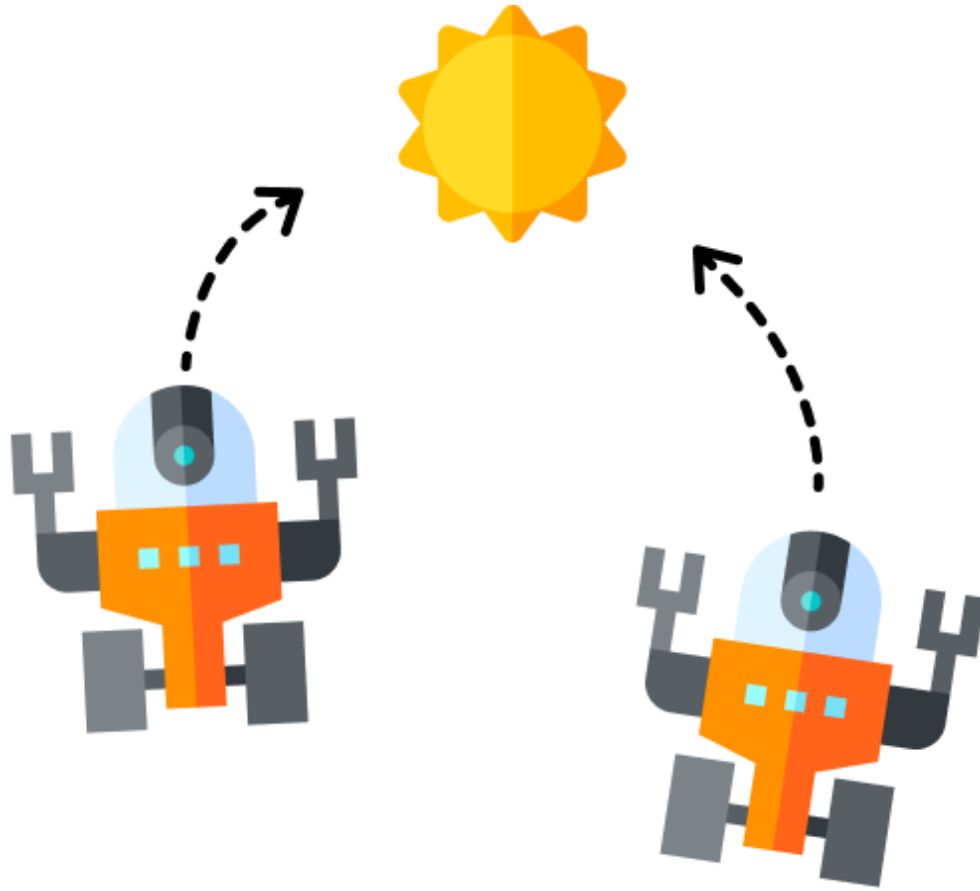
Figure 2-5 Forager Robot

Each state represents a particular set of behaviors or actions. The forger robot can be in only one of these statements at any given instant in time and transitions between states may be caused by either internal or external events[20, 37].

Odometry refers to the technique of self-localization in which a robot measures how far it has traveled by, for instance, counting the revolution of its wheel. One of the problems with Odometry is that since omnidirectional wheel leads to full errors, odometric localization estimates are generally false and of limited value unless combined with other localization techniques. In turn, mobile robot and robot

are used interchangeably. A mobile robot is a vehicle or human-made device capable of sensing its environment and purposefully moving through and acting within that environment. The robot may be teleoperated, semi-autonomous, or fully autonomous[20, 24].

2.8 Proposed Method

Despite different techniques and methods, the above approaches to the search problem are led by the common idea, namely, to define a behavior of the robots to develop them further or specify their activity. In contrast, foraging theory addresses the process of search from the opposite point of view. It starts, observing search activity of the living Organisms followed by a concentration on formal modeling of their behavior. Moreover, as indicated by Pyke in his critical review of the theory (Pyke, 1984), proponents of optimal foraging theory seek to predict the behavior of animals while they are foraging[20].

In general, the foraging theory deals with two different problems:

1- a problem of search for prey or food.

2- a problem of deciding to hunt or not to hunt the found prey. This discourse is restricted by considering the search activity.

The first attempts to informally consider foraging behavior were made at the end of the 1950s. These studies were published in 1966 in the papers by MacArthur and Pianka (1966) and by Emlen (1966). They ushered in the main directions of further studies in foraging theory. In particular, regarding the specification of a forager behavior, MacArthur and Pianka (1966) suggested taking into account its movement in a patchy environment using specific optimization techniques.

In his famous paper, Charnov (1976) formulated a model of optimal foraging by patches and derived a condition under certain assumptions regarding predator's behavior and energy depot. This condition governs whether searcher has to stay in the current patch or leave it for search in the other patch. This result is widely known as marginal value theorem and forms a basis for classical optimal foraging theory[20]. In 1977, Oaten applied the Charnov approach to the foraging in a stochastic environment while Green (1980) suggested a simple model of such foraging, elaborating on its application. During the subsequent years, similar optimization techniques were applied to the analysis of foraging processes in different conditions; the resulting models were developed, for example, in the papers by McNamara (1982), by Stephens and Charnov (1982), and by Mangel and Clark (1986). A summary of the methods and results obtained during these 20 years of the development of optimal foraging theory was presented by Stephens and Krebs in their book (Stephens & Krebs, 1986). A detailed contemporary

review of the mathematical methods and optimization techniques used in classical optimal foraging theory was given in a book by Pirolli (2007).

The optimal foraging theory has been mainly formulated from the biological perspective. The models mentioned above are based on several assumptions regarding the foraging process itself as well as a link between evolution and foraging. Pyke lists a summary of these assumptions in his already mentioned critical review (Pyke, 1984). Analysis of these assumptions and a historical overview of the theory appear in the first section of the recent book (Stephens et al., 2007). In parallel to the studies explicitly dealing with the foraging behavior, several models of animals' movement were suggested based on scientific random walk processes[20]. Probably, the first results regarding trajectories were reported by Wilkinson (1952) who investigated the possibility of a random search in the birds' wandering. Following the Wilkinson results and based on the work published in 1951 by Skellam (1951), Patlak (1953) developed mathematical techniques for modeling animals' migration via Brownian random walks.

These models formed a basis for a new perspective on foraging, using a methodology different from the optimization techniques, which are used in classical foraging theory[20, 22]. In particular, Hoffman (1983) studied the optimality of Brownian search or foraging via a Brownian random walk, based on search theory and stochastic processes. Five years later, Bovet and Benhamou

(1988) applied a correlated Brownian motion for modeling of foraging in the stochastic environment, indicating a good correspondence between the modeled trajectories and the observed trajectories of foraging ants. Details of such models and underlying theories were published in the book by Turchin (1998). A group led by Viswanathan suggested another approach to the studies of forager motion[31, 33]. A study conducted in 1996 found that the trajectories of albatrosses are better described by Lévy flights rather than by Brownian walks. This finding initiated intensive research on Lévy flights, and accordingly, Lévy walks applied to animal motion as well as the modeling of individual trajectories of the foragers. In 1999, the same research group considered optimality of search via Lévy flights, and then this behavior was studied in a broad context of foraging activity in particular, in comparison with the Brownian walks' search[31]. A review of the results in this regard up to the recent time has been presented (Viswanathan, da Luz, Raposo& Stanley, 2011). Despite the successful application of Lévy flights and Lévy walks to the models of animals foraging, during the last years, there have been several studies which do not meet the results provided by these models[16, 30]. A summary of the main critics of Lévy flights models has been presented in a series of papers published by Plank and colleagues (Codling, Plank, & Benhamou, 2008; James, Plank, & Edwards, 2011). Such inconsistencies gave rise to a reconsideration of biased Lévy walks (Marthaler, Bertozzi, & Schwartz, 2004) and Lévy flights in a random environment. In the same vein, another alternative to the

Lévy models was put forth by Bénichou and colleagues in 2005–2007. This approach deals with the alternate search strategies, which combine the strategies of optimal search and screening with the strategies specified by optimal foraging by patches. The resulting walks consist of the movements with low and high velocities and can model the motion in different environments[29].

2.9   Search and Forager

The studies of living organisms inspire foraging drawn from the pioneering ideas of von Neumann and Wiener, the progress made in the development of computers and intelligent machines. Logical schemes, perceptron, neural networks, and storage modification machines and fields of cybernetics follow biology. What is a living organism? How can we recognize intelligent behavior?

That a system is intelligent implies that we cannot produce sufficient evidence for determining its behavior in certain "problem solving" situations. Note how many computers drop in I.Q. We must know in full deterministic detail what we are doing to build a complex machine; Should a machine be called an intelligent machine, it requires that we forget or ignore our knowledge of just how it DOEs what it DOEs[29, 35].

Probably, an intuitive awareness of this problem was a driving factor in turning to the mathematical description of morphogenesis, a method which is entirely

deterministic and allows for complex, unpredictable behavior. At the time, this crucial paper was considered as a kind of nonsense. However, the Turing system became one of the basic models in the studies of self-organization and nonlinear dynamics, determining the essence of mathematical biology research. Being able to observe a working brain, we can derive various characteristics, indicating its functionality[17]. However, we cannot directly observe the working mind or human intelligence. All we can do is to build a mathematical model and hopefully implement it on a device which demonstrates an activity similar to that of intelligence. Even if such a device passes the Turing test and if we consider the activity of living organisms to be different from humans, we even do not know what we are going to formalize and what the test is. Introducing the Tsetlin works in mathematical biology, Israel Gelfand (1969) wrote as follows: What should the degree of formalization be in biology in a study on living systems? Given quantum mechanics, one can distinguish two stages in its formation. At the time, the formulas did not make sense yet, and even if they did, they were not entirely as they should be[17, 28]. They were sometimes utterly wrong. The second stage was a period of quantum mechanics, and rapid growth became an exact branch of physics with a vast number of precise formulas. However, this stage was possible only after the first stage had taken place. By comparison, in biology, the first stage has not occurred yet[4, 36].

Unfortunately, this opinion dating back to more than 40 years ago is still considered correct. We have various mathematical models of different activities of humans and animals, but we still cannot present a testable device that implements the Kurzweil optimistic predictions, or, at least, can be compared with the living organism in its most fundamental activity. The theory of foraging provides a fortunate exception[2, 6].

The theory of foraging addresses the behavior of individual animals and their swarms while seeking for food. We do not understand whether their behavior is optimal from an abstract mathematical point of view, but like any natural behavior, it is certainly optimal from an evolutionary point of view. Viswanathan and his colleagues initiated the mathematical modeling of the forager trajectories, using Lévy flights. Commenting on the findings of this group, Mark Buchanan noted as follows: They show to be on the track of a new domain of ecology, demonstrating that this way of moving is, under some conditions, theoretically the best way for insect and animal to discover scarce prey[2, 5].

Probably it is the first model of the external directly observable behavior of living organisms that are built without any specific knowledge about the internal activity of its brain and intelligence. There is a highly developed theory of search and screening, which was initiated in 1942 in response to the German submarine threat in the Atlantic. How best to search for an object when the amount of searching

effort is limited, and when probabilities of the object's possible positions are only given. Here the search robots are equipped with artificial intelligence, so their abilities and structure are known, and internal activity can be planned and programmed. In the case of search in the stable unvarying environment, optimal search plans can be obtained, using standard optimization techniques. However, if the environment is changing during the search, then a globally optimal solution cannot be found, and the search plans have to be optimally corrected online for the task that is above the abilities of modern computers, but living foragers naturally solve that[6].

Hence, if we can build and program artificial search robots in such a way that they will demonstrate the same behavior as that predicted by the foraging theory for living organisms, we will achieve two goals. For cybernetics, we will obtain the techniques suitable for the best online search planning in varying environments. For biology, we will get reasonable insights regarding the internal activity of living organisms performing foraging tasks[2, 4].

Foraging and search and screening theories are considered in the same mathematical and algorithmic framework. The following section overviews the main ideas and methods of foraging and search theories; considers Lévy flight models of individual foraging and corresponding diffusion models and algorithms of search and foraging in the random environment both by single and by multiple

robots. The results of laboratory verifications and the active Brownian motion model for swarm dynamics with corresponding Fokker-Planck equations are also presented[28].

2.10 Robot Foraging

Statistical models of random searches do not assume any particular implementation of searchers and targets. The searcher is usually taken to be a biological organism or, in the case of DNA searches, a natural enzyme or macromolecule and biological, but the searcher could also be robotic. Because the behavior of random searches is independent of implementation details, successful robotic searches closely resemble natural and biological searches. Robot foraging and evolutionary robotics, in particular, is an expanding field of scientific research. Although robot behavior has traditionally been studied via the microscopic analysis of systems composed of a single or only multi-robot, more recently swarms of robots have been studied.In contrast, macroscopic robot analysis focuses on averaged quantities[3, 28]. In a study, a model of robot foraging was analyzed, with results showing that successful robots forage like Lévy walk foraging. Another attractive phenomenon is micro-movements. A movement is a quick, simultaneous movement of both eyes that occurs when, e.g., the viewer wants to remain focused on a single spot (visual fixation) or to engage in a rapid eye movement. Micro-movements are

involuntary, smaller versions of fixation movements and their role has been a topic of much debate[18].

Foraging by robotics and animals involves the movement of searching mainly for collecting or capturing food for consumption or storage. Robot foraging is a broader definition of searching for and collecting any objects, then returning it to the point of objects collection. Of course, when the robot forger engages in searching and discovering to reach energy resources, both robot foraging and animal/ human foraging will have the same meaning. Based on cooperative, mobile robotics, "In foraging, a group of robots must pick up objects scattered in the environment[3].

2.11 Swarm Robotics

The term "swarm robotics intelligence" describes the purposeful collective behaviors in nature found mainly in social animals, fish, and insects. Swarm intelligence is the discovery of those collective behaviors, in both artificial and natural systems of multiple robots as well as how they emerge from the local action and interaction of the robots with each other and their environment.

'Search' and 'search problem' arises in some problems in many fields of applied mathematics and robotics[7, 19]. Explain the meaning or the notion of a search problem which corresponds to the class of situations in which an agent or robot is

looking for a target by screening a certain defined unknown environment. Search, and movement problem has been formulated under various restrictions with respect to search implementation and the target and the robot functionalities. To illustrate the potential complexity that might be considered in a search problem, let us start with a simple search example and gradually add to it various assumptions and conditions. The figure below presents a simple schematic view of a search problem where the target (a robot) is located at some point in a given environment, and the robot or robot's aircraft robot is looking for it. An initial classification of the problem depends on the definition, which can be either discrete or continuous. We mainly consider the former case, which implies that the target and the robot move to well-defined points in the environment. These discrete positions can also be modeled in a graph. This type of presentation is popular in the artificial intelligence (AI) literature where cost parameters or weights are often added to the edges of the graph so that the overall cost of the search is obtained by accumulating these costs along the search path over the edges[19].

If the weights are distributed unevenly, the search procedure can account for different considerations, such as non-homogeneous search distances or search efforts. We consider some of these cases second critical feature of the problem is related to the ability of the target to move in the environment[19]. In the case of a moving target, several versions exist for representing its type and path. Some of the

most popular schemes are random moves, Markovian moves, and Brownian moves. In general, optimal solutions exist for static search problems, but there is often no optimal solution for dynamic search problems with a moving target. We consider both approaches, and in particular, we propose a general search scheme that applies to both cases. A third feature of the search is related to the information available to the robot. If the location of the target is known, then a complete-information search problem can be mapped to a relatively more straightforward path planning or chase planning problem. These types of problems often appear in the literature on operations research. The origin of these deterministic search problems for a moving target was the pursuit problem formulated in the eighteenth century. This class of problem is computationally tractable and often focuses on capturing the target with a minimal number of search moves[8, 41].

We focus almost entirely on the robotics search, where the exact location of the target is generally an unknown environment to the robot. Note that there are several methodological concepts for addressing the incomplete-information search (e.g., rough-set theory, fuzzy logic, or on probability theory). However, response surface methodology is used in this thesis. We follow the probabilistic search approach and model the incomplete information on the target location using a function that quantifies the probability of the target to be located at any point in the environment. The search then becomes a probabilistic search and, in many cases an

adaptive one, where the results of the search up to a particular time are used to update the location probability distribution over the environment, often by using a Bayesian statistics approach as we do here. The problem is probabilistic not only in terms of the location of the target but also with respect to the distribution of the search efforts that are applied continuously by the robot to the search environment[41].
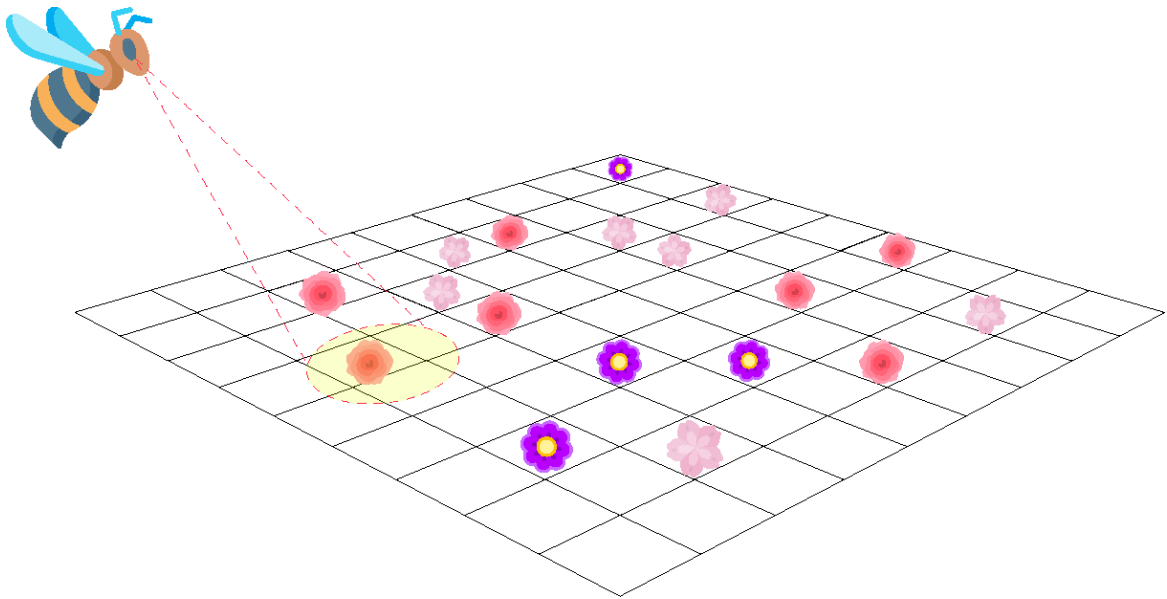
Figure 2-6 Animal forager

This approach is followed in this study, although we do not use the notion of distributed efforts. Instead, it is assumed that the search can be applied to discrete points in the search environment. An essential extension of the distributed search efforts in a discrete search environment is the group-testing search. In group testing, the robot can look for the target in a sub-environment of the search environment, obtaining an indication of whether the target is located somewhere in

this sub-environment. The allowed size of the sub-environment is treated as an input parameter. The search terminates if the sub-environment contains only a single point, thus representing complete information on the target location. We explicitly consider the methods of group testing. If the target is static, the search can be modeled by a coding theory process (where the code DOEs not represent the location of the target in the environment), while the coding procedures can be easily mapped to obtain the optimal search policy. These cases are often represented by decision trees that have become extremely popular in data-mining applications. In dynamic search, when the target is moving, such isomorphism between coding theory and search is no longer valid, so we propose a methodology that can also extend to these cases. There are several variants of the incomplete-information search[42]. We often assume that the target is unaware of the search robot. When this DOEs not hold, the search process turns in a search game and relies on some game theory concepts. We will shortly address these search games. Another conventional and realistic assumption is that the robot's observations are prone to some observation errors. In these cases, two types of statistical errors have to be considered – either missing the target even though the robot has searched the right point (a false harmful error), or falsely indicating that the target has found at a certain point (a false positive error). Of these two errors, the false-negative one is much more popular.

Another version of the incomplete-information search also addresses the situation of side information, where the robot obtains some (incomplete)indication during the search of the target location A natural extension to all of the above methods obtained when assuming that there is more than one target or robot in the search environment. In such a case, a question arises regarding the amount of cooperation among the targets or search robots. A simple example of such cooperation is an information sharing between the robots in order to better estimate the location probability distribution and to better utilize the joint search efforts[42].

We must stress the fact that the general formulation of the search problem as presented DOEs not distinguish between a search for existing physical objects. such as cell (mobile) phones, people, and devices or a search for abstract entities, such as records in a database. An e-commerce customer on the Internet, a targeted customer type, or a search for feasible solutions of a given problem within a predefined solution environment. Some favorite tools for such search procedures can be found in the data-mining and statistics literature. We draw clear lines of similarities between search procedures for physical entities and those found in problem-solving procedures, typically in stochastic local search methods that are used to obtain feasible solutions to a given schematic problem[42].

Even from the above simple example, it can be understood that the search problem in its general form can be very complicated, highly variant, calling for different

solution schemes, which are partially covered. It is worth noting that despite the similar properties of the variants mentioned above of the search problem, no formal and unified search theory captures all these points. Instead, one can find different search procedures and considerations in various research areas, such as operations research, coding theory, information theory, graph theory, computer science, data mining, machine learning, statistics, and AI. It is not to say that the proposed theory is a unified search theory. However, we try to bridge some of these gaps by formalizing the main properties, procedures, and assumptions that are related to many of these search problems and their variants[42, 43].

