Theoretical estimation of the upper limit of critical current density by flux pinning in superconductors under the influence of kinetic energy

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The upper limit of critical current density by the flux pinning mechanism is theoretically investigated with taking account of the influence of kinetic energy. The upper limit is estimated to be 67 % of the depairing current density in the London limit, which is about 23 % higher than the estimation by Tinkham. The reason for the higher value is attributed to the enhancement of the order parameter that reduces the kinetic energy caused by the current. The possibility of such a strong flux pinning is discussed for nano-rods introduced to REBCO coated conductors.

Critical current density J_c, which is the maximum nondissipative current density, is the most important factor for application of superconductors. Theoretically estimated maximum value of J_c is the depairing current density j_d. It is considered that the departing current density can be achieved for a superconducting wire with the transverse sizes comparable to or smaller than the coherence length ξ under zero magnetic field. Experimental results have been compared with the theoretical estimation.[1-5] In this case, however, a large superconducting current cannot be obtained due to limited cross-sectional area of superconductors, even if a high critical current density is achieved.

Practical critical current density in a superconductor with a much larger cross-sectional area is governed by the flux pinning mechanism. Recently introduction of artificial pinning centers to REBCO superconducting films is commonly employed to appreciably enhance the J_c-value.[6-16] Great interest in this case is how J_c-values can be improved and observed J_c-values have been frequently compared with j_d.[17]

However, any theoretical investigation has not been given for the upper limit of the critical current density attained by the flux pinning mechanism. The factor that restricts the enhancement of the critical current density is the increase in kinetic energy, which is similar to the limit of depairing current density. In this paper the critical current density at low magnetic fields is theoretically estimated as a function of the flux pinning strength using the Ginzburg-Landau theory. It is also investigated how close we can approach this limit for the case of flux pinning by nano-rods introduced to REBCO coated conductors.

First, we discuss the depairing current density, since it helps to understand the difference from the pinning current density. The depairing current density is discussed in the absence of external magnetic field for a superconductor with small transverse sizes normal to the current direction. The Ginzburg-Landau free energy density is given by

$$\mathcal{F} = \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \frac{1}{2m^*}|(-\mathrm{i}\hbar\nabla + 2e\boldsymbol{A})\Psi|^2, \tag{1}$$

where m^* and $-2e(e>0)$ are the mass and electric charge of superconducting electron and $\boldsymbol{A}$ is the vector potential. In the condition of our interest the magnitude of the order parameter $|\Psi|$ in the superconductor is expected to be equal to that in the equilibrium condition, $|\Psi_\infty|$. Then, we have $\nabla\Psi = \nabla|\Psi|\mathrm{e}^{\mathrm{i}\varphi} = \mathrm{i}(\nabla\varphi)\Psi$ with φ denoting the phase of

Ψ and the kinetic energy density in the Ginzburg-Landau theory is

$$\mathcal{F}_{\mathrm{k}} = \frac{1}{2m^*}|(\hbar\nabla\varphi + 2e\boldsymbol{A})\Psi|^2 = \frac{1}{2m^*}(\hbar\nabla\varphi + 2e\boldsymbol{A})^2|\Psi_\infty|^2 \equiv \mathcal{F}_{\mathrm{k}\infty}. \tag{2}$$

The superconducting current density is

$$\boldsymbol{j}_\infty = -\frac{2e}{m^*}(\hbar\nabla\varphi + 2e\boldsymbol{A})|\Psi_\infty|^2. \tag{3}$$

Then, Eq. (2) is rewritten as

$$\mathcal{F}_{\mathrm{k}\infty} = \frac{m^*{\boldsymbol{j}_\infty}^2}{8e^2|\Psi_\infty|^2}, \tag{4}$$

and the Ginzburg-Landau free energy density is

$$\mathcal{F}_\infty = \alpha|\Psi_\infty|^2 + \frac{\beta}{2}|\Psi_\infty|^4 + \frac{m^*{\boldsymbol{j}_\infty}^2}{8e^2|\Psi_\infty|^2}, \tag{5}$$

where the magnetic energy is omitted. In this equation, the magnitude of the sum of the first and second terms gives the condensation energy density:

$$|\mathcal{F}_{\mathrm{c}\infty}| = \left|\alpha|\Psi_\infty|^2 + \frac{\beta}{2}|\Psi_\infty|^4\right| = \frac{1}{2}\mu_0{H_{\mathrm{c}}}^2, \tag{6}$$

where H_{c} is the thermodynamic critical field. The critical current density at which the transition to the normal state occurs with satisfying $\mathcal{F}_\infty = 0$ is obtained as

$$j_{\mathrm{d}} = \left(\frac{4\mu_0{H_{\mathrm{c}}}^2e^2|\Psi_\infty|^2}{m^*}\right)^{1/2} = \frac{H_{\mathrm{c}}}{\lambda}, \tag{7}$$

where λ is the penetration depth. This is the depairing current density in the London limit.[18] The depairing current density is also discussed in a different way as will be shown later.

The practical critical current density is the maximum non-dissipative current density determined by the mechanism of flux pinning interactions in magnetic fields for superconductors with much larger cross-sectional area. At magnetic fields sufficiently higher than the lower critical field H_{c1}, the penetration depth, a characteristic length of spatial distribution of the current, is larger than the spacing of quantized flux lines. Hence, it can be regarded that the current flows uniformly with the density $\boldsymbol{J}$ in the superconductor. As a result, the influence of the increase in kinetic energy appears globally.