2.12 Foraging Model of Robot Foraging

Foraging robots are a type of mobile robots which can search and transport objects to one or more collection target. Foraging robots may be a single robot operating exclusively or multiple robots operating collectively. The Single foraging robot may be remotely teleoperated or semi-autonomous; multiple foraging robots are more likely to be fully autonomous systems. Robotics foraging is essential for several reasons:

It is a simile for a broad class of problems integrating exploration Navigation, and object identification, manipulation, and transport in multi-robot systems, foraging is a canonical problem for the study of robot forager cooperation. The actual real-world or many potential applications for robotics are instances of foraging robots,

for cleaning, instance harvesting, search for rescue robot, land-mine clearance or

planetary exploration[15, 20].
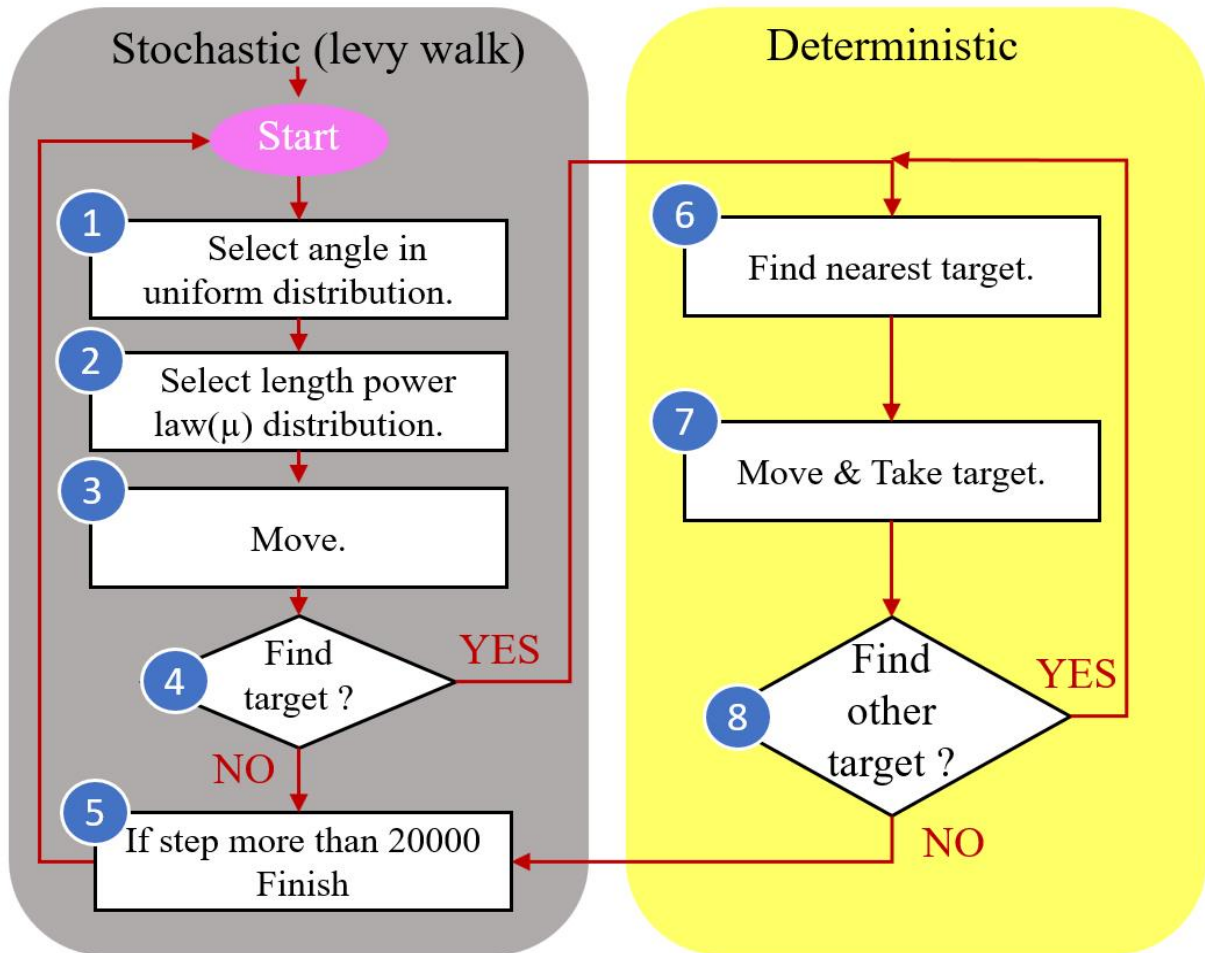
Figure 2-7 Algorithm of robot behavior
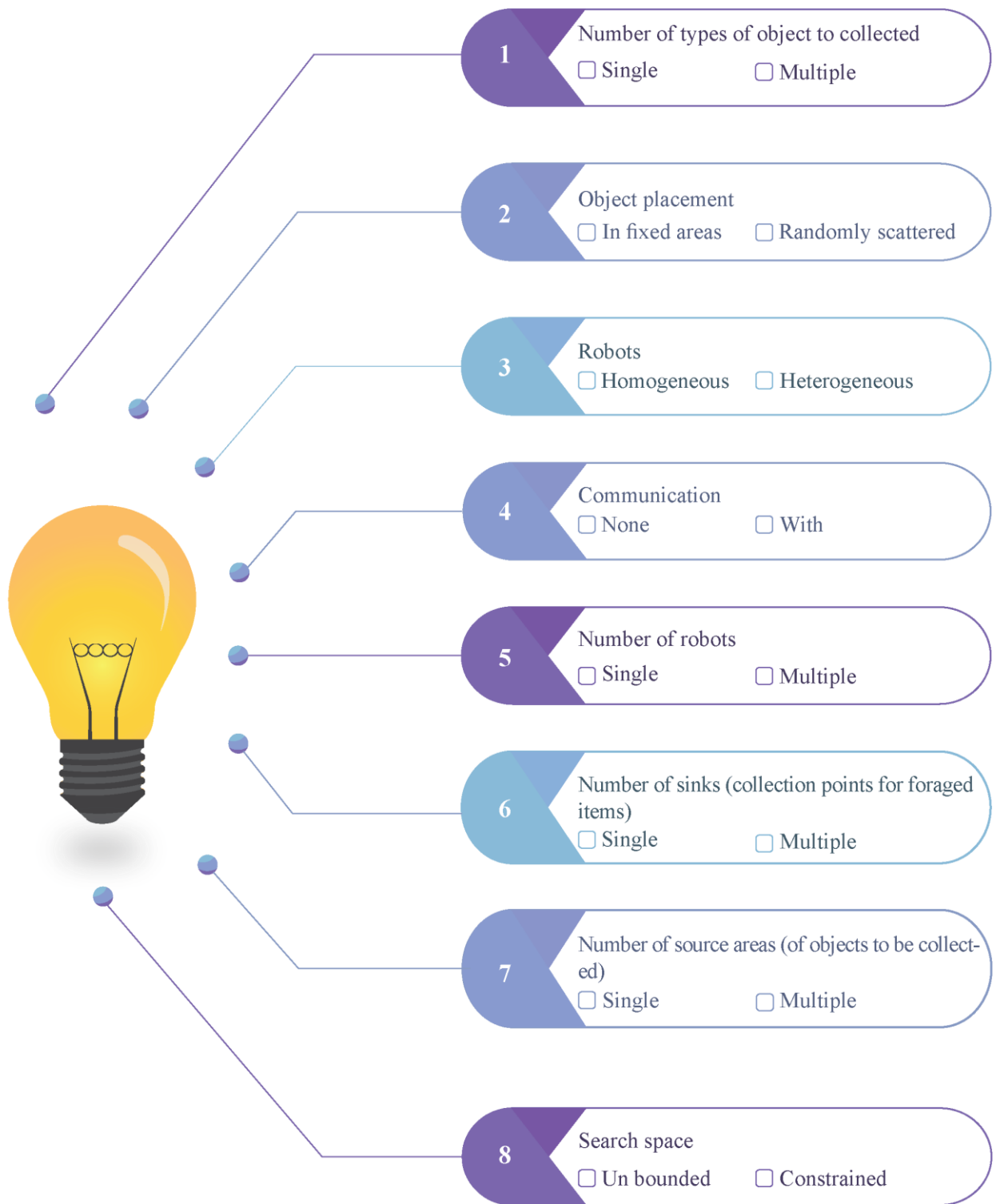
Figure 2-8. The condition of the all system.

2.13 Single Robot Foraging

A review of the literature shows that the design of any foraging robot, whether functioning alone or operating as a member of a multi-robot team, will necessarily follow a pattern[4]. The robot will need one or more sensors, with which it can both sense its environment for safe navigation and detect the food or objects items it is seeking. Actuators for locomotion through the environment and for physically collecting, holding and putting down its prey[37]. A control system is at the disposal of the robot, allowing for at least a set of basic reflex or movement behaviors. Since robots are machines that perform work and need energy, power management is of great importance. The robot is foraging for its energy. Then a balance should be made between its energy needs and the energy cost of foraging. Therefore, a complex set of interconnected subsystems is needed and although its system-level structure is likely to use a standard pattern, the shape, and the form of the robot will vary significantly based on its intended environment and application. We will present techniques for sensing, actuation, communications, and control, within the context of robot foraging. Moreover, given that the current research focuses on enhancing specific capabilities within each of these domains of interest, some examples of single robot foraging are presented, including real-world applications[20, 37].

2.13.1 Obstacle Avoidance and Path Planning:

There are many sensors to food and comprehensive search robot designers for robotics. Foraging robot will typically need short or medium range closeness sensors for obstacle avoidance, such as infrared return signal intensity or ultrasonic or laser or another crucial sensor time for systems[44]. The most versatile and widely used 2D or 3D laser scanning machine can provide the robot with a set of radial distance measure and hence allow the bot to plan a safe route through the obstacles. Localization, foraging robots require sensors for localization. They make it possible for the robot to estimate its position in the environment[20, 37]. The availability of external reference signals including fixed beacons through which a robot can use radio trilateration to fix its position relative to those beacons, or the capability of satellite navigation such as the Global Positioning System (GPS) paves the way for straightforward Localization. When there is no external infrastructure, then a robot will typically resort to multiple sensors including odometry, an inertial measurement unit (IMU) and a magnetic compass, often integrating the data from all of these sensors, such as laser scanning data, to reach an estimate of its position. As a well-known stochastic approach, Concurrent Mapping and Localization (SLAM) typically makes use of Kalman filters to allow a robots (or a team of robots) to both fix their position

relative to observed landmarks and at the same time to map those landmarks so that the confidence will increase as the robot(s) move(s) by the environment[32].

2.13.2   Object Detection

Vision is often one of the sensors required for object detection by foraging robots in laboratory experiments. If, for example, the object in question has a distinct color which makes it outstanding in the environment, then the robot can use standard image processing techniques to detect, then steering towards the object. However, when the environment is visually vague, unknown, or poorly illuminated, vision is problematic. Different approach to object detection is artificial odor sensors: Hayes et al. developed a multi-robot approach to localizing an odor source. An artificial whisker modeled on the Rat my special vibrissae has recently been demonstrated. Such a sensor can be of particular value in dusty or smoky environments[20].

The means for physical locomotion of a foraging robot can come in different forms, depending on the environment where the robot is supposed to operate. Wheels, tracks or legs are typically used in ground robots.

2.13.3   Communications

Communication plays an essential role in robot foraging. Even in the simplest case of a single foraging robot, communication is unnecessary. As for single robot

teleoperation, radio communication between operator and robot is a necessity. More importantly, in multi-robot foraging robot, communication is used continuously to enhance multi-robot performance; all six axes of strategy in the classification of Table 3(search, grabbing, transport, homing, recruitment, and coordination) need some form of robot-robot communication. Arai et al. refer to the critical difference between explicit and implicit communication required by robots to exchange information directly. Radio is the physical medium of communication (but not necessarily). Wireless local area network (WLAN) technology is highly fit for terrestrial multi-robot systems. This is partly because a spatially distributed team of wirelessly networked robots makes an ad-hoc network, providing the team with sufficient connectivity[9, 45]. Thanks to this connectivity, any robot can communicate with any other via multiple hops. Situated communication comes into play when "both the physical properties of the signal that carries the message and the content of the message contribute to its meaning."

2.13.4   Implicit Communication

Implicit communication is used when robots engage not in direct communication but indirectly via the environment, also known as stigmergic communications. Therefore, one robot makes some changes to the environment, and another robot senses the change, altering its behavior accordingly. Beckers et al. demonstrate that

stigmergic communication alone can bring about the desired overall group behavior. In their investigation on multi-robot communication, Balch and Arkin indicate that while stigmergy may be adequate for the completion of the task, direct communication can enhance efficiency. Trail following, through which a robot follows a short trail left by other(s), is an instance of implicit communication[9].

2.14 Multi-Robot Foraging

Foraging is a task that lends itself to multi-robot systems and, even if a single robot can accomplish the task, foraging should be done with careful design of strategies aimed at enhancing cooperation among the multiple robots. Swarm intelligence has to do with the investigation of natural and artificial systems of multiple robots. In this system, there is no centralized or hierarchical command or control. Rather, global swarm behaviors result from local interactions among the robots, and interaction between robots and the environment as well. Swarm robotics is related to the design of artificial robot swarms by drawing on the principles of swarm intelligence. Therefore, control is wholly distributed, with robots typically having to select actions based only on local sensing and communications. Consequently, swarm robotics is a subset of multi-robot systems[1, 6].

Figure 2-9 Foraging Robots (Slugbot)

The Slugbot: a proof-of-concept robot predator foraging is, therefore, a benchmark problem within swarm robotics, not least because of the active crossover between the study of self-organization in social insects and their artificial counterparts within swarm intelligence. This section presents some examples of multi-robot foraging, taken from the field of swarm robotics. Below, three cooperation strategies will be discussed:

information sharing, environmental cooperation, labor division[40].

2.14.1 Swarm Robotic Systems for Search and Foraging

Collective behavior of robots is a basis for any automated system, especially, of computer-integrated manufacturing systems, which require the synchronized activity of a large number of controlled manipulators. However, when the autonomous mobile robots are considered, the main questions concentrate on swarming itself and self-organization of the swarm regarding the robots' abilities

and the task to be solved[40]. Food search criteria AI problem, especially for multiple robots system. It is an essential problem for several reasons:

sophisticated foraging observed in social insect inspires artificial system-level model.

Foraging is a complex work that is related to the coordination of multiple tasks, each of them being tricky.

Efficient multi-robot foraging requires cooperation between individuals involving either communication to signal to others where the objects may be found or cooperative transport of objects too large for a single individual to transport[40].

Because of the complexity of the problem of search and foraging by the robot swarms, a variety of methods and techniques are often considered under a distinct theory of social foraging (Andrews, Passino, & Waite, 2007a, b). For a very brief overview of mathematical models used in this theory and swarm robotics in general, see the report by Muniganti and Pujol (2010) and a survey by Chung, Hollinger, and Isler (2011). The most popular taxonomy of the multi-robot systems was suggested in 2001 by Iocchi, Nardi, and Salerno (2001)Notice that if the robots are not aware of the other group members, then the actions of each robot can be considered separately and the group behavior is a result of parallel independent activities of the members. If the robots are not coordinated, they

execute their tasks in parallel, but the actions of one robot can depend on the results of the actions of the other robot (e.g., in the production lines)[40].

Due to weak coordination, the robots do not apply the coordination protocol, acting in parallel. In this context, they undertake certain corrections of the behavior about the other robots, for example, for collision avoidance[24]. In contrast, strong coordination implies that the robots support the coordination protocol and consider their actions and their influence on the behavior of the other robots. In the strongly centralized systems, the decision making is conducted by a single leading robot, which obtains information about the other robots and accordingly prescribes their behavior. This leading status remains during the mission. Weak centralization also assumes that the leading robot controls the activity of the group but allows for changing the leader during the mission. Finally, in the distributed systems, the robots make their decisions autonomously according to the activities of the other robots[24].

2.14.2   Mathematical Modeling

Multi-robot foraging is typically a stochastic nonlinear dynamical process and therefore challenging to mathematical. Experiments in a computer simulation or with real robots (which provide in effect an embodied simulation) show that limited environment exploration permit parameter which at best is only a weak inductive proof of accuracy. A mathematical model of the other parameter

complements space analysis and optimation parameters identification. Finally, in real-world applications, foraging robot systems are credited for safety and reliability, using a wide range of formal methods such as mathematical modeling[24].

Martinoli, Lerman, and coworkers proposed a microscopic approach to studying the collective behavior of a swarm of robots engaged. In cluster aggregation and collaborative stick-pulling, in which a robot's interactions with other robots and the environment are modeled as a collection of stochastic movement with simple geometrical considerations and the possibilities of regular experiment are determined with one or two real robots[24].

Martinoli, Lerman, and coworkers have also put forth a practical approach widely employed in physics, chemistry, biology, and social sciences. This approach directly demonstrates the collective behavior of the robotic swarm. A group of macroscopic models has been applied to examine the impact of interference in a swarm of foraging robots and collaborative stick-pulling. More recently, macroscopic models are given in Lerman et al. 's study on the probability model of success with macroscopic dynamic allocation. In the band, the robots at work will need to decide whether to pick up red or green puck based on local information. Methods of Optimal Foraging assume that a predator is hunting for prey[24]. The hunting consists of three main processes:

The process is a search for prey.

Deciding whether to terminate a search and start a chase for prey or to omit the chase and continue searching.

The process would be a pursuit for the prey if such a decision were made. The theory of optimal foraging for a robotic system In contrast to the theory of search, where the studies start with the formulation of the optimization problem subject to the given characteristics of search robotic system and yield the solution (which prescribes the robot's behavior), in the foraging theory, the consideration follows an opposite direction. It starts with the observed or expected behavior of the forager and then, based on certain meaningful assumptions regarding forager's goals and abilities, attempts to formulate an optimization problem so that its solution corresponds to the observed motion of the forager. Basic model of the forager's behavior, which represents an observed movement and a decision-making process, is based on the assumption that the prey items are distributed differently in different regions, or patches in the environment and the predators hunt in the patch during a specified period and then pass it for the other patch (MacArthur & Pianka, 1966). Such a model is known as a model of foraging in a patchy environment[24].

Regarding the patches, it is assumed that each patch is characterized by the availability of the prey items or, in the simplest case, by the number of items, which is known for the forager. The optimization problem regarding the forager's behavior follows the general assumption that the forager acts as economically as possible. Usually, the problem is formulated either as a problem of minimization of the time spent for capturing the prey item, including the time of the search for a patch and the time of hunting in the patch (MacArthur & Pianka, 1966), or as a problem of maximization of the utilized energy per prey item (Charnov, 1976). Then, the prey model deals with making a decision as to whether to stay in the patch or to continue search, while the patch model addresses a question: how long should the forager stay and hunt in a particular patch, or when should the forager leave the current patch and continue searching (Stephens & Krebs, 1986). In the deterministic setup, the solution of the patch problem was found in the form of the marginal value theorem (Charnov, 1976). It assumed that the resources in the patches are not renewed and that the times of movements between the patches are proportional to the distances. Then, given the requirement that the forager maximizes the net rate of the energy intake, the theorem states that the forager should leave the patch if the marginal rate of gain in the patch becomes equivalent to the long-term average rate of energy intake in the habitat (Charnov, 1976). Notice that according to the model, since the overall energy rate

depends on the rates in the patches, the forager will return to the already-explored patches up to their complete depletion (Stephens & Krebs, 1986).

All requirements of the deterministic setup of the problem cannot be satisfied in practice: the forager cannot know an exact rate of the energy intake in the patch and certainly cannot know the long-term rate of energy intake over the habitat. To overcome these difficulties; the problem should be formulated in a more realistic probabilistic setup (Oaten, 1977). In such setup, it is assumed that the captures of the prey in the patches are random events and that the forager is not informed of the number of prey items in the patches. However, the forager keeps a distribution of the prey items over the patches such that it defines the probability that the patch includes a certain amount of prey. Then, the strategy of the forage are specified regarding probabilities of gain and energy rates, but the resulting solution is not necessarily optimal (Oaten,1977), and the forager may terminate hunting and leaves a patch before reaching the threshold value of the energy rate. Such forager's behavior can be represented by patch sampling and assessing the potential gain in this patch (Stephens & Krebs, 1986) implying specific predictive abilities of the forager[24]. Regarding foraging in random environment, such abilities are represented by the potential function (McNamara, 1982), which specifies relative advantages of continuing hunting in the patch given that the future behavior of the forager is optimal[24]. An optimal policy is also defined,

using the potential function and it prescribes staying in the patch as long as the value of potential is positive and leave the patch when it drops to zero[24].

The trajectories of the robots and foragers are obtained as a solution to the specific optimization problem, depending on the implemented constraints and assumptions. The search and foraging in the opposite direction: it start with the class of trajectories, which are postulated as optimal. Then, it considers feasible models and algorithms, resulting in such trajectories. It has been successfully applied in biological and ecological studies and is used in this approach, given their relationship with the probabilistic algorithms of search and screening. The trajectories of the searching and foraging robots were considered as direct results of the algorithms of search and path planning, serving as traces of the robot foraging in the patchy environment[24]. According to the task, the obtained trajectories satisfy the following specific optimality criteria: the maximal probability of detection of the target or minimal search time up to the specific detection of the target in the case of search and screening problems or maximal expected intake energy rate in the case of foraging. The other approach to the search and foraging problems follows the opposite direction. The consideration starts with the observed trajectories of the foraging animals, birds, or insects, which given their abilities and habitat are postulated as optimal evolutionary foragers (Pyke, 1984. This is followed by the development and analysis of a simple

formal model, which allows for the detection of such trajectories. Indeed, different living organisms follow different paths, which are described by different stochastic processes. The random Brownian motion of the robot is walking, that fits within the framework of the theory of search and random search formula. The movements of the foraging ants usually are modeled by the corresponding Brownian motion, which demonstrates a right consistency with experimental results. The relevance of Lévy flights is possibly right for flying insects. The trajectories resulted in the indicated processes[2, 24]. For illustrative purposes, all shown trajectories (both simulated and observed) include 1000 Points with the coordinates normalized for the square arena 100×100 units. In Brownian motion, the step length is three units. In the Lévy flight, parameter $\mu = 1.6$, minimum step length is1 unit, and maximal step length is 100 units that are a rounded value of the arena diagonal. In the corresponding Brownian motion, the correlation coefficient between the following directions of the steps. Notice that by large, the trajectory specified by the Lévy flight looks similar to the trajectories generated by the algorithms of foraging in the patchy environment considered[2]. However, in a more close resolution, it is seen that the Lévy flight trajectory is scaled invariantly, while in the trajectory of foraging in the patchy environment it is not the case, and its long-distance jumps are defined by the location of the patches. The Lévy flights demonstrate good correspondence with the long-distance wanderings. Also, in the models which consider the short distance movements and, mainly, the movements

of the animals and insects, the dependence on the environment is defined. Also, mixed models are used to combine different strategies of search and foraging. The indicated methods are developed at most in the framework of contemporary foraging theory and are aimed at providing formal models for the observed movements of living organisms. At the same time, similar trajectories of the search robots are specified by the methods of the real-time probabilistic search and path planning, developed in the framework of search and screening theory. The following section describes the temporary cost methods used in the foraging theory and search theory. Moreover, the search and foraging algorithms used for implementing these methods are discussed. For a detailed overview of general methods, used for modeling spatial trajectories, see a recent publication by Brillinger[2].

3　Design of Experiment (DOE)
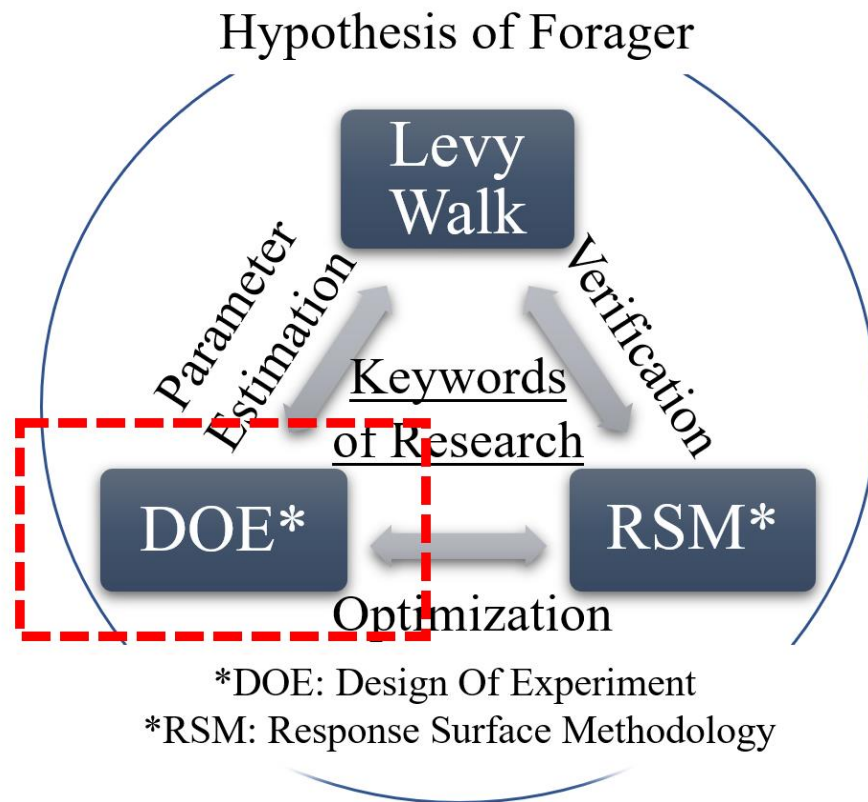
Figure 3-1Design Of Experiment

Design of Experiment methodology was proposed by a British statistician, Sir Ronald A Fisher, as early as 1926. The pioneer work on statistical methods was used in the field, and the concepts and procedures are still in use today. In particular, Fisher and colleagues found that experimental design involves multiple measurements to the level of fluctuation in the measurements. During World War II, DOE was beyond its roots in agricultural experimentation, as it was a way to evaluate and improve the performance of weapons systems. Immediately after World War II, the first industrial period marked the boom of DOE. Quality

management is a comprehensive and continuous improvement of management techniques, which was later used by the US military industry. Bowes and Kitsch simultaneously developed some effective plans for estimating several significant impacts in 1940. In 1944 Plecote Berman plans were presented, which are still unclear[38, 40]. At the time, Rao introduced the concepts of orthogonal arrays as experimental designs. Today, this theory is based on advanced topics in linear and hybrid algebra[46].
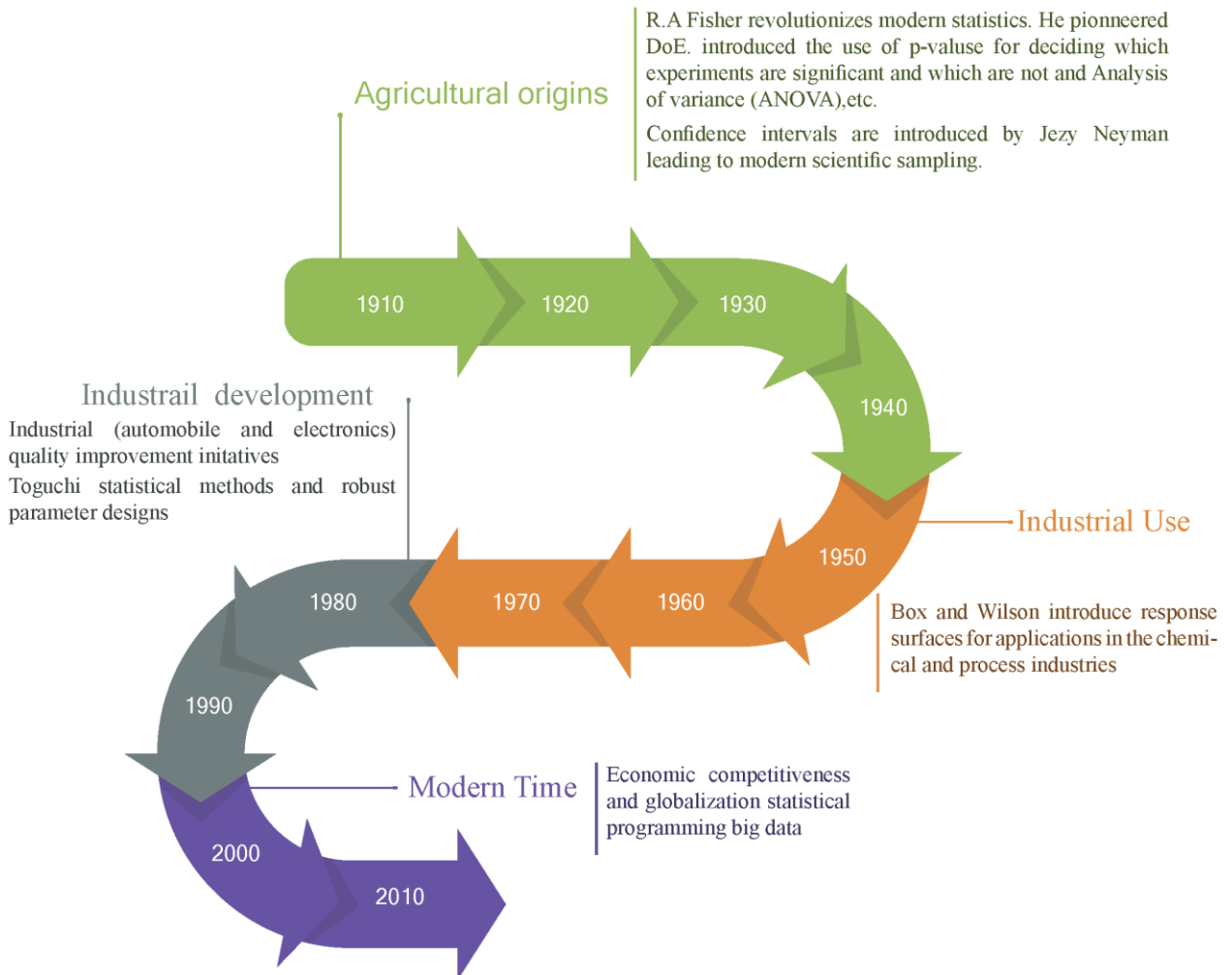
Figure 3-2 History of design of experiment

Design of the experiment is one of the most powerful techniques used to improve quality and increase productivity. In this way, some changes are made to the processor system by doing some tests, and the influence of those tests on performance characteristics or process response to them are considered. The design of experiments means to systematically manipulate some variables to assess the impact of these manipulations. This proposed processor system can be identified, using the model shown in the following[46, 47].

3.1 Traditional Experimental Design Versus DOE

DOE is not an alternative approach to experimental research. DOE is instead a methodology that provides stringency to the classical approach for doing research. DOE can be used to assist with the statistical section of the research process, as shown briefly. Before identifying the private parts of a DOE methodology, it is worthwhile looking the defects of the traditional(one factore at the time) optimization approach briefly. In the most straightforward traditional approach to optimizing experiments, one parameter is varied while all others are defined. The experiments performed traditionally are out of range, leading to conclusions, and sometimes even worse, wrong conclusions. Further, the traditional setup DOEs not take into account that experimental parameters, which can be dependent on each other (parameter interaction). In ion-exchange chromatography, optimum will change when conductivity is changed[12].

So, with the one factor at the time(OFAT) experimental setup, there is a substantial danger that the exact optimum for the studied process is not recognized. At last, a study with the wrong setup cannot be saved or evaluated by even the most advanced statistical programs and approach. On the contrary; process parameters are allowed to vary simultaneously, allowing for the effect of each parameter, particularly in combination[13].

3.2   Why DOE?

The design of experiments is one of these sophisticated, specialized tools. Explore the relationship between several explanatory variables and one or more response variables. Unlike the usual methods, the interaction between process variables can be determined using statistical techniques. Inferential statistics are used in the processing of raw data in order to achieve optimal rather than emotional planning and decisions. This knowledge is continually increasing all over the world. The use of this knowledge is for the following applications, among others: statistical quality control, the design of experiments, data mining, and prediction[13, 32].

Given the modern technical approaches, products and processes are becoming extremely complicated. As the price of experimentation goes up rapidly, it is getting more and more unmanageable for the psychoanalysts, who are already restricted by resources and time, to investigate numerous ingredients that bear upon these complex processes using trial and error methods. Preferably, a

technique is needed to identify the "critical few" factors most efficiently, and then to guide the process to its best setting to match the ever-increasing need for improved tone and increased productivity[32]. DOE techniques provide powerful and effective methods to accomplish these aims. This procedure is a technique for optimization of any process or product but is better, faster and cheaper than other engineering methods, such as A/B tests (which are known as OFAT or any agent) and "expert guess." When studying the effect of two or more factors on a process, the control and arrangement of the DOE experimental setup allow for the collection of sufficient information with fewer experiments, compared to the traditional approach[32].
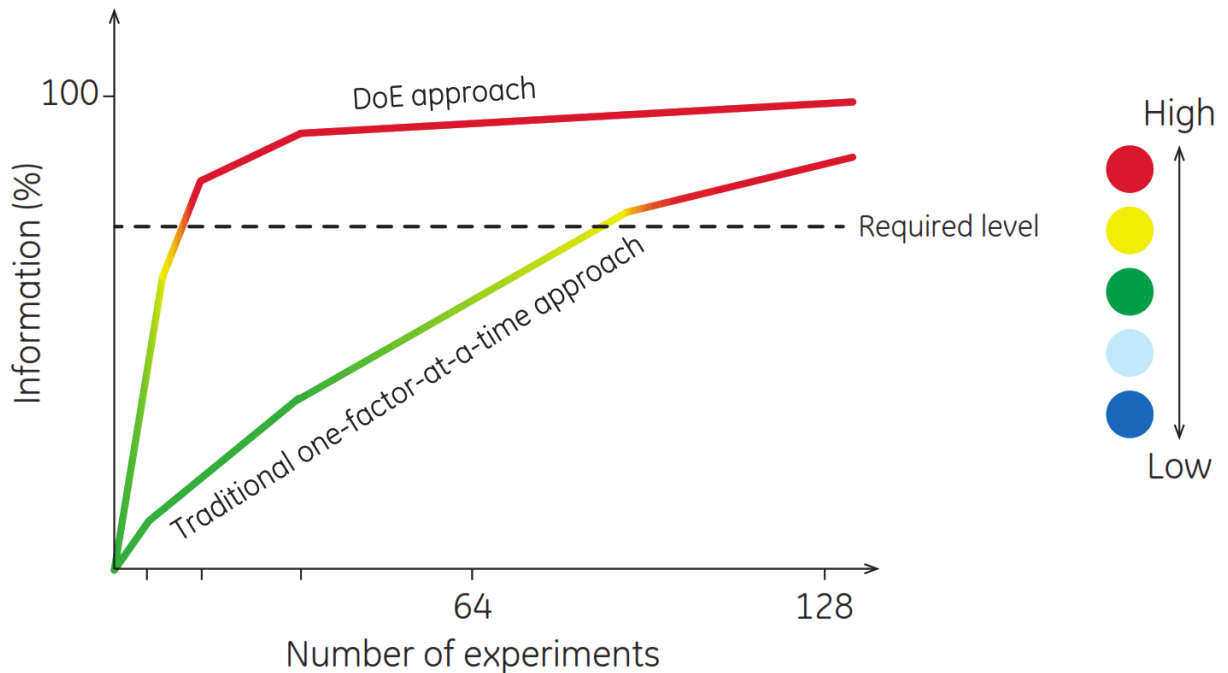
Figure 3-3 schematic comparison of the number of experiments required to reach an acceptable level of information in an experimental study[47].

Designed experiments are, by far, more effective than single-factor-at-a-time experiments, which involve converting a single component at a time to examine the upshot of the ingredient on the product or procedure. While (OFAT)experiments are easy to read, they do not permit the investigation of how a gene affects a product or process in the presence of other elements[48]. An interaction is a relationship whereby the result that a factor has on the product or process is changed due to the presence of ace or more other elements. Often interaction effects are more significant than the outcome of individual ingredients. This is because the product or process software environment involves the presence of many factors together instead of events separate from one of the factors at different times[47, 49]. Traditional Experimentation studies one factor at a time (OFAT), holding all other factors constant. Serial experimentation is uneconomical in terms of time, money, and energy. Moreover, unfavorable & unpredictable complete fulfillment of the correct optimal product or robust process can never be guaranteed due to the presence of multiplication/ interactions of factors. impact of one or multiple factors on others, which OFAT and Design of Experiments (DOE) can not deal with as they study multiple factors at once as a systematic series of parallel experiments simultaneously. In contrast, parallel experiments are considered economical in terms of time, money and efforts, yielding maximal information with minimum runs. Input factors sometimes undergo some changes in order to identify causes for significant changes in the output responses. In this

way, the relationship between factors is determined, and an optimized product/ robust process is found. Accounts for interactions between factors estimate the effect of each factor regardless of another factor effect, using multiplication[48].

An interaction is supposed to continue between the two factors regarding the robotic. The central concepts in inferential statistics deal with the development of the variance index, expected value, random variable, probability distributions, and all of the Concepts of probability. The extent of inferential statistics should be introduced or reviewed in all studies of the sciences so that all scientific experiments should be taken into account. DOE can be defined as a systematic means of changing experimental parameters (components) to create solutions that can be methodically analyzed, providing useful information about the process studied[50, 51].

The DOE methodology ensures that all agents and their interactions are systematically investigated. Thus, the data received from a DOE analysis is, by far, more authentic and complete than the results from (OFAT)experiments that ignore interactions and hence may lead to wrong conclusions.

This led to the emergence of disciplines such as operations research, ergonomics, and design of the experiments. These three sciences helped the Allied to make significant gains. During the war, a form of experimental design called "operating

plan" was considered a significant weapon to speed up the development of the industry. These projects consisted of two levels of each factor and only a fraction of all compounds. After the war, a statistician in Imperial Chemical Industries, named George Box explained how has created response surfaces for optimization. Later design of experiments was applied in simulation processes resources such as time, speed, radius detection, forager number, and mixing that were easily manipulated. It was also used in the fields of science, including biology, agriculture[52, 53]. Later Mr. Fisher expanded the concept of experimental design. He was responsible for analyzing the data in an experimental agricultural center in London. The chemical industry in the United States, the UK, and many developed countries still make the best use of the design of experiments. In recent years this science has been used in many fields of engineering and computer science. Moreover, it was known as a competitive tool in the industrialized world. Now that we are somewhat familiar with the history of science, we will refer to its position in the academic fields[53].

Undoubtedly, the industry has realized the importance of quality. Today, quality is considered as a business strategy to increase market share, with organizations using designed experiments to achieve global quality. The design of experiments has been developed as Quality science and statistical quality control in England and the United States, respectively[48].

The first step includes defining the objective of the study and the factors that should be systematically varied. The range of variation is also defined in this step. The second step involves defining relevant responses (a type of analytical methods and data)[48].

A DOE experiment is set up rationally to cover the intended experimental space. The axes of the cube represent the different factors (X_1, X_2, and X_3...., represent three different factors, e.g., Using DOE, multiple factors dealt with in a single series of experiments can be viewed in arrangements called hypercube as the setup becomes multidimensional. Different types of designs are available, depending on the study to be performed[48].

After performing the experiments according to the selected design, step 5 in the workflow involves using DOE software for deriving a mathematical model that describes the investigated process or system. A relevant model tells us, for example, which factors have a significant impact on the response and which factors do not. It is essential to evaluate the model to determine its relevance, using DOE software. The model is often visualized as a response surface plot and is used for evaluation of other parameter settings or process outputs within the experimental space. While performing the DOE study, it should always be carefully verified that the model is relevant. Verification of the model is preferably done through verification experiments within the experimental space[54]. The existence of a

relationship between a factor and the response is a critical requirement for the relevancy of the model.
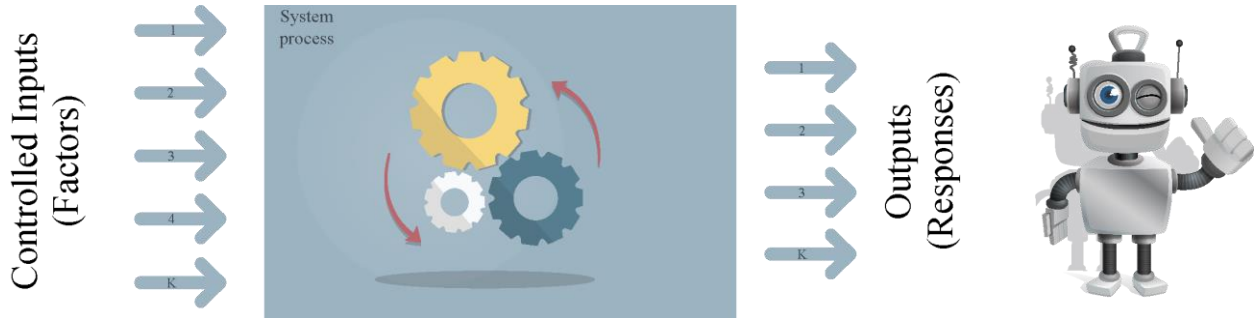
Figure 3-4 Parameter estimation to provide the best efficiency

3.3   Modeling the search

In this study, we model a two-dimensional environment that contains targets. Each target is a point (i.e., it has radius zero). A robot is modeled as a point that moves across the environment.   The robot DOEs not have a fixed detection radius, because when designing a robot, it is essential to select the sensor detection range within which it can detect resources. This detection radius is assumed small relative to the size of the environment. When a robot detects a target, it stops its current movement, moves directly to the target, and the target is removed from the environment. This model is the "destructive search," which is appropriate for situations where targets are objects to be consumed, collected, or otherwise eliminated. Depending on the application, targets may be more appropriately modeled as discs, rather than points. Fortunately, this is equivalent to the model

where the targets are points, and the target radius is added to the robot's detection radius.

A natural metric for search efficiency is the number of targets encountered divided by the total time spent on searching, and this is the metric we use in this study. We assume that a robot moves with a speed while performing search and that its movement pattern is continuous (i.e., the robot never jumps from one location to another). Finally, we assume that the robot has no a priori information about the location of targets.

Suppose you belong to a hunter-gatherer tribe whose habitat is located in a remote part of the sprawling African plains, and you are hungry. How do you proceed with searching for something to eat?

Based on a new study, we are likely to make use of the same food-scouring technique as that used by animals and cells, and organisms. How can the behaviors of living cells and organisms serve as a basis for programming robots? These behaviors are assumed to be well-honed to their purpose by natural selection. In the case of stochastic search strategies, one can look to the foraging behavior of animals, including seabirds, sharks, fruit flies, fish, bacteria, large mammals, etc. One can even look at the behavior of cells in the human immune system, which can be thought of as "foraging" for disease cells[7, 40].

The studies conducted by scientists have shown that a diverse number of species (e.g., organisms) seem to use a mathematical pattern of movement to move via a random walk while foraging for food. Consequently, they can be interpreted as implementing stochastic search strategies.Each successive move in a random walk is chosen randomly, uninfluenced by any previous move. Take a drunk stumbling along, where a step may be taken either to the right or left, with no memory of the route he/she has taken.

In a simple random walk, a searcher moves a fixed distance (the step-length) in a randomly chosen direction, stops, randomly chooses another direction, and moves a distance equal to the step-length in that new direction, and so on. In a Gaussian random walk, step-lengths are chosen from a Gaussian distribution (a simple random walk can be viewed as a Gaussian random walk with variance zero). At the sufficiently large time and distance scales, all Gaussian random walks converge to Brownian motion. The mean-square displacement of Gaussian random walks and Brownian motion both scale linearly with time. Random walks with this property are called diffusive. When the step directions are selected from a non-uniform distribution, the result is a biased random walk. In a correlated random walk, the probability distribution of each step direction is concentrated around the direction of the previous step direction. Searchers that move via correlated random walks are

said to display directional persistence. A random walk can be both biased and correlated[2].

An investigation (published in the Proceedings of the National Academy of Sciences) made use of GPS-tracking devices attached to the belts or arms of forty-four members of the Hadza, a group of hunter-gatherers living in the northern region in Tanzania. These hunters wore these devices from dawn to dusk, who walked several kilometers per day to find food[61]. Most of the Hadza foraging treks can be best characterized by Lévy Walk, as opposed to alternative statistical models of motion.In recent decades, scientists have observed this Lévy-like behavior in various creatures ranging from bacteria to penguin in their search to find a meal. These observations have been made across the natural world among the animals with varying degrees of complexity. This same pattern was seen among all of them[21, 22].

Lévy Walk is associated with many small moves integrated with a few longer trajectories, with most of the steps being made within a small area and longer routes took on occasion.

Observations have been made of the Lévy-like pattern in insects, sea predators such as sharks and tuna, terrestrial mammals. More interestingly, evidence of Lévy

Walks has even been gathered of people's wandering through university campuses and urban areas[21].

Specialists of human locomotion evolution studied hunter-gatherers to find clues on how ancient man moved. Human foragers are likely to use a different search methodology, which is different from animals since human enjoys high cognitive ability to use memory and environmental cues. Humans may use the same technique as other species. In this study, we use the robot to find the object or food[61].

An extensive investigation conducted on sea predators such as sharks and tuna showed that Lévy Walks alternated with another kind of movement called Brownian motion, keeping searcher within a smaller area without the longer trajectories. Here, food distribution is a likely factor. In the case of prey abundance, it seems that Brownian is the right choice for picking through a closed area at random, collecting the bounty[61].

In contrast, in the case of food scarcity, Levy patterns do the job better. "It facilitates searching for widely and randomly distributed food without returning to the same patches, compared to something like a Brownian walk." In the case of the Hadza, food is distributed in patches, with the subjects having a set plan for the day. The women forage in groups, engaging in hot debates beforehand about where

to go. They are eager to proceed with more detailed data collection (e.g., chatting with the subjects to figure out their intentions, following them on treks and taking note of what they bring back). Ballistic motion is the term for straight-line movement, involving a random walk with infinite step-length. In the ballistic motion, a searcher selects a direction at random and travels in that direction indefinitely the mean-squared displacement of ballistic motion scales with the square of time. Such a movement pattern emerges if its mean-square displacement scales with time at a faster-than-linear rate; hence ballistic motion is superdiffusive.

In natural systems, food resources (targets) are often distributed in clumps. If a forager encounters a food item, likely, another food source is nearby. Hence it makes sense to carefully search the nearby area, using a movement pattern such as Brownian motion. On the other hand, a search strategy like Brownian motion is inefficient, because it involves revisiting previously explored areas. On the other extreme, a forager employing ballistic motion DOEs not revisit previously explored terrain but might be unlucky, moving in a direction away from a clump of food resources. Lévy walks a trade-off between these two phenomena. Lévy walk foragers are likely to take small steps (similar to Brownian motion), but will occasionally take very long steps, preventing them from wasting time intensively searching a barren region.

In ecology, the Lévy foraging hypothesis has been extremely controversial; nonetheless, it serves as an excellent motivation for programming autonomous robotic search. In the current study, an attempt is made to find an answer to the following critical question: what are the optimal parameter values for a random walk stochastic search strategy? The reply to this question hinges on many characteristics of the system, such as the detection radius and the speed at which the searcher moves as well as the overall spatial distribution of targets on the landscape. Here, the coupling between move length and time is discussed. We start with the formal dynamical coupling of the particle position and current time via a constant velocity of the particle. There are two closely related models which incorporate finite velocity of random walkers. Lévy Walks use stochastic processes that provide a versatile tool for modeling robot movement.

$$f(x)=Cx^{-\mu}$$

A Lévy Walk with parameter μ is a random walk with step lengths x drawn from a Power-law distribution, p (l) ~l type equation here $1<\mu<3$ and C between 0 degrees and 360 degrees selected, using random method. Different values of μ yield different types of random walks.Given that $\mu\rightarrow1$, the resulting random walk approaches ballistic (i.e., straight-line) motion. A random walk whose step lengths are drawn from a Power-law distribution with $\mu \rightarrow 3$ acts like Brownian motion.

Consequently, Lévy Walk can be characterized as a spectrum of movement behavior, ranging from ballistic motion ($\mu \rightarrow 1$) on one extreme to Brownian-like motion ($\mu = 3$) on the other. A reason for using Lévy Walks to model robot movement is that they are ''superdiffusive''.

3.4   Design of Experiment

Variations occur in nature be it the distribution of a particular grade of food in the unknown environment, robot content in the large environment or the distance traveled by the vehicle. Mutations are also picked up in the observations recorded during multiple executions of a process, even when all elements are strictly kept at their respective levels, and all the executions are run as identically as possible. Fundamental changes that occur in the process are often called noise when all conditions are maintained at the same point. Some statistical methods are available to achieve this. The presumption of the normal distribution is widely used in the analysis of experiments design. Design of experiments is a technique for planning experiments and studying the information received. The technique permits us to employ a minimum number of experiments, in which we systematically vary several experimental parameters simultaneously to get sufficient data. The example can be applied to see the influence of the experimental parameters on the outcome and to recover an optimum for the process. Modern software is applied to produce the experimental designs, to obtain a model, and to visualize the generated data.

DOE approach can significantly improve the efficiency in screening for suitable experimental conditions (for simulation of robotic movement, optimization of a process)[54].

The focus is on DOE for simulation and robotic, but the theory can be applied in many other applications. This theory is essential to gain a clear understanding of hypothesis testing because this concept is directly applied in the analysis of designs experiments, determining whether or not a particular factor is significant . A lot of our knowledge about the processes and products in the scientific and engineering disciplines emanates from the experiment. A test was conducted in a series of tests systematically to increase knowledge of an existing processor to be explored. Then, the design of experiments is considered a tool to develop an experimental strategy that maximizes learning using a minimum of resources, data, and experiment. DOE is widely used in many fields with broad applications across all the natural and computer sciences. It was in widespread use by engineers and scientists involved in the improvement of processes to maximize performance and reduce production variability[54]. Most engineers are also working on products or processes to which scientific theories or principles are applied directly. The experimental method plays an essential role in studies on the cost-effective and confident development of new products and processes. Those designs that are based on inferential statistic methods are called classic designs. Classic designs

include designs such as balanced and unbalanced ANOVA designs block designs, factorial designs, fractional factorial designs, Latin square design, and nested designs, and so on. Also, the RSM designs can be made out of the factorial designs[48, 55].

Most applications of classic design are in industries such as electronics, mechanics, and engineering materials and robotic. Moreover, fractional factorial designs are also used in the robotics, petrochemical industry, and airspace. The design and analysis of experiments revolve around the understanding of the effects of different variables on another. The aim is to demonstrate a cause and effect relationship between some independent variables and a dependent variable of interest. The subject variable in the context of DOE is called the response, and the independent variables are called genes. The treatments of an experiment are limited by the number of factor levels being investigated. For instance, if an experiment with two elements is to be executed, it can be understood that the size of an experiment expands rapidly as the number of factors (or the number of the stories of the factors) increases[48, 54].

3.4.1   Examples Of Application Of DOE & Its Advantages

Most of the valid companies are using the design of experiments methods, benefitting from the annual profit of many economic savings brought about by these methods. Some companies that continually take advantage of these

techniques(e.g., BMW, Audi, Samsung, Sony, Henkel, and popular airlines such as Boeing and Airbus and especially organizations like NASA) are some examples. All of these firms act as a pioneer in the work field, seeking creative innovations in their industry. Currently, advanced courses of 2 to 6 DOE which is called MDOE are held in NASA[48, 52].

So in this way, the volume of used resources is reduced, and the quality of their products increase. Application of DOE such as the parameter design and tolerance design has helped the Samsung company to capture the first place in terms of Plasma TV (PDP) manufacturing all over the world, enjoying a mass production line of this product. New concepts for small size LCD has given rise to TV panels called UFS (Ultra Find and High Speed) with the highest resolution in the world[53].

Table 2. Several of projects using DOE methods in different companies.

Company Name	Project conducted by the DOE	Results
BMW	Design of experiments for identification of nonlinear dynamic systems	Reduce emissions and fuel consumption
HYUNDAI	The optimization of the airbag for knee	Make Optimization Airbag
SAMSUNG	Design of Experiments optimization for the NBR composites	Increase the performance of NBR
SAMSUNG	Improvement of Compressor performance (International conference of Compressor Engineers)	Increase performance and noise reduction Suction Muffler
BOEING	Reduce variability cars OSU	Find Significant and affected parameter on variability

DOE method reduces the number of performers dramatically, allowing for reviewing the primary effect of the interaction between agents as well. Another example has to do with a 250 passenger airplane wing design. Design engineers often use numerical optimization techniques to evaluate and compare the use of a new configuration plane. Though the application of numerical optimization has been very successful, the existence of irregularities in the optimization of real engineering problems often precludes the use of optimization techniques based on the gradient. Irregularities caused wrong chaos gradient calculation, slowing or

stopping the convergence of the optimization process to an optimal solution. This problem is particularly acute when a structural analysis of the actual configuration of the aircraft aerodynamic aircraft design may require thousands of hours of CPU time on a supercomputer. The method was developed to make two kinds of the mathematical model, consisting of irregularities factors in the use of optimization to build aircraft wings[53].

3.4.2   Simulation Software

Since the simulation software yields the results for reality simulation, the correct use of simulation software, like Taylor and Arena, can be helpful in the recognition process. The correct choice of the DOE profit rate depends on organization definition of the word Profitability that defines Profitability as optimization of the product (reducing waste and using cheaper materials) or innovations (design engineering) or quality Enhancement (better performance). Each of the points of view will undoubtedly bring about much profitability. DOE can help achieve all these outcomes. Energy has been extensively covered in Engineering, with many many ways to save energy being proposed. Today the world is using clean energy, and every country is seeking to achieve it. One application of design of experiments is in reducing variability in processes as well as waste. Design of experiments is a useful tool to determine the specific factors that affect the product. DOE can reduce the time required for the development of the product[53, 56].

The pharmaceutical companies use DOE for the development of the formulation, which allows the evaluation of all potential risk factors simultaneously, quickly and systematically. DOE can identify any of the formulas on the response (probability of interaction between the two factors), helping to evaluate the statistical analysis of the critical factors. After the identification of the critical factors, the formula can be improved, using the DOE to optimize all critical factors. The manufacturing process can also be developed and optimized in the same way[56].

3.4.3 Mechanical Engineering

Mechanical engineering needs to design all pieces at first. Most designs are very complicated, and issues such as the strength of materials statics, and applied mathematics are considered. Ski Company has produced K2 in Washington, using a complex design for its product (a top rate of 30% waste was produced). DOE found the cause and solution and downtime pressing (skate producer) from the 250-hour workweeks to 2.5 hours. In this field of engineering, Taguchi methods and classical DOE methods are widely used[56].

3.4.4 Petroleum & Gas Industry

Given the rising oil prices and demand for oil, many companies in the oil and gas industry are seeking to improve control process and data analysis in order to optimize their operations and gain a competitive edge. DOE experiments are used

to minimize the expensive trial and error test. The experimental design methods are widely used in the exploration extraction and refining of crude oil. (e.g., maximization of light olefins from ethanol). Useful parameters such as the amount of water, entry of water, catalyst type and WHSE speed, the reactor shape, temperature, and many other parameters that affect the system can identify and determine the levels. There are many irregularities in the oil and gas industry. Therefore it is not easy to find the optimal control parameters. Most of the data types used in the oil and gas industry(especially when the project is related to the area of operations) include a series of interconnected data, both input variables, and output variables. If there is no description of data, a tool such as regression or combination of regression methods, including stepwise regression can be used[48].

3.4.5   Mining & Material Engineering

DOE Techniques are an integral part of Materials and Mining Engineering. Several projects have been done, using these techniques. For example, temperature and the type of material are determining factors in building a battery, i.e., Whether two factors interact to influence the manufacturing of batteries together or not, and to what extent is it useful in the useful life of the battery or the impact of each factor on the response. To be linear or nonlinear as well as the useful life of the battery are justified by the two factors, or other factors affecting the battery life. These are questions which can be answered accurately only with the accurate design of

experiments. The design of experiments technique has many applications in mining engineering such as determining the optimum operating conditions of Copper ore flotation. Irregularities in determining the optimum operating conditions can have a significant effect on experimental design in minimizing these irregularities. In this industry, more RSM and mixture designs for response experiment design are used[48].

3.4.6   Agriculture

Since its beginnings in agriculture, DOE has been very useful across many sectors of the robotics and industry. Six Sigma is a technique that uses various tools, including DOE, to derive statistics-based quality improvements. One of the first applications of DOE methods was in the agricultural industry. Just two simple examples of the application of this method have been documented in the agriculture industry. For example, we can use these methods to maximize the size of basil leaf (factors such as the sprinkling of Irrigation, temperature, ambient noise, and fertilizer use can be useful in the growth and size of basil leaves)[54].

3.4.7   Would Be a Case of Button Mushrooms

Making compost takes a long time, and utilization of mushrooms compost per square meter (the compost) in a country like Iran is 15% to 17% and in advanced countries such as the Netherlands (one of the leading exporters of this product in the world) is 30%. However, why is this the case? It is quite evident that they have

found critical parameters in the production process and achieved high performance in production. Also, they have reduced the time required for the production of compost by one third lower than the required time. It has provided many benefits, such as the lower cost of production and the reduced Portion of their time. Many factors are involved in making the compost, which can be produced in various ways. However, since compost can be produced with some slightly different formulation, DOE methods can be used for this purpose and different sectors. Classical DOE methods and RSM are widely used in this industry[54].

3.4.8   Design for Computer Experiments

The choice of design for a computer simulation experiment presents some exciting alternatives if the experimenter is considering a polynomial model to describe the underlying relationship. In this way, an optimal design such as a design for the specified model is a possibility to choose. Various types of space-filling design have been suggested for computer experiments. There are several reasons why space-filling designs are thought to be particularly appropriate for deterministic computer models:

1. Recall that the estimation codes methods are often used for developing models based on computer experiments from deterministic codes, having the characteristic of no uncertainty at an input location. However, uncertainty increases as we move farther away from any observation. Hence, space-filling designs are desirable

since, in general, they spread the design points out nearly evenly or uniformly throughout the region of experimentation[57].

2. Most space-filling designs do not contain any replicate runs, even when the design is projected into lower-dimensional spaces. This is desirable for a definite computer model since a single computer model at the design point provides all the information about the response at that point. If the design were to contain replicates when projected into lower dimensions and some of the factors were not active, this could result in the same response value being obtained for multiple runs. Since it is unknown a priori which factors are active, this could be costly duplication if obtaining those runs requires a great deal of computational effort.

3. Region of the design space may be known to be unacceptable based on underlying science or engineering knowledge. Space-filling designs can be easily adapted to fit into nonstandard shaped regions[57].

4   Response Surface Methodology
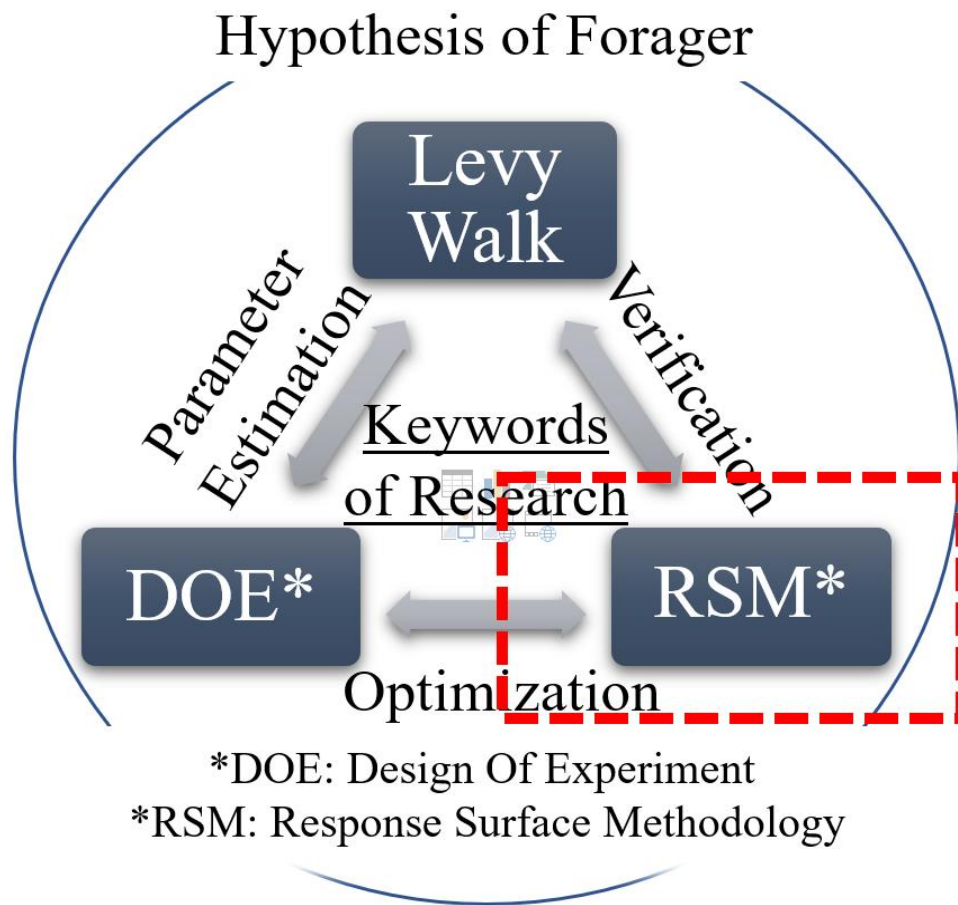
Figure 4-1 Response Surface Methodology (RSM)

In the 21$^{st\ century}$, engineers are now taking on projects of unprecedented complexity. For example, consider the state of the art aircraft now versus the plane made by Wright brothers about 100 years ago. They did many experiments on the wing design, the configuration of the propeller, and so on. After all these pioneering works, experiments and mistakes, Wright finally landed. Today, much of the development of airplanes and other advanced equipment is done through experiments on high-power computer simulations[10, 55]. The approach is to

sample some number of times within the experimental space randomly. Following

a more systematic array of points requires first segmenting the region into a given

number of rows and columns. It is possible to do sampling in such a way that 1

point appears in each row and each column(no more, no less)[58].
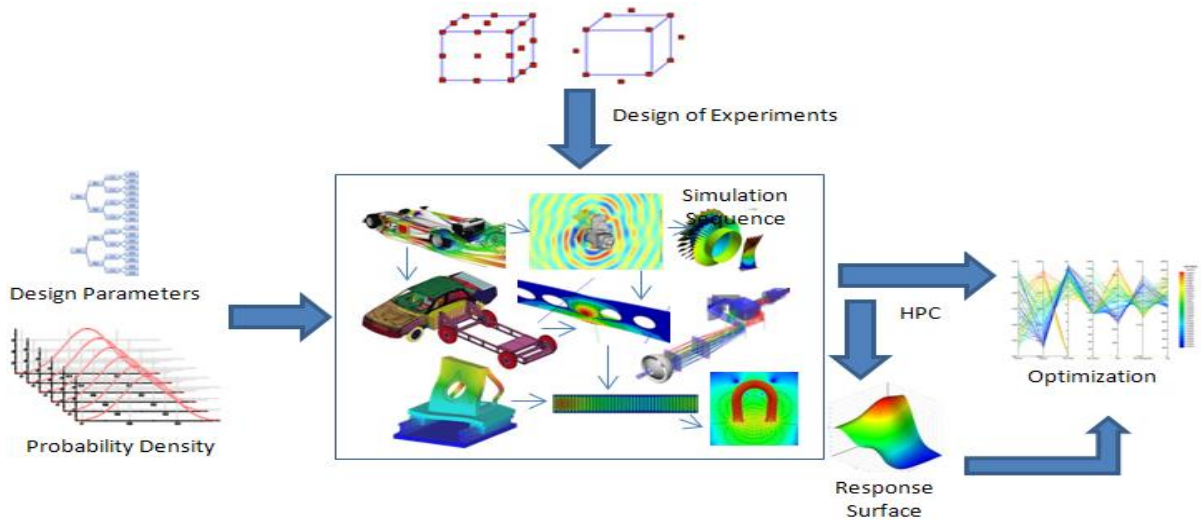
Figure 4-2. Sample for Response Surface Methodology

Response Surface Methodology is a classification of statistical and mathematical

parameters used in the development and optimization of an adequate functional

relationship between a response of interest, y=efficiency, and the number of

associated control (or input) variables denoted by x_1, x_2, x_k. In general, such a

relationship is unknown but can be approximated by a technique which comprises

a body of methods for exploring optimum operating conditions through

experimental methods. central-composite and Box-Behnken designs are useful for

building empirical models and Functions. The aim of these designs is optimizing

the response (output variable) that is affected by several independent variables (input variables). One experiment is a series of tests called the Implementation. In each experiment, changes are made to input variables to determine the causes of variations in the response variable. For analysis of the resulting data, the response-surface methods provide an estimate of the response surface, testing its lack of fit, displaying an ensemble of contour plots of the surface, and doing follow-up analyses such as steepest ascent, ridge analysis, and legal analysis. The design of functions is hoped to provide an intuitive and useful user interface relationship between multiple explanatory variables and one or more response variables. Response Surface Methodology makes use of a sequence of designed experiments to obtain an optimal response. It uses statistical models. Therefore, practitioners need to be aware that even the best statistical model is an approximation of reality[58, 59].

RSM was developed initially for Experimental responses model (Box & Draper, 1987). Later it was extended to model the numerical experiments. The error on physical examinations can occur in different shapes(e.g., evaluating errors when irregularities or error are caused by incorrect convergence). RSM assumes that the errors are random. Application of RSM for design optimization leads to a decrease in the cost of expensive analysis methods and numerical irregularities connected with them. The RSM convergence is made toward the optimal element because

they reduce the effects of irregularities. Constructing response surface models in response to surface designs is an iterative process. Once an approximate model is obtained, it is tested whether the answer is satisfactory or not, using the goodness of fit method. If not approved, estimating of the process starts again and further tests are done.

Different grades or values of the operating conditions comprise the ingredients in each experiment. Some may be more or less categorical, and others may be quantitative (speed, radius detection, and mu). In practice, categorical variables must be managed on an individual basis by comparing our best-operating conditions for the quantitative variables across different combinations of certain singles. The basic methods for quantitative variables require fitting first-order (linear) or second-order (quadratic) functions of the predictors to single or multiple response variables. This is followed by analyzing the characteristics of the fitted surface to determine what activity is appropriate[59, 60].

Since it may seem like a response-surface analysis, it is considered merely a fixation problem. However, there are some intricacies in this analysis as well as in how it is commonly used and given its difference from routine regression problems, some special help is warranted. These intricacies have to do with the everyday function and importance of coded predictor variables; the assessment of the fit; the different follow-up analysis that is used depending on what type of

model is fitted as the result of the psychoanalysis. Visualizing the response surface methods also involve some unique experimental design issues. Given the emphasis put on iterative experimentation and the need for relatively sparse designs that can be built up piece by piece according to the developing demands of the experimenter, these designs cover only the most standard first-and second-order methods. Although they are aimed for one response variable, they cover those variables reasonably well. Foremost, it provides functions and data types that bring home the bacon for the coding and decoding of factor levels, given that appropriate coding is considered as an indispensable factor of response-surface analysis. Second, it provides parts for generating standard designs and building blocks thereof, as well as examining their variance function. Standard response-surface models provide appropriate summaries. They provide a means of visualizing a fitted response surface. It guides further experimentation, e.g., along with the path of steepest ascent. Most RSM functions take advantage of formula capabilities to provide intuitive and transparent ways of obtaining the needed result[48, 60].

There is commercial software on the market, facilitating the design and analysis of RSM, with JMP (SAS Institute, Inc. 2009) as the most popular software. This study makes use of JMP. This makes it possible to visualize them. These programs generally go beyond RSM capabilities (for instance, more types of designs,

provisions for mixture experiments, and so on); but RSM makes the most important methods available. RSM may go beyond the capacities of these plans in the generality of central-composite designs that it can produce. The destination of this overview of RSM and how its parts may be used to plan and analyze response-surface experiments are discussed[60].

The mechanism of some scientific phenomena is understood sufficiently as they use mathematical models that flow from the physical mechanism. Although some essential statistical problems arise in the building and study of such models, our discussion will be appropriate for the phenomena that are not sufficiently well understood to permit the mechanistic approach. Response surface methodology comprises a group of statistical and mathematical techniques for empirical model building and model exploitation, which are useful for developing, improving, and optimizing processes. The careful design and analysis of the tests reveal that they are concerned with the following: responding or output variables, the levels of some predictors or input variables affecting it, essential applications in design, development, and formulation, new robot design, as well as upgrading existing robotic design. RSM is widely used in the industrialized world, especially in situations where multiple input variables potentially affect performance or qualitative product or process characteristics. These performance measures or quality characteristics are called the response[48, 60]. They are usually measured

on a continuous scale, although correct answers, ratings, and sensory responses are not abnormal. Most real RSM applications include more than one answer. Input variables are sometimes referred to as independent variables, and they are under the control of an engineer or scientist, at least for testing. The field response methodology involves practical strategies for exploring the space of the processor independent variables[48].

The conduct of an experimental investigation seems to be a highly arbitrary and uncertain process. In this thesis, it is supposed that six parameters of experimenters competent in a particular field of robotics are collected, each parameter is locked, all experimenters are presented with the same general robotics problem, and each parameter is asked to submit a plan that could lead to a solution for the problem. For sure, no two parameters would present the same plan[50].

4.1   Which Input Variables Should Be Studied?

A robotic movement reaction was studied. Most investigators would regard six parameters as being essential, but there might be a diversity of opinion about which should be included among other input variables, e.g., the initial rate of addition of the reactant, the ratio of certain parameters, the agitation rate mu, and so on. The similar and perhaps even stronger disagreement might occur in a psychological experiment[44].

When an input variable, such as energy increases, the robot may increase its linear response, such as perceived loudness of noise(when it is varied on a regular scale or, equivalently, when ln is varied on a linear scale). It is simpler to express such a relationship; therefore by first transforming the input into its logarithm. Another input might be related to the response by an inverse square law, suggesting an inverse square transformation, the inverse square root examples can lead to transformations such as the square root, and the reciprocal. A choice of a transformation for a single variable is often called a choice of metric for that variable. More generally, a transformation on the input variables can involve two or more of the original inputs. Suppose, for example, that the amounts and Greek zeta of 12 two nitrogenous fertilizers were being investigated[44]. Rather than employing themselves as the input variables, their sum, the total amount of nitrogenous fertilizer, and their ratio might be used if it were likely that the response relationship could be more simply expressed. In some instances, the theory of dimensionless groups can be used to indicate appropriate transformations of this kind, but usually the best choice of metrics and transformations is not clear-cut and, initially at least, will be subject to conflicting opinion.

4.2 Approximating Response Functions

Good response surface designs have been constructed to perform well based on a particular assumed model. They have also been structured to evaluate the

assumptions of the model being analyzed in order to determine if the experimenter's initial impressions of the robotic system under study match the right underlying relationship which produced the data to be analyzed. Hence the experimenter should think carefully about the goals of a particular experiment and what the anticipated analysis will involve before selecting the design for data collection[44].

Most applications are sequential. It means that at first some ideas are created to figure out which factors or variables are probably important in the response surface study. This usually results in an experiment developed to examine these factors, with a view toward verifying the contribution of the factors to the response as well as to eliminating the unimportant ones[50, 56].

4.3 Objectives and Typical Applications of RSM

Response surface methodology is useful in the solution of many types of robotics.

 In general, these problems can be divided into three categories, as follows:

1. Mapping a Response Surface over a Particular Region of Interest. This process would be typically performed at a specific set of reaction mu and reaction efficiency and another parameter used in a robot. However, it may sometimes be necessary to make some changes to these normal operating levels,.That is, to design a manufacturing robot that meets the criteria. The approximation of the

correct unknown response function over a region around the current operating conditions having a suitable fitted response surface (second-order surface) provides the process engineer with a chance to predict in advance the changes in yield that will emanate from any readjustments to the input variables, namely, time and speed.

2. Optimization of the Response. In the industrial sector, a fundamental problem is determining the conditions that optimize the process. This requires determining the levels of time as well as the speed that lead to maximum efficiency. Then, a second-order model can be employed to approximate the effective response in the context of a narrow region around point B. Based on this approximating response surface, the optimum levels or condition for time and speed could be selected.

3. Choosing Operating Conditions for the Achievement of Specifications or Customer Requirements. In the case of most response surface problems, multiple responses should be simultaneously taken into account[59].

4.4 RSM and the Philosophy of Quality Improvement

During the last few decades, robotics system has become most interested in quality and process improvement. Statistical methods, including statistical process control and design of experiments, play a vital role in this activity. Quality improvement is considered as the most effective when it is obtained early in the product and

process development cycle. It is very difficult, costly, and it is considered inefficient to manufacture a poorly designed robot such as control and electronics, mechanic automotive, and hardware devices, software, robotics. Processes are some examples where experimental design methodology has resulted in shorter design and development time for new products. Also, a robot which is easier to produce has higher reliability, enjoys enhanced field performance, and satisfies or goes beyond goal point and optimization efficiency. In this respect, RSM is considered as an essential branch of experimental design as well as a critical technology in the development of new processes, allowing for the optimization of their performance as well as the improvement of the design and formulation of a new robot. It is frequently an essential concurrent engineering tool, in that product design, process development, quality, manufacturing technology, and operations personnel often work together in a team-work environment to apply RSM. The targets of quality improvement, including a decrease of variability and improved product and operation performance, can often be achieved directly using RSM[59].

4.5 Iterative Nature of the Experimental Learning Process

Faced with so many indeterminacies and uncertainties, one can easily be disappointed in finding a successful outcome of any kind. However, he should keep high morale. Fortunately, practical experimentation is frequently successful. How is DOE involved with a robot? The position seems more promising when we

remember that experimental runs are usually only one part of an iterative sequence and that an investigation strategy should be directed at the overall furthering of knowledge rather than just the success of any single group of tests. Our problem is to organize such matters to reach the right conclusions even though our initial choices of the area of interest, the metrics, the transformations, and stages of the input variables may not all be right. Our strategy must allow any poor initial choices to be rectified as we proceed. The way to success is not unique, although it may seem so.to the first investigator in a study. Thus, it is not the uniqueness of the path that we should try to accomplish, but instead, the probable and rapid convergence of an iterative sequence to the correct conclusions should be the priority. This iterative process of learning by experience can roughly be formalized. It consists mainly of the continuous and repeated use of the sequence[56].

It often happens at the beginning of an investigation that there is preferably a long list of variables..., which could be of importance in terms of their effect. One way to reduce the list to a manageable size is to sit down with the investigator the biologist, robotics, psychologist, etc. and ask him/her to pick out the variables he/she believes to be the most important. To press this too far, however, is dangerous because, not infrequently, a variable initially believed unimportant turns out to have a significant effect. A good compromise is to employ a preliminary

screening design such as a two-level Fractional factorial to pick out worthy variables. In one investigation, for instance, the original list of variables that might have affected the response contained many candidates. Three of these were, after careful thought, eliminated as they turned out to be unimportant. Therefore, they were safely ignored. A 16 run two-level fractional factorial design was run on the remaining eight variables, and four of the eight were designated as probably influential over the ranges studied. Three of these four had already been selected by the investigator as likely to be critical, confirming his judgment. The fourth was unexpected and turned out to be of great importance. Screening designs are often carried out sequentially in small blocks and are very useful when performed in this way[56].

4.6 Empirical Model-Building HOW Stage

When input variables are quantitative, and the experimental error is not too large. It may be more beneficial to attempt to estimate the response function within some area of immediate interest rather than the range covered by the observed responses. In many problems, the form of the real response function is strange and cannot economically be obtained, but may be capable of being locally approximated by a polynomial or some other type of graduating function. Suitable experimental designs for this purpose have been modernized. The fundamentally iterative nature of response surface methodology RSM would ensure that as the investigation

proceeds, it would be possible to learn about the amount of replication needed to achieve sufficient precision. Locating the experimental region of most interest, one can use appropriate scaling and transformations for the input and output variables, and the degree of complexity of an approximating function, and hence of the designs are needed at various stages[56].

Ideally, we like to use the right function to represent the response instead of approximating it by a graduating function. In some problems, we can be sure to achieve useful working mechanistic models which, at least, take account of the main characteristics of the mechanism. These examples are often most naturally expressed via differential equations or other no explicit forms, but modern developments in working out facilities and the theory of nonlinear design and estimation have made it possible to make out with the ensuing problems. A mechanistic model has the following advantages:

It leads to our scientific understanding of the phenomenon under study.

It commonly offers a sounder basis for extrapolation of at least two conditions worthy of further experimental investigation (if the entire ranges of all input variables are not considered).

It is considered to be parsimonious, i.e., frugal in the use of parameters, providing better estimates of the response[59].

Results from fitting mechanistic models have sometimes been disappointing because not enough care has been paid to finding out what an appropriate model form is. It is easy to accumulate data that never ''place the postulated model in jeopardy,'' and so it is common, e.g., in chemical engineering to finding different research groups, each advocating a different model for the same phenomenon and each proffering datum that ''prove'' their claim. In such instances, methods that discriminate between the various candidate models must be used[59].

It sometimes finds, e.g., in investigations of industrial plant processes that a large amount of past operational data is usable. It may then be tempting to think that no experimentation is needed because it ought to be possible to extract information related to the response of interest to changes that have occurred naturally in the input variables. Such investigations are often valued as preliminary studies, but the existence of such data rarely eliminates the need for further planned experimentation. There are several reasons for this[59].

1. Significant input variables affecting the response are not altered.

2. Dealings between the response variable and several input variables may be induced by unrecorded ''lurking'' variables that involve both the reaction and the input variables. These can give a lift to ''nonsense correlations.''

3. Historical operating data often contain gaps and omit important ancillary information.

4.7   Desirable Properties of Response Surface Designs

A review of the literature reveals many experiments design classifications as well as many criteria against which designs have been developed. Indeed, many computer packages offer optimal designs based on particular criteria and input from the user. Particular design criteria and critical issues associated with the computer-generated design of experiments have been discussed[56]. However, it is essential for the reader first to review a set of properties that should be taken into account while choosing a response surface design. Some of the essential characteristics are as follows:

yielding an acceptable fit of the model to the data.

Providing reasonable model parameter estimates.

Providing a proper distribution of prediction variance of the response, Variance throughout the region of interest.

Providing an estimate of "pure" experimental error.

Giving sufficient information to allow for running a fit test.

Checking the homogeneous variance assumption being insensitive (robust) to the existing outliers in the data.

Being robust to errors associated with the control of design levels.

Allowing for the models of increasing order to be constructed sequentially.

Allowing for experiments to be done in blocks.

Being cost-effective.

To help organize our thinking about the characteristics on this list, we can divide the list into several categories[56]. The assumption of items 1– 4 is that the practitioner makes the right assumptions regarding the nature of the underlying relationship between the inputs and the response. It assumes that given the correct model, the goal is to obtain an estimation of model parameters as well as a prediction of new observations, using the model. The reader has been exposed to the notion of prediction variance. Now the importance of stability of prediction variance is discussed[59]. This is often the most common category on which emphasis is placed when selecting a designed experiment. Items 5 and 6 seek to provide ways in which the assumptions of the model can be evaluated. All models are based on some assumptions. The data collected in the experiment are very helpful in that they allow for the evaluation of feedback about the suitability of

these assumptions moreover, items 7 and 8 focus on how the experiment will be affected if something goes wrong[56].

The goal is to design an experiment that can withstand some less-than-ideal outcome and still generate useful results. Item 7 is particularly important given the possible existence of outliers. Items 9 and 10 are aimed at the flexible implementation of the experiment and leveraging of the results as a part of the sequential nature of many experiential learning cycles. Finally, item 11 is a reminder concerning the existence of cost constraints for experiments. More extensive experiments can often lead to the improved characteristics of the first ten items though they increase the total cost of the experiment, preventing those resources from being used for other purposes. The introduction of the eleven-characteristic list at this point is aimed at achieving multiple goals. The reader needs to be reminded that designing an experiment is not necessarily secure, given that it is a complex undertaking. The design of the experiment should involve striking a balance among the multiple objectives, not just focusing on a single characteristic. If the optimization of a product or process involves taking int account multiple aspects, designing an experiment usually involves balancing multiple objectives. Indeed, some of 11 items may be important, and yet the researcher may not be completely aware of the relative importance of those items. Some items do conflict with each other. As a result, there are trade-offs that almost

always exist when one chooses an appropriate design. For example, good choices of where to allocate are based on the assumption that the model selected is correct. The selection is of enormous importance when we are trying to evaluate the correctness of the model[59].

Similarly, assuming that the model is correct, protecting against model misspecification would lead to different choices of designs. Another potential trade-off has to do with whether we think that the implementation of the experiment will run smoothly or whether there might be some complications. Finally, there must be cost trade-off with most of the other categories: A more massive experiment can help improve the results for most of the first four categories, yet at the cost of using additional resources[59].

4.8   Experiment With Computer Model

We usually think of applying RSM to a physical process, such as a chemical process in manufacturing and machining. However, RSM can be also successfully applied to computer simulation models of physical systems. Computer models are becoming increasingly common, and they can be used as proxies for many complex processes that are difficult or expensive to manipulate. In some applications, there are restrictions on what conditions can be explored with a physical experiment because of cost, safety, or regulations. In other cases, having a computer model (or code) allows for much more rapid exploration of alternatives,

development of prototypes and new products, bringing increasing competitive advantage and speed to market. Computer models can often yield a large number of inputs and result in multiple responses, which can be either scalar (a single value) or functional (a collection of values connected over time or space). Hence the design of experiments has a vital role to play in the selection of a preferred set of input combinations to be explored. In such computer modeling applications, the role of RSM is different, as the data obtained from runs of the computer model can be used to build a model of the system being modeled by the computer simulation, which is called a metamodel or emulator. Given that for some computer models, obtaining one observation from the code may take considerable computer runtime, the characteristics of the system can be understood from exploring the estimated model,with optimization carried out on the metamodel. It is presumed that if the computer simulation model is deemed as a faithful representation of the real system, it follows that the RSM optimization will determine the optimal conditions for the actual organization[59]. Simulation models can be divided into two categories: stochastic and deterministic. In the former, the output responses are random variables. Examples are systems simulations including the factory planning and scheduling models employed in the semiconductor industry as well as traffic flow simulators used by civil engineers. Another example is Monte Carlo simulations, sampling from probability distributions to study complex mathematical phenomena that lack direct analytical solutions. In deterministic

simulation models, the output responses are not random variables; they are entirely deterministic quantities whose values are determined by the (often extremely complex) mathematical models upon which the computer model is established. Today, deterministic simulation models are used by many engineers and scientists as computer-based design tools[59]. Typical examples are circuit simulators used for designing electronic circuits and semiconductor devices, finite element analysis models employed for mechanical and structural invention, and computational models for physical phenomena including the robotics system. Before discussing what designs and analysis to use when studying computer models, it is helpful to compare data obtained from physical experiments versus those obtained from computer models. Information from both types of experiments can be expensive: for physical experiments, the cost emanates from the frame-up and running the experiment as well as from the quantification of the response values. For computer experiments, the development of the codes takes intensive labor and time, often requiring subject matter expertise. However, in this study it is assumed that the computer model is already available, and we use it to explore the underlying relationship of interest. Obtaining the data itself is also often expensive, and due to the code complexity, it may need enormous amounts of computer power and runtime to gain results even for a moderate number of input combinations. Hence, for both types of experiments, the ability to obtain a reasonable estimate of the relationship from small to moderate amounts of data is essential[59]. A key

difference between physical and computer experiments is the range over which experimentation may take place. For physical experiments, the goal is often to restrict the region of interest to a local region where a low-order polynomial well characterizes the relationship between inputs and response. However, for computer experiments, an experiment might be sought to characterize the relationship over much larger regions of the input space, since the place where possible values may be obtained in the region of operability s not subject to any limitation. The boundaries for where observations can be obtained from a computer code are often dictated by changing science or engineering mechanisms that have been built into the code[59]. Data from physical experiments represent observations from the actual process and typically are thought to be unbiased, relying a good measurement device for measuring the response. However, due to imperfect measurement devices as well as the natural variability of the process, we typically do not expect to observe identical values for replicate runs of the experiment. Accordingly, our statistical models for physical processing are defined with an error term to capture and estimate these differences. On the other hand, data from computer models are the result of a human's characterization of the relationship, based on the best available science and engineering of the mechanisms driving the process. Depending on the maturity of the code and the depth of underlying knowledge, these codes can range from very accurate to just coarse representations of the main mechanisms. Consequently, the user should know that the results of an

experiment used to explore a computer model are only as good as the underlying knowledge available to build the code. Hence, bias or systematic differences between the code and the exact process that it is intended to represent are possible. If the computer model is deterministic, then repeated runs with the same inputs will result in identical values, making this feature undesirable for this type of computer experiments. It is helpful to have an estimated model or emulator for the code interpolation between observed points. This makes sense since we believe that the results from the code represent the best available knowledge of the underlying process, and we use this information directly. The emulator is aimed at allowing for the estimation of other locations in the input space that have not been directly evaluated. If the computer model is stochastic, then it may be of value to obtain replicates to understand the natural variability[50]. Another key aspect to consider when comparing physical experiments and computer models is the nature of the underlying relationship being characterized. As already mentioned throughout previous chapters, RSM is predicated on the assumption that many physical systems are smooth and continuous and can be well approximated, at least in the region of interest, by low-order polynomials. This assumption might not be appropriate for many complex computer codes[52]. For example, sometimes polynomials of higher-order than the usual quadratic response surface models are used. Johnson et al. (2010) compare the performance of different designs when an analysis uses higher-order polynomials. Because different options are sometimes

needed to describe the underlying relationship, strategies for designed experiments based on different models may require some specialized techniques. Since we have an exact value for the computer code results at that location, the uncertainty for the emulator is zero at that point, and the uncertainty bands around the function shrink to zero. Note that the uncertainty for the estimated curve increases as we move away from any observation. As a result, a strategy to minimize the worst-case prediction variance of a deterministic computer code anywhere in the design space is to try to have points as spread out as possible throughout the design space. This will minimize the distance between any new location where we wish to predict and an observed observation[52]. Using computer experiments, this assumption of sparsity may be more local: Namely, in some region of the design space, only a small number of factors are actively influencing the response. However, in a computer experiment, as we move throughout the design space, different subsets of the input factors play a role in influencing the response. This is different from the physical experiment, in which it is often reasonable to assume that the same subset of factors is active[56].

4.9   Iterative Nature of The Experimental Learning Process

We used Lévy Walk method for stochastic movement since many organisms such as bacteria, animals, and human search in practically observable in an unknown environment via Lévy Walk. The response surface methodology is a practical

modeling method that defines the relation between various useful parameters and their respective responses with multiple favored criteria while determining the importance of useful parameters upon the Coupled responses. The experimental costs and the variability around the target are minimized when replacing the target value with the performance value is decreased, using the response surface methodology[56].

The cases where resources do not occur in well-defined patches, these models are not directly applicable, taking on more common spatial distributions. Random search theory is more appropriately used in the optimal robot on unique landscapes have given that robot' last data encounter relies more strongly on visual or vibratory clues than the elapsed time when deciding when to quit a searching area. Some animals use sensory cues to determine search mode. It should be noted that one difficult problem is to detect discrete behavioral status from useful movement data. Fortunately, considerable progress has been made in this area[53, 54].

The satisfaction of the self-similarity condition yields provided fractal dimension (D); it is possible to assemble mathematical objects. It presents an intimate connection between such the fractal dimensionality and classified behavior in space or time of these processes. A robot hops instantly from the start to the end of each step length, continually moving along each step length respectively. While Lévy flights model movement of the salutatory, Lévy Walk, simulate cruise

movement. Most models of the Lévy Walk(those considered in this literature) are technically truncated Lévy Walks: Achieving a resource diagnosis or when the maximum time of the simulation elapses, Step lengths are terminated. Fortunately, truncated Lévy Walks retain many significant features of the Lévy Walks, including general properties of the mean-square displacement. The criticisms against the Lévy Walk concept are addressed in our model[58].

5   Results and Discussions

5.1   Simulated environment characteristics
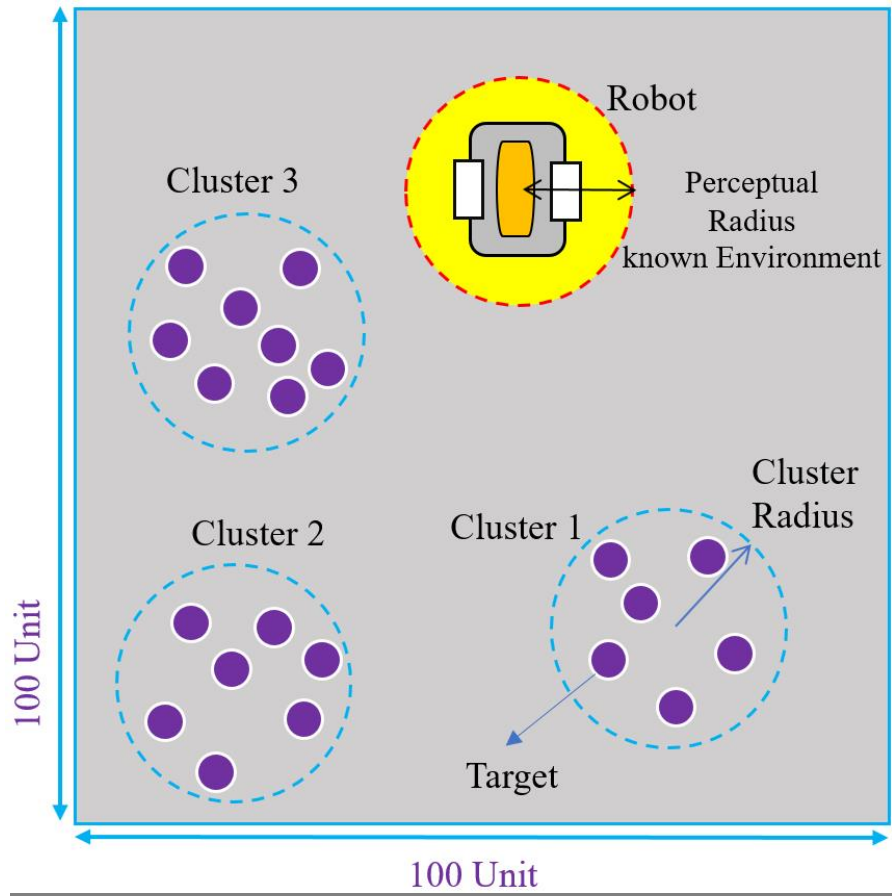
Figure 5-1 Condition of parameters in the target environment

In this study, simulations are used to assess the relative efficiencies of different Lévy walk stochastic search strategies in a range of environments. These simulations helped to identify the optimal parameters for a searcher in a given type of environment. Such information will be useful for future studies, which will use

real robots (as opposed to simulating ones). All of the simulations in the case study were performed in NetLogo simulation[2, 5].

In our simulations, an environment is a rectangular region, 100 units width and 100 units height (units are arbitrary). The spatial coordinates of targets and robots are recorded as floating real point numbers, and hence location is substantially continuous. Targets are distributed across the environment according to a Neyman-Scott spatial point process. This is a useful model for targets that occur in clumps. There are two parameters for the Neyman-Scott process: the total number of targets in the environment and the radius of each cluster. High cluster radius corresponds to the relatively homogeneous distribution of targets, while a low cluster radius corresponds to highly aggregated targets[5]. The technical details for the simulated environments are the same as those already described where boundary conditions, how edge effects are controlled for, and how the Neyman-Scott process is used to generate locations for targets are explained.

Our simulations model is destructive search, which means that targets are eliminated when a robot finds them. This is an appropriate model for applications like contaminant clean-up or resource harvesting. An environment in our simulations can adequately be described by two parameters the total number of resources and the cluster radius. These two parameters allowed for examining a variety of different situations that a robot could encounter, (e.g., a highly clumped,

target-rich environment, or a highly clumped, poor target environment, or a very homogeneous, target-rich environment, etc.). Note that a new environment was generated for each run of the simulation. The stochastic is so even if these parameters are held constant, the generated environment will not be the same[5].

The simulated robot moves through the environment via a Lévy walk. The parameter μ determines the correct type of Lévy walk. The robot travels at speed. It has no memory and no information about targets outside of their detection radius. If a target falls within its detection radius, it truncates the current step of the random walk, moves directly to the target, and removes it. A robot is mainly characterized by four parameters: its detection radius, and its Lévy walk parameter μ and speed and number of robot. The general procedure for the RSM is explained in detail. In particular, it is made clear if RSM is used to hone-in on optional parameter combinations or if it is just used for the pre-determined parameter values. Value stream mapping, as well as Full Factorial Design rotatable design, are explained. In this study, the functional optimal procedure setup that optimizes the parameters of the design is the quadratic model of RSM incorporating the Full Factorial Designs. This design is employed by considering three levels and six factors[5].

Table 3 parameters in the target environment

	Variable	Distribution/Experiment		
	Field size	100*100 Unit		
	#Resources	100	1000	DOE*
Environment	Cluster Radius	4	64	DOE
	#Clusters	GD*($\bar{c}$=5)		
	μ (Lévy Walk)	1.3	2.9	DOE
Robot	Perceptual Radius	1	10	DOE
	Speed	1	10	DOE
	#Robots	1	11	DOE

*GD: Gaussian Distribution

*DOE: Design Of Experiment

The regressive analysis improves the relevant mathematical models, and then these models are used to measure its similar accuracy, followed by an examination through analysis of variance (ANOVA)[48]. The quadratic model is usually sufficient for industrial applications. For n-factors, the complete quadratic model is shown in the following equation:

$$Y = \beta_0 + \sum_{j=1}^{K} \beta_j X_j + \sum_{i<j=1}^{K} \sum \beta_{ij} X_i X_j + \sum_{j=1}^{k} \beta_{jj} X_j^2 + \varepsilon$$

Equation 1

Where Y is the predicted response and X_i are the coded form of the input variables, which in this study represent the search efficiency and six search parameters (as reported respectively. The term β_0 is the intercept term, β_i are the long terms, β_{ii} are the squared terms, and β_{ij} are the interaction terms between the input variables. The Full Factorial Design rotatable design was used to evaluate the six factors above. Three levels of points being were analyzed, with the range being determined based on extensive screening experiments.

According to the proposed model, six factors were used, each at the predefined level and range, as shown in table-3. Thus, two dimensions of the surface will be as follows: Resource number, Cluster radius, μ(Lévy Walk), Robot perception radius, Robot speed, and number robot, and the response y represents the search efficiency. The total number of experimental combinations to be conducted is based on the concept of Full Factorial Design. Given the use of Full Factorial Design, a central point had to be considered and measured for each factor. So, each factor is divided into three levels: a min, a max, and a central point. Therefore, we have 3^6=729 cells for full factorial design. Deciding the central point can be either a mathematical process, or it can be a predefined number set by a specialist.

The predefined number should be approximately in the center of the numerical range of each factor levels so that the best results are obtained[54]. The response variable is continuous, and so are our factors in the real world. As a correction for

this categorization of continuous variables (in which we know may reduce the hypothesis tests power) and given the ability of simulation of Lévy Walk with the computer, replication for each cell was selected way above the ordinal sample size and set it to 10. Thus, the data set has 7290 rows and six measures at the start of the analysis. The linkage function is the polynomial function of all factors and their squares. The FFD analysis was done using SAS JMP; analysis steps are discussed[54].

The response studied from the experiments was the stochastic movement. The qualitative result obtained from analysis indicates the complete design matrix of the experiments, performed together with the obtained results. The responses were used to develop an empirical model for the stochastic movement robot via the Lévy Walk method. After executing the experimental design, analyses of the experimental data were performed using ANOVA at a 5% level of significance and the P-value. The P-value a simple arithmetical method that sorts the components of variation in a given set of data and provides the test for significance.

Where Y is the predictable reaction or dependent variable, Xi and Xj are the independent variables, while bi and bj are constants. In this situation, the quantity of independent factors is four, and therefore, k=6: Eq. (1) becomes Eq. (2):

$$Yu=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_5x_5+\beta_{12}x_1x_2+\beta_{13}x_1x_3+\beta_{14}x_1x_4+\beta_{15}x_1x_5+\beta_{23}x_2x_3+\beta_{24}x_2$$

$$x_4+\beta_{25}x_2x_5+\beta_{34}x_3x_4+\beta_{35}x_3x_5+\beta_{45}x_4x_5+\beta_{11}x_{11}+\beta_{22}x_{22}+\beta_{33}x_{33}+\beta_{44}x_{44}+\beta_{66}x_{66}$$

$$(Eq.\ 2)$$

Where Y is the predictable reaction, and X_1, X_2, X_3, X_4, and X_6 are the coded type of the input variables. The term β0 is the intercept term; β_1, β_2, β_3, and β_6 are the linear terms; β_{11}, β_{22}, β_{33} and β_{66} are the squared terms; β_{12}, β_{13}, β_{14}, $\beta_{15,}$ β_{21}, β_{23}, β_{24} β_{25} and β_{56} are the interaction terms between the eight variables. The focal composite rotatable design was utilized to assess the previously stated eight components. Their levels of points were investigated, with the range being resolved using huge screening analyses and writing survey.
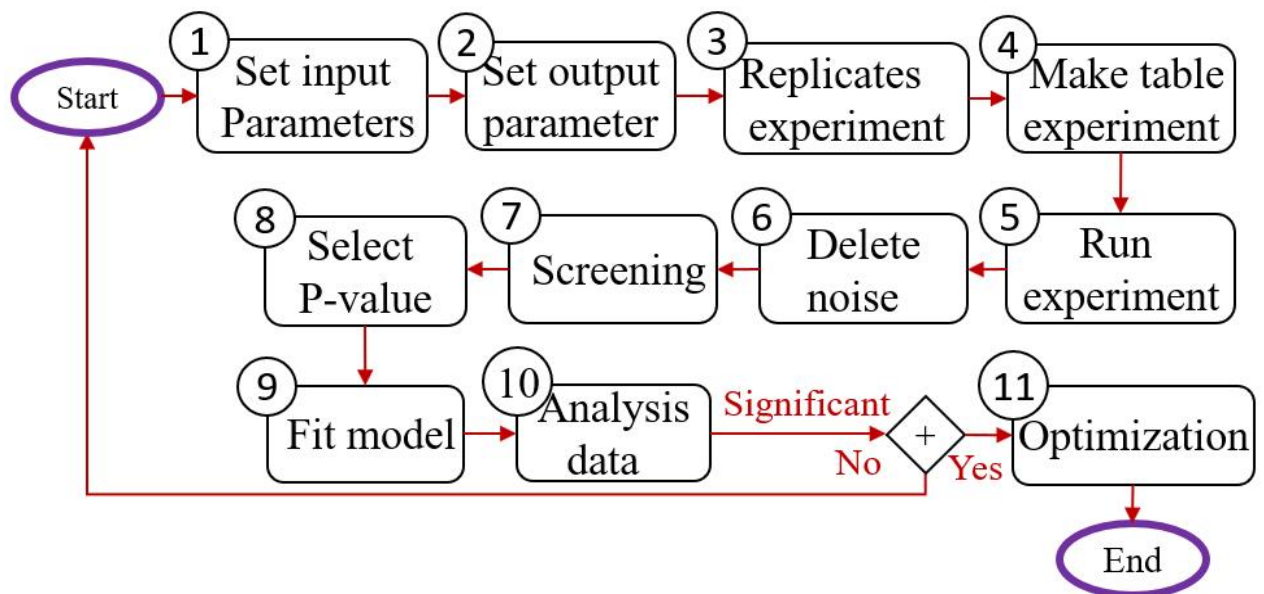
Figure 5-2 Flow chart of statistical

The first step selects the input parameter.

Table 4 Independent variables and their coded and actual values and *α=1*.

Parameter to be studied	Independent Variable	Symbol	Actual Levels		
			−1	0	+1
Environment	#Resource	Nr	100	550	1000
	Cluster Radius	Rc	4	34	64
Robot	μ (Lévy Walk)	μ	1.3	2.1	2.9
	Perceptual Radius	Rp	1	5.5	11
	Speed	V	1	6	11
	#Robot	Nf	1	5	11

The second step selects the out-put parameter.

Table 5 Responses to Be Measured.

Responses (Effects)		Goals for Individual Responses
Y1	Count Source	To Achieve Maximum Recounter
Y2	Distance	To Achieve Minimum Distance

$$Ef = \frac{Y1}{Y2} = \frac{\# \text{ Obtained Resources}}{\text{Moved Distance}}$$

The third step replication experiment and make a table for experiment.

The aggregate number of test combination ought to be determined, taking into account the comparable idea by applying Eq. (2) Six components in full factorial with three levels brought about 729 exploratory runs, where k spoke to the number of independent variables or elements chosen. Six focus point tests were used to assess the unadulterated mistake enlargement, using eight hubs and 729 factorial trial runs.

$$\text{Number of experiments} = \text{level}^{\text{factor}}$$

$$\text{Number of experiments} = 3^6 = 729$$

In this thesis use stochastic movement and need replication for the experiment.

$$\text{Replication experiment} = 10$$

$$\text{Number of experiments} = 3^6 = 729*10 = 7290$$

The reaction was investigated through the stochastic movement. The outcome was obtained subjectively. Table 4 shows the complete outline network of the investigations, performed together with the achieved results. The reactions were utilized to build up an observational model for the stochastic development of robot by the toll walk technique. After testing out the configuration, elucidations, and examination of the test information was resolved to utilize ANOVA at a 5% level of centrality, using the P-value. The P-value is an arithmetical technique that breaks down the variance parts in a given arrangement of information, giving the

test for significance. Stochastically Optimization of efficiency using response surface methodology Examinations of the best variation of the medium of response resource, span for search, and addition μ in the combined states of stochastic was performed to expand the essential vitality of separation amid the development robot technique. Eq. 1 shows an observational relationship, depicted as a numerical model for the measurement fractal (μ) and the test variables in a coding unit. Indeed, the exact model was created in Eq. 1 by applying the various relapse strategy which was fitted to the exploratory results. Dimension fractal (μ) model concurred with the empirical results.

5.2 Screening

Testing many responses to the effects of factors can be challenging. Response Screening automates the process of conducting tests across. Test results and summary statistics are presented in data tables, rather than reports, to enable data exploration. Rate approach guards against incorrect declarations of significance. Plots of p-values are scaled, making them easily interpretable. Because large scale data sets are often Response Screening presents methods that address irregularly distributed and missing data.When having many observations, even differences that are of no practical interest can be statistically significant. Response Screening presents tests of practical difference, where specify the difference that is interested in detecting. For this purpose, Response Screening presents equivalence tests.

Table 6. Screening for Ef

Term	Contrast	Lenth t-Ratio	Individual p-Value	Simultaneous p-Value
Nr	0.100638	87.26	<.0001*	<.0001*
Pr	0.061825	53.61	<.0001*	<.0001*
Cr	0.046995	40.75	<.0001*	<.0001*
Nf	0.043091	37.36	<.0001*	<.0001*
V	0.039568	34.31	<.0001*	<.0001*
µ	-0.010449	-9.06	<.0001*	<.0001*
Nr*Nr	-0.019633	-17.02	<.0001*	<.0001*
Nr*Pr	0.034162	29.62	<.0001*	<.0001*
Pr*Pr	-0.051359	-44.53	<.0001*	<.0001*
Nr*Cr	0.032986	28.60	<.0001*	<.0001*
Pr*Cr	0.029460	25.54	<.0001*	<.0001*
Cr*Cr	-0.001147	-0.99	0.3246	1.0000
Nr*Nf	0.020723	17.97	<.0001*	<.0001*
Pr*Nf	0.022284	19.32	<.0001*	<.0001*
Cr*Nf	0.000414	0.36	0.7274	1.0000
Nf*Nf	-0.014011	-12.15	<.0001*	<.0001*
Nr*V	0.033088	28.69	<.0001*	<.0001*
Pr*V	0.039605	34.34	<.0001*	<.0001*
Cr*V	-0.012301	-10.67	<.0001*	<.0001*
Nf*V	-0.003580	-3.10	0.0022*	1.0000
V*V	-0.014413	-12.50	<.0001*	<.0001*
Nr*µ	-0.004463	-3.87	0.0002*	0.5701
Pr*µ	0.000186	0.16	0.8751	1.0000
Cr*µ	0.001481	1.28	0.2014	1.0000
Nf*µ	0.000120	0.10	0.9172	1.0000
V*µ	-0.007227	-6.27	<.0001*	<.0001*
µ*µ	-0.085621	-74.24	<.0001*	<.0001*
Nr*Pr*Cr	0.003662	3.18	0.0018*	1.0000
Nr*Pr*Nf	0.003522	3.05	0.0027*	1.0000
Nr*Cr*Nf	0.000878	0.76	0.4516	1.0000
Pr*Cr*Nf	0.001050	0.91	0.3703	1.0000
Nr*Pr*V	0.004656	4.04	<.0001*	0.3416
Nr*Cr*V	-0.002169	-1.88	0.0593	1.0000
Pr*Cr*V	-0.001167	-1.01	0.3157	1.0000
Nr*Nf*V	0.002140	1.86	0.0624	1.0000
Pr*Nf*V	0.000913	0.79	0.4325	1.0000
Cr*Nf*V	-0.000874	-0.76	0.4530	1.0000
Nr*Pr*µ	0.000299	0.26	0.8010	1.0000
Nr*Cr*µ	0.001462	1.27	0.2078	1.0000
Pr*Cr*µ	-0.001871	-1.62	0.1009	1.0000
Nr*Nf*µ	-0.000685	-0.59	0.5591	1.0000
Pr*Nf*µ	0.000171	0.15	0.8839	1.0000
Cr*Nf*µ	-0.000621	-0.54	0.6012	1.0000
Nr*V*µ	0.000696	0.60	0.5529	1.0000
Pr*V*µ	0.000306	0.27	0.7962	1.0000
Cr*V*µ	0.001367	1.19	0.2385	1.0000
Nf*V*µ	-0.002154	-1.87	0.0611	1.0000

Table 7. Select P-value Screening for Ef

Contrasts						
Term	Contrast			Lenth t-Ratio	Individual p-Value	Simultaneous p-Value
Nr	0.100638			87.26	<.0001*	<.0001*
Pr	0.061825			53.61	<.0001*	<.0001*
Cr	0.046995			40.75	<.0001*	<.0001*
Nf	0.043091			37.36	<.0001*	<.0001*
Pr*V	0.039605			34.34	<.0001*	<.0001*
V	0.039568			34.31	<.0001*	<.0001*
Nr*Pr	0.034162			29.62	<.0001*	<.0001*
Nr*V	0.033088			28.69	<.0001*	<.0001*
Nr*Cr	0.032986			28.60	<.0001*	<.0001*
Pr*Cr	0.029460			25.54	<.0001*	<.0001*
Pr*Nf	0.022284			19.32	<.0001*	<.0001*
Nr*Nf	0.020723			17.97	<.0001*	<.0001*
Nr*Pr*V	0.004656			4.04	<.0001*	0.3416
Nr*Pr*Cr	0.003662			3.18	0.0018*	1.0000
Nr*Pr*Nf	0.003522			3.05	0.0027*	1.0000
Nr*Nf*V	0.002140			1.86	0.0624	1.0000
Cr*μ	0.001481			1.28	0.2014	1.0000
Nr*Cr*μ	0.001462			1.27	0.2078	1.0000
Cr*V*μ	0.001367			1.19	0.2385	1.0000
Pr*Cr*Nf	0.001050			0.91	0.3703	1.0000
Pr*Nf*V	0.000913			0.79	0.4325	1.0000
Nr*Cr*Nf	0.000878			0.76	0.4516	1.0000
Nr*V*μ	0.000696			0.60	0.5529	1.0000
Cr*Nf	0.000414			0.36	0.7274	1.0000
Pr*V*μ	0.000306			0.27	0.7962	1.0000
Nr*Pr*μ	0.000299			0.26	0.8010	1.0000
Pr*μ	0.000186			0.16	0.8751	1.0000
Pr*Nf*μ	0.000171			0.15	0.8839	1.0000
Nf*μ	0.000120			0.10	0.9172	1.0000
Cr*Nf*μ	-0.000621			-0.54	0.6012	1.0000
Nr*Nf*μ	-0.000685			-0.59	0.5591	1.0000
Cr*Nf*V	-0.000874			-0.76	0.4530	1.0000
Cr*Cr	-0.001147			-0.99	0.3246	1.0000
Pr*Cr*V	-0.001167			-1.01	0.3157	1.0000
Pr*Cr*μ	-0.001871			-1.62	0.1009	1.0000
Nf*V*μ	-0.002154			-1.87	0.0611	1.0000
Nr*Cr*V	-0.002169			-1.88	0.0593	1.0000
Nf*V	-0.003580			-3.10	0.0022*	1.0000
Nr*μ	-0.004463			-3.87	0.0002*	0.5701
V*μ	-0.007227			-6.27	<.0001*	<.0001*
μ	-0.010449			-9.06	<.0001*	<.0001*
Cr*V	-0.012301			-10.67	<.0001*	<.0001*
Nf*Nf	-0.014011			-12.15	<.0001*	<.0001*
V*V	-0.014413			-12.50	<.0001*	<.0001*
Nr*Nr	-0.019633			-17.02	<.0001*	<.0001*
Pr*Pr	-0.051359			-44.53	<.0001*	<.0001*
μ*μ	-0.085621			-74.24	<.0001*	<.0001*

Table 8. Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2128461	0.006702	31.76	<.0001*
Nr	0.0002739	3.217e-6	85.14	<.0001*
Pr	0.0168267	0.000322	52.30	<.0001*
Cr	0.0019186	4.826e-5	39.76	<.0001*
Nf	0.010555	0.00029	36.45	<.0001*
V	0.0107691	0.000322	33.47	<.0001*
μ	-0.015996	0.00181	-8.84	<.0001*
(Nr)*(Nr)	-2.057e-7	1.238e-8	-16.61	<.0001*
(Nr)*(Pr)	0.0000253	8.756e-7	28.90	<.0001*
(Pr)*(Pr)	-0.00538	0.000124	-43.45	<.0001*
(Nr)*(Cr)	3.6651e-6	1.313e-7	27.90	<.0001*
(Pr)*(Cr)	0.0003273	1.313e-5	24.92	<.0001*
(Nr)*(Nf)	1.3815e-5	7.881e-7	17.53	<.0001*
(Pr)*(Nf)	0.0014856	7.881e-5	18.85	<.0001*
(Nf)*(Nf)	-0.001189	0.0001	-11.85	<.0001*
(Nr)*(V)	0.0000245	8.756e-7	27.99	<.0001*
(Pr)*(V)	0.0029337	8.756e-5	33.50	<.0001*
(Cr)*(V)	-0.000137	1.313e-5	-10.41	<.0001*
(Nf)*(V)	-0.000239	7.881e-5	-3.03	0.0025*
(V)*(V)	-0.00151	0.000124	-12.19	<.0001*
(Nr)*(μ)	-1.859e-5	4.925e-6	-3.78	0.0002*
(V)*(μ)	-0.003011	0.000493	-6.11	<.0001*
(μ)*(μ)	-0.283797	0.003918	-72.43	<.0001*

$$Y = \beta_0 + \sum_{j=1}^{K} \beta_j X_j + \sum_{i<j=1}^{K} \sum \beta_{ij} X_i X_j + \sum_{j=1}^{k} \beta_{jj} X_j^2 + \varepsilon$$

Make the mathematic model:

Y=0.21+Nr (0.00027) +Pr (0.016) +Cr (0.0019) +………. +ε
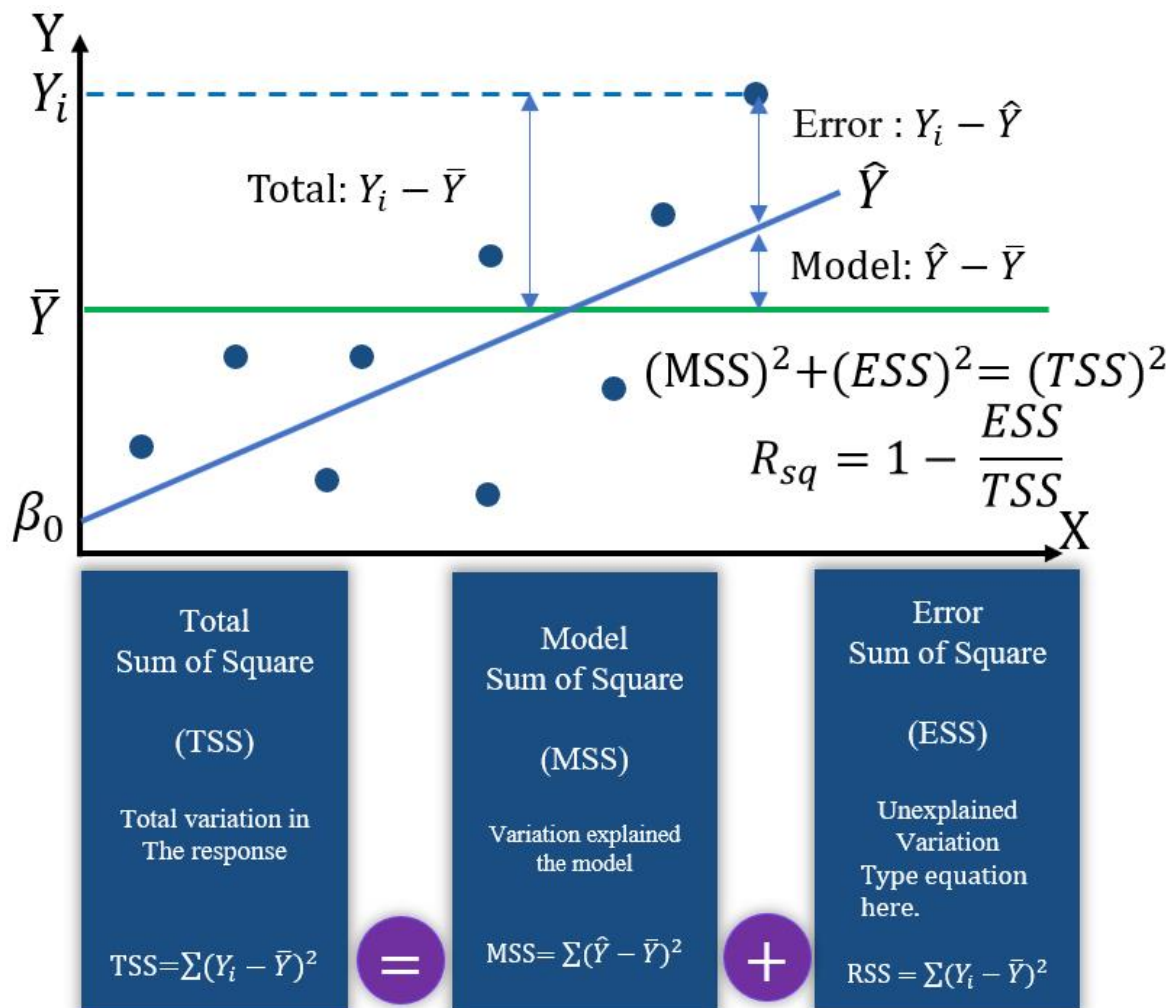
5.3   Analysis of Variance

**Figure 5-3 Analysis data calculation of sum of square error.**

Often an experimenter is interested in whether individual factors or interactions among factors have a significant effect on a response. The most widely-used analytic method is the analysis of variance (ANOVA), which can be used to analyze data collected from many types of experimental designs, including those previously described. Analysis of variance is used to analyze experimentally collected data to test the differences between the group means for more than two groups. ANOVA works by partitioning the observed variance into that which can be explained (based on the data and an associated regression model) and that which cannot be explained. Using sum-of-squares decomposition and statistical tests comparing the explained and unexplained variance, one can determine the significance of model terms (whether they are single main effects or interaction effects). ANOVA is based on the following three assumptions: the response variable is normally distributed, each group has equal variance (i.e., homoscedasticity) and observations are independent.

Table 9 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	22	272.62725	12.3921	1216.512
Error	7267	74.02620	0.0102	Prob > F
C. Total	7289	346.65345		<.0001*

Table 10 Summary of Fit

|	
R Square	0.786455
R Square Adj	0.785808
Root Mean Square Error	0.100929
Mean of Response	0.348557
Observations (or Sum Wgts)	7290
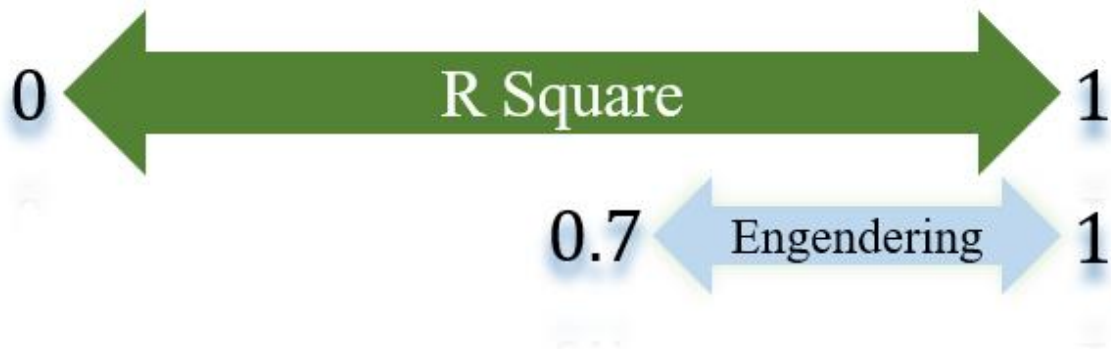
$$R^2 \rightarrow 1 \text{ Best model fit}$$

It should be noted that truly customarily distributed data are rarely seen in practice, and that ANOVA can still provide useful information with deviations from the normality assumption. Additionally, the most straightforward use of ANOVA requires equal numbers of observations at each factor-level, using Type I sum-of-squares. Type II and III sum-of-squares can be used with unequal numbers of factor-level observations. While ANOVA is relatively independent of the

experimental design, alternative design of experiment techniques, such as the response surface methodology (RSM) use experimental designs that are complementary to the analysis methods.
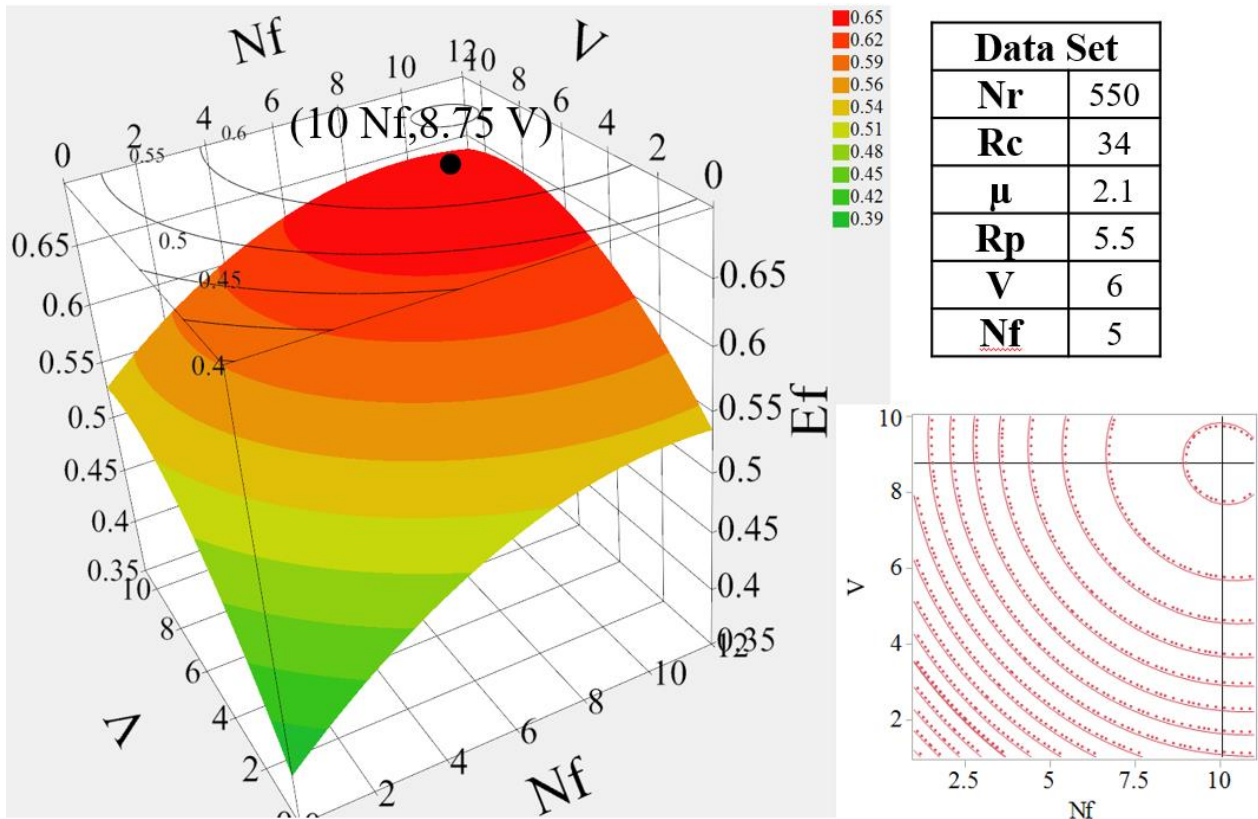
**Figure 5-4 Surface Plot Between Ef and V and Nf.**

Data set	
Nr	550
Rc	34
μ	2.1
Rp	5.5
V	6
Nf	5
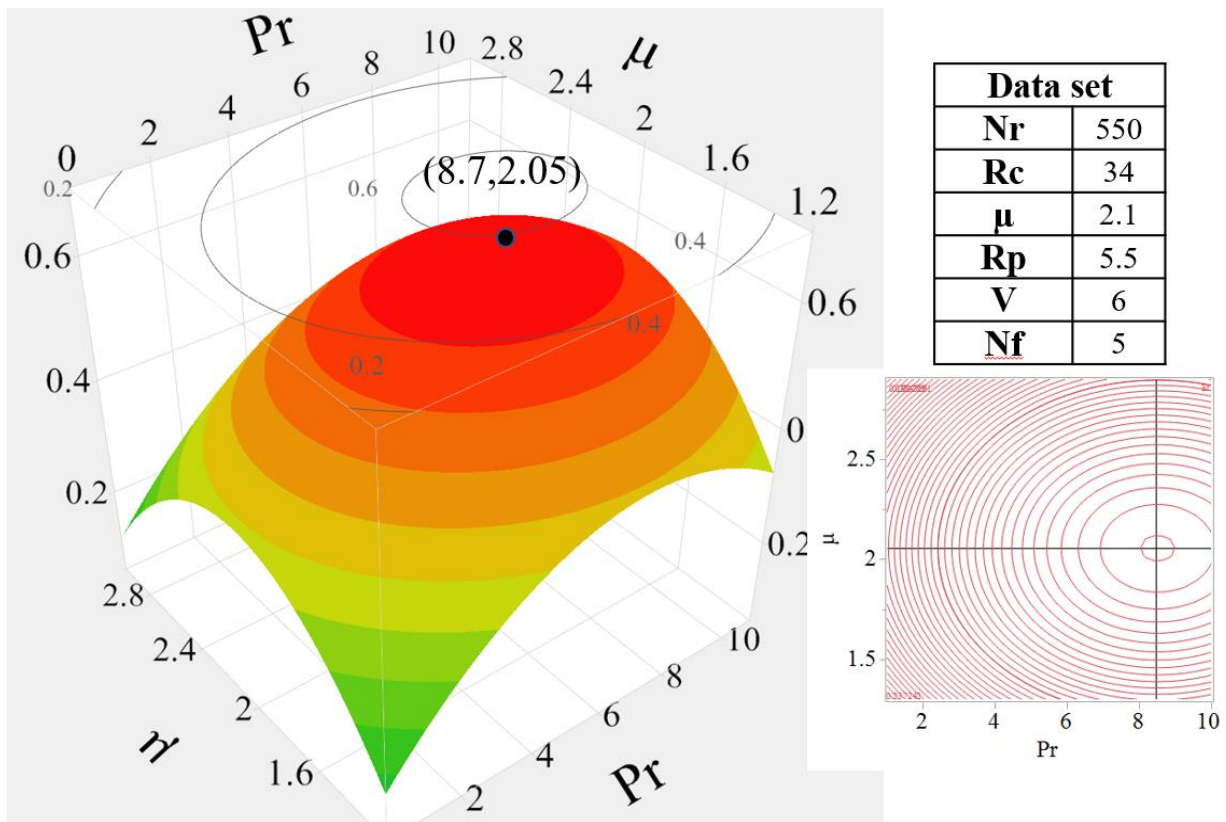
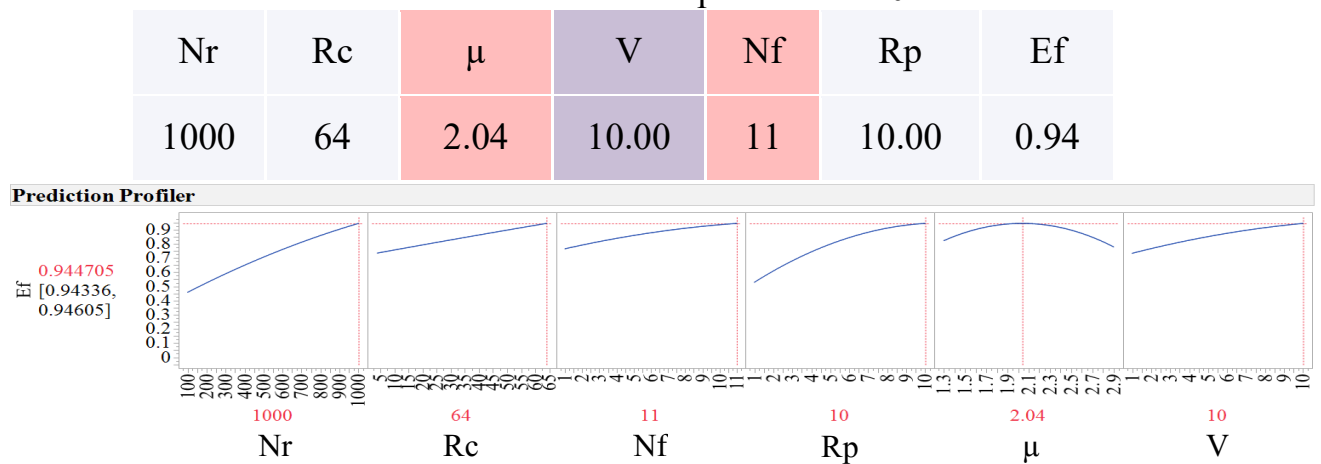**Figure 5-5 Surface Plot Between Ef and μ and Pr.**

A surface plot is a three-dimensional plot with efficiency and dependent variables represented by a smooth surface. The initial Surface Plot report shows the surface plot, the Independent Variables controls, the appearance controls, and the Dependent Variables controls. The Surface Plot platform creates a stand-alone report that contains a surface plot for formulas. The formulas can be formula columns in a data table or mathematical formulas that do not involve any data points. The Surface Profiler option in RSM fitting platforms produces a surface plot for the fitted model in the existing platform report.

5.4 Comparison between the Design Of Experiment (DOF) and One Factor At the Time (OFAT).

DOE: number of the experiment: 7290

Nr	Rc	μ	V	Nf	Rp	Ef
1000	64	2.05	9.66	11	8.04	0.88

| Nr | Rc | Nf | Rp | μ | V |
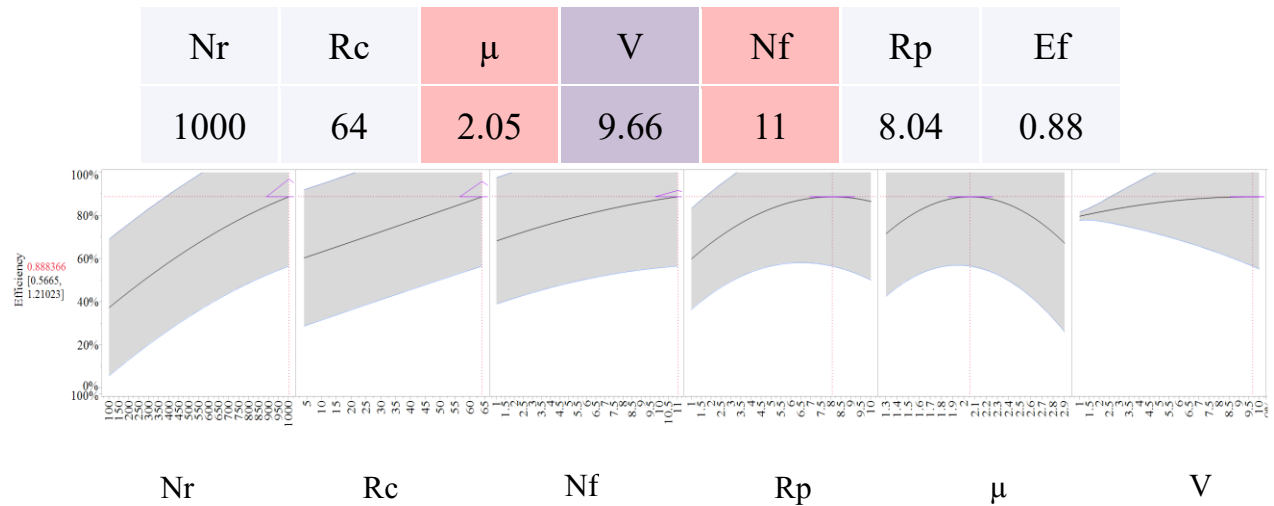
OFAT: number of the experiment: ~10^7

Nr	Rc	μ	V	Nf	Rp	Ef
1000	64	2.04	10.00	11	10.00	0.94

Prediction Profiler

If used DOE minimum experiment and cost.

5.5 Optimization

Table 11. Optimization

R	C	F	PR	MU	S	Efficiency
1000	64	11	8.04	2.05	9.66	0.88
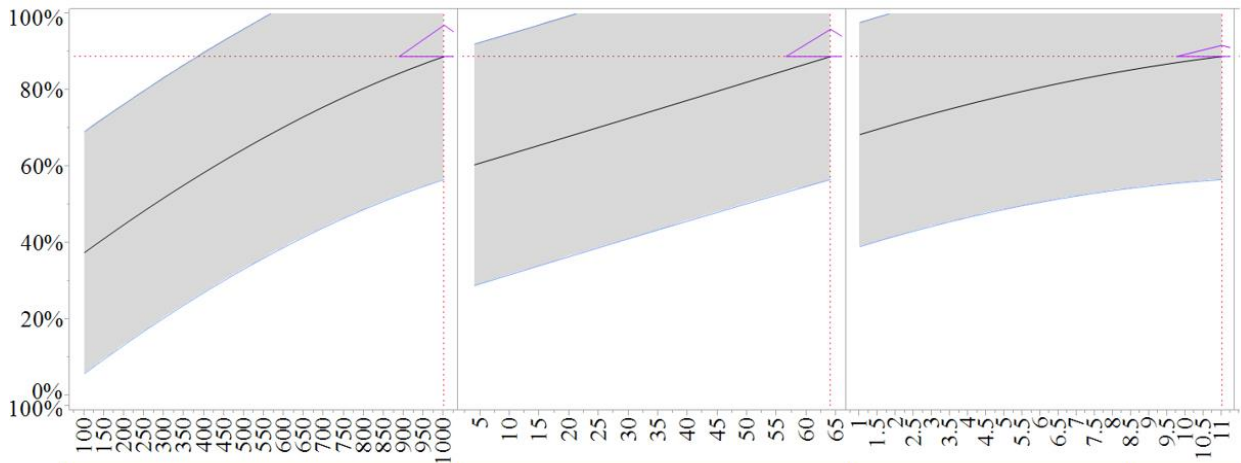
Nr	Rc	Nf	Ef
1000	64	11	0.88
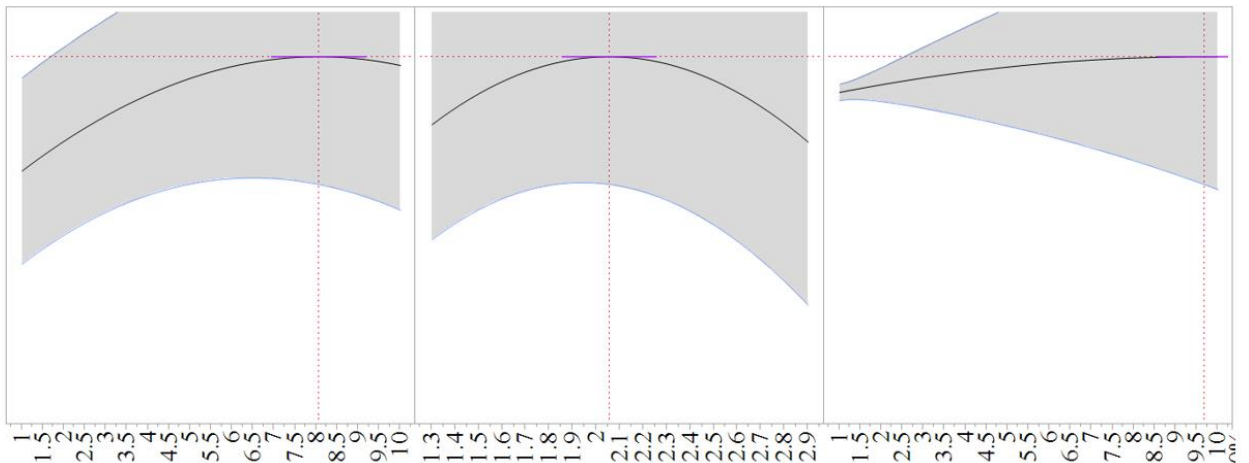
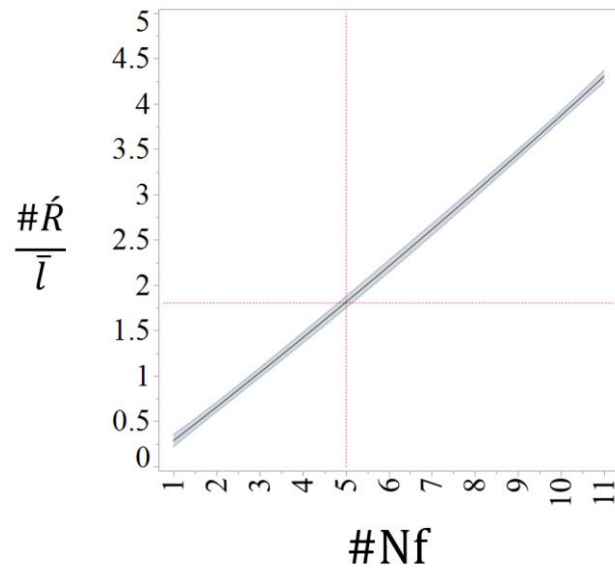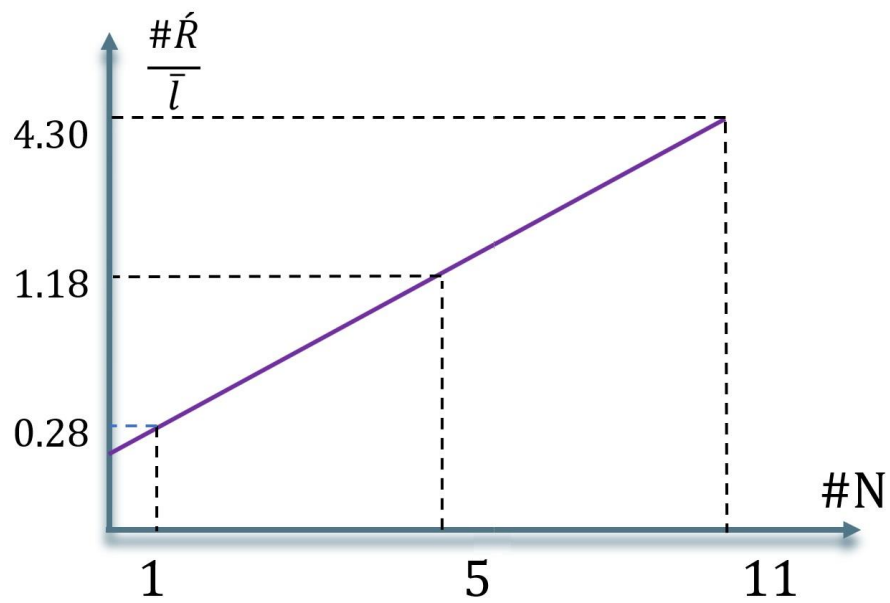| Rp	μ	V	Ef
8.04	2.05	9.66	0.88

5.6 Cost function

Figure 5-6

Figure 5-7

$$Ef = \frac{\#\acute{R}}{\sum_{R=1}^{Nf} \sum_{j}^{T} l_j^R} = \frac{\#\acute{R}}{Nf.\bar{l}}$$

$$\frac{\#\acute{R}}{\bar{l}} = Ef * Nf$$

$$\bar{l} = \frac{\sum_{R=1}^{Nf} \sum_{j}^{T} l_j^R}{Nf}$$

Where $\bar{l}$: average total length of a robot.

$\#\acute{R}$:#obtained target.

T:simulation step (20000).

Nf: #Robots.

l_j^R : length of R- th robot.at j- th stop.

$\bar{l}$: obtained target with respect to the average length.

6   Conclusion

In this study, we showed how mathematical models of stochastic movement could be used to analyze foraging behavior in multi-robot systems. This study helps a robot designer or programmer to decide proper design variables. A large number of possible combinations of different types of environments and robot's parameters necessitate a systematic statistical approach. Therefore, we employed Full Factor Design (FFD) and Response Surface Methodology (RSM) to analyze the relationship between these parameters and search efficiency. Our study was limited to the dependency of search efficiency on the robot sensor perception radius and robot speed. However, it can extend to other parameters. Experimental perceptions indicate that an assortment of species uses stochastic movement models to locate resources. In this study, we used mathematical models of stochastic movement to analyze foraging behavior to study how the stochastic processes of robot movement and resource distribution combine to influence search success. We showed how search efficiency depends on the robot sensor perception radius and robot speed, parameters that a designer need for robot design. We also investigated the best variant of speed, perceptual radius, μ (Lévy walk), resource number, and cluster radius of robot movement in the environment to increase the efficiency of robot movement during the stochastic approach method. It is worth mentioning that the determination coefficient $R^2 = 0.78$ is relatively lower than the μ (Lévy

walk) of the stochastic approach model. Thus, the model exhibits a better fit than the movement activity model. The primary use of this modeling is for prediction, optimization, or model tuning. The effects of the explanatory variables on the movement with minimum required energy were also studied using the response surface methodology (RSM).

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Reference

1.	Andersson, M., *Optimal foraging area: size and allocation of search effort.* Theoretical population biology, 1978. **13**(3): p. 397-409.

2.	Nolting, B.C., *Random search models of foraging behavior: theory, simulation, and observation.* 2013.

3.	Viswanathan, G.M., et al., *The physics of foraging: an introduction to random searches and biological encounters*. 2011: Cambridge University Press.

4.	Kagan, E. and I. Ben-Gal, *Search and foraging: individual motion and swarm dynamics*. 2015: Chapman and Hall/CRC.

5.	Hinkelman, T., et al., *Composite random search strategies based on non-proximate sensory cues.* Ecol. Model.(submitted for publication), 2013.

6.	Ariel, G., et al., *Swarming bacteria migrate by Lévy Walk.* Nature communications, 2015. **6**: p. 8396.

7.	Bartumeus, F. and J. Catalan, *Optimal search behavior and classic foraging theory.* Journal of Physics A: Mathematical and Theoretical, 2009. **42**(43): p. 434002.

8.	Ramos-Fernández, G., et al., *Lévy walk patterns in the foraging movements of spider monkeys (Ateles geoffroyi).* Behavioral ecology and Sociobiology, 2004. **55**(3): p. 223-230.

9.	Bond, A.B., *Optimal foraging in a uniform habitat: the search mechanism of the green lacewing.* Animal Behaviour, 1980. **28**(1): p. 10-19.

10.	Puri, S., Q.K. Beg, and R. Gupta, *Optimization of alkaline protease production from Bacillus sp. by response surface methodology.* Current microbiology, 2002. **44**(4): p. 286-290.

11.	O'brien, W.J., H.I. Browman, and B.I. Evans, *Search strategies of foraging animals.* American Scientist, 1990. **78**(2): p. 152-160.

12.	Sarsfield, M., *Design of Experiment Approach to the Optimization of Gold Nanoparticle Synthesis on a Microfluidic Mixer Platform*. 2018, Cleveland State University.

13.	Maaseidvaag, L., S.K. Skrivanek, and A. Gao, *Computer-implemented system and method of performing a design of experiment, generating a product thereof, generating designed experiment options, and a graphical user interface*. 2018, Google Patents.

14.	Giraldeau, L.-A. and G. Pyke, *Group foraging*, in *Encyclopedia of animal behavior*. 2019, Elsevier Academic Press. p. 191-200.

15.   Satin, S. and A. Gangal, *Random walk and broad distributions on fractal curves.* Chaos, Solitons & Fractals, 2019. **127**: p. 17-23.

16.   Chawla, M. and M. Duhan, *Levy Flights in Metaheuristics Optimization Algorithms–A Review.* Applied Artificial Intelligence, 2018. **32**(9-10): p. 802-821.

17.   Gallotti, R., et al., *A stochastic model of randomly accelerated walkers for human mobility.* Nature Communications, 2016. **7**: p. 12600.

18.   Yang, X.-S., *Firefly algorithm, Levy flights and global optimization*, in *Research and development in intelligent systems XXVI.* 2010, Springer. p. 209-218.

19.   Sims, D.W., et al., *Scaling laws of marine predator search behaviour.* Nature, 2008. **451**: p. 1098.

20.   Pyke, G. and D.W. Stephens, *Optimal foraging theory: application and inspiration in human endeavors outside biology*, in *Encyclopedia of animal behavior.* 2019, Elsevier Academic Press. p. 217-222.

21.   O'Keeffe, K., et al., *Urban sensing as a random search process.* arXiv preprint arXiv:1901.08678, 2019.

22.   Murakami, H., C. Feliciani, and K. Nishinari, *Lévy walk process in self-organization of pedestrian crowds.* Journal of the Royal Society Interface, 2019. **16**(153): p. 20180939.

23.   Heinrich, B., *10. Optimal Foraging Strategy: II. Foraging Movements of Bumble Bees.* Insect Behavior: A Sourcebook Of Laboratory And Field Exercises, 2019: p. 10.

24.   Brabazon, A. and S. McGarraghy, *Foraging-Inspired Optimisation Algorithms.* 2019: Springer.

25.   Borgers, C. and C. Greengard, *On the mean square displacement in Levy walks.* arXiv preprint arXiv:1903.10696, 2019.

26.   Xu, H., et al. *Application of Elephant Herd Optimization Algorithm Based on Levy Flight Strategy in Intrusion Detection.* in *2018 IEEE 4th International Symposium on Wireless Systems within the International Conferences on Intelligent Data Acquisition and Advanced Computing Systems (IDAACS-SWS).* 2018. IEEE.

27.   Reynolds, A., et al., *Lévy foraging patterns of rural humans.* PLOS ONE, 2018. **13**(6): p. e0199099.

28.   Edelhoff, H., J. Signer, and N. Balkenhol, *Path segmentation for beginners: an overview of current methods for detecting changes in animal movement patterns.* Movement ecology, 2016. **4**(1): p. 21.

29.   Musco, C., H.-H. Su, and N.A. Lynch, *Ant-inspired density estimation via random walks.* Proceedings of the National Academy of Sciences, 2017. **114**(40): p. 10534-10541.

30. Boczkowski, L., et al. *Random walks with multiple step lengths*. in *Latin American Symposium on Theoretical Informatics*. 2018. Springer.

31. Cohen, S.E. and P.M. Todd, *Relationship foraging: DOEs time spent searching predict relationship length?* Evolutionary Behavioral Sciences, 2018. **12**(3): p. 139.

32. Khalilpourazari, S. and S. Khalilpourazary, *A Robust Stochastic Fractal Search approach for optimization of the surface grinding process.* Swarm and Evolutionary Computation, 2018. **38**: p. 173-186.

33. Hayden, B.Y., *Economic choice: the foraging perspective.* Current Opinion in Behavioral Sciences, 2018. **24**: p. 1-6.

34. Gill, G. and P. Straka, *A Universal Algorithm for Continuous Time Random Walks Limit Distributions.* arXiv preprint arXiv:1808.05737, 2018.

35. Namboodiri, V.M.K., et al., *Rationalizing spatial exploration patterns of wild animals and humans through a temporal discounting framework.* Proceedings of the National Academy of Sciences, 2016. **113**(31): p. 8747-8752.

36. Zhao, K., et al., *Explaining the power-law distribution of human mobility through transportation modality decomposition.* Scientific Reports, 2015. **5**: p. 9136.

37. Farhad Maroofkhani, K.I., *Design and Optimization of Bio-Inspired Robotic Stochastic Search Strategy.* International Journal of Reasoning-based Intelligent Systems, 2019.

38. Xiong, Y., et al., *Design and experiment of grain mass flow sensor based on embedded system.* Transactions of the Chinese Society of Agricultural Engineering, 2018. **34**(5): p. 39-46.

39. Guillen-Perez, A. and M.-D. Cano, *Flying ad hoc networks: A new domain for network communications.* Sensors, 2018. **18**(10): p. 3571.

40. Butt, A.A., et al. *Optimization of Response and Processing Time for Smart Societies Using Particle Swarm Optimization and Levy Walk*. in *International Conference on Advanced Information Networking and Applications*. 2019. Springer.

41. Bartumeus, F., et al., *Helical Lévy walks: adjusting searching statistics to resource availability in microzooplankton.* Proceedings of the National Academy of Sciences, 2003. **100**(22): p. 12771-12775.

42. Viswanathan, G.M., et al., *Optimizing the success of random searches.* Nature, 1999. **401**(6756): p. 911-914.

43. Bertoin, J., *Lévy processes*. Vol. 121. 1996: Cambridge university press Cambridge.

44. Wang, J., et al., *Design and experiment of spray and rotary tillage combined disinfection machine for soil.* International Journal of Agricultural and Biological Engineering, 2019. **12**(1): p. 52-58.

45. Robinson, S.K. and R.T. Holmes, *Foraging behavior of forest birds: the relationships among search tactics, diet, and habitat structure.* Ecology, 1982. **63**(6): p. 1918-1931.

46. Wang, Q., et al., *Statistical Design of Experiment (DOE) based development and optimization of DB213 in situ thermosensitive gel for intranasal delivery.* International journal of pharmaceutics, 2018. **539**(1-2): p. 50-57.

47. Wang, Q., et al., *Design and experiment of chopping-type maize straw returning machine.* Transactions of the Chinese Society of Agricultural Engineering, 2018. **34**(2): p. 10-17.

48. Dean, A., et al., *Handbook of design and analysis of experiments*. Vol. 7. 2015: CRC Press.

49. Yang, R., et al., *Design and experiment of vertical circular separating and conveying device for potato combine harvester.* Transactions of the Chinese Society of Agricultural Engineering, 2018. **34**(3): p. 10-18.

50. Gigliotti, C., et al. *Design of Experiment Optimization of Artificial Bone Construct Fabrication Via Direct Ink Writing of Hydroxyapatite*. in *12th Pacific Rim Conference on Ceramic and Glass Technology*. 2018. Wiley Online Library.

51. Cosenz, F. and G. Noto, *A dynamic business modelling approach to design and experiment new business venture strategies.* Long Range Planning, 2018. **51**(1): p. 127-140.

52. Bordonaro, G., et al., *Design of Experiment as a powerful tool when applying Finite Element Method: a case study on prediction of hot rolling process parameters.* 2018.

53. Anderson, V.L. and R.A. McLean, *Design of experiments: a realistic approach*. 2018: Routledge.

54. Jensen, W.A., *DOE Simplified: Practical Tools for Effective Experimentation, Second Edition.* Journal of Quality Technology, 2008. **40**(1): p. 124-125.

55. Myers, R.H., *Response surface methodology—current status and future directions.* Journal of quality technology, 1999. **31**(1): p. 30-44.

56. Myers, R.H., D.C. Montgomery, and C.M. Anderson-Cook, *Response surface methodology: process and product optimization using designed experiments*. 2016: John Wiley & Sons.

57. Hill, W.J. and W.G. Hunter, *A review of response surface methodology: a literature survey.* Technometrics, 1966. **8**(4): p. 571-590.

58. Liyana-Pathirana, C. and F. Shahidi, *Optimization of extraction of phenolic compounds from wheat using response surface methodology.* Food chemistry, 2005. **93**(1): p. 47-56.

59. Khuri, A.I. and S. Mukhopadhyay, *Response surface methodology.* Wiley Interdisciplinary Reviews: Computational Statistics, 2010. **2**(2): p. 128-149.

60. Ghanbarzadeh, S., H. Valizadeh, and P. Zakeri-Milani, *Application of response surface methodology in development of sirolimus liposomes prepared by thin film hydration technique.* BioImpacts : BI, 2013. **3**(2): p. 75-81.

61. Raichlen, D.A., et al., *Physical activity patterns and biomarkers of cardiovascular disease risk in hunter-gatherers.* American Journal of Human Biology, 2017. **29**(2): p. e22919.