The equilibrium condition under the flux pinning interactions is obtained by minimizing the Gibbs free energy density:[19, 20]

$$\mathcal{g} = \mathcal{F} + U_{\mathrm{p}} - (\boldsymbol{J} \times \boldsymbol{B}) \cdot \boldsymbol{u}, \tag{8}$$

where U_{p} is the pinning energy density, $\boldsymbol{J} \times \boldsymbol{B}$ is the Lorentz force and $\boldsymbol{u}$ is the displacement of flux lines. The minimization of $\mathcal{g}$ with respect to $\boldsymbol{u}$ leads to the force-balance equation in the critical state model:

$$\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{F}_{\mathrm{p}} = 0, \tag{9}$$

where $\boldsymbol{F}_{\mathrm{p}} = -\partial U_{\mathrm{p}}/\partial \boldsymbol{u}$ is the pinning force density. Note that this minimization holds in the region of reversible flux motion up to the critical state.[19] Hence, the principle of determination of the maximum pinning current density is essentially different from the determination of depairing current density in Eq. (7). The free energy density $\mathcal{g}$ should also be minimized with respect to $|\Psi|$. This will be discussed later.

We treat the flux pinning by nano-rods introduced in REBCO thin films, since the flux pinning interaction is considered to be strongest. The pinning mechanism of such normal precipitates is the condensation energy interaction. We assume that a low magnetic field is applied along the c-axis and all flux lines are pinned completely by parallel nano-rods. The diameter of nano-rods is assumed to be slightly larger than the diameter of the normal core of each flux line $2\xi_{ab}$. Then, the elementary pinning force in a unit length of the flux line is given by

$$\hat{f}_{\mathrm{p}} = \frac{\pi \xi_{ab}}{2} \mathcal{F}_{\mathrm{c}}, \tag{9}$$

where $\mathcal{F}_{\mathrm{c}}$ is the magnitude of the condensation energy density:

$$\mathcal{F}_{\mathrm{c}} = \left|\alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4\right| \tag{10}$$

The current density by this pinning mechanism is theoretically estimated as

$$J = \eta\frac{\hat{f}_{\mathrm{p}}}{\phi_0} = \eta\frac{\pi\xi_{ab}}{2\phi_0}\mathcal{F}_{\mathrm{c}}, \tag{11}$$

where η is a constant called the pinning efficiency and ϕ_0 is the flux quantum. Equation (11) shows that the critical current density is independent of the magnetic field as observed at low magnetic fields. In the ideal case where we can neglect the influence of kinetic energy, we can assume $|\Psi|^2 = |\Psi_\infty|^2$ and Eq. (11) leads to

$$J = \eta\frac{\pi\xi_{ab}\mu_0 {H_{\mathrm{c}}}^2}{4\phi_0} \equiv J_{\mathrm{c}\infty}. \tag{12}$$

For simplicity we normalize as

$$x = \frac{|\Psi|^2}{|\Psi_\infty|^2}, \qquad y = \frac{\lambda_{ab}J}{H_{\mathrm{c}}} = \frac{J}{j_{\mathrm{d}}}, \qquad \frac{J_{\mathrm{c}\infty}}{j_{\mathrm{d}}} = \eta\frac{\pi\xi_{ab}\mu_0 H_{\mathrm{c}}\lambda_{ab}}{4\phi_0} \equiv k. \tag{13}$$

The corresponding penetration depth is that in the a-b plane. Then, the normalized current density is written as

$$y = k(2x - x^2). \tag{14}$$

Thus, the free energy density $\mathcal{F}$ is given by

$$\left(\frac{1}{2}\mu_0 {H_{\mathrm{c}}}^2\right)^{-1}\mathcal{F} = -2x + x^2 + k^2 x(2-x)^2. \tag{15}$$

Note that whereas only the expression of k changes depending on a kind and size of pinning centers, the form of Eq. (14) is unchanged so far as the elementary flux pinning mechanism is the condensation energy interaction. Then, we use k as a mathematical parameter representing the flux pinning strength which can be extended to infinity.

Exactly speaking, $\mathcal{F} + U_{\mathrm{p}}$ is to be minimized with respect to $|\Psi|^2$. Since the volume fraction of the pinning region is very small, however, the pinning energy can be approximately neglected in determination of the equilibrium condition for $|\Psi|^2$. Hence, $\mathcal{F}$ is minimized with respect to x. This leads to

$$3k^2x^2 + 2(1 - 4k^2)x - 2 + 4k^2 = 0. \tag{16}$$

This can be easily solved as

$$x = \frac{1}{3k^2}\left[-1 + 4k^2 + (1 - 2k^2 + 4k^4)^{1/2}\right] \equiv x_{\mathrm{c}}. \tag{17}$$

Substituting Eq. (17) into Eq. (14), we have the critical current density:

$$y = \frac{2}{9k^3}\left[-1 + 2k^2 + 2k^4 + (1 - k^2)(1 - 2k^2 + 4k^4)^{1/2}\right] \equiv y_{\mathrm{c}} = \frac{J_{\mathrm{c}}}{j_{\mathrm{d}}}. \tag{18}$$

Obtained Eqs. (17) and (18) satisfy the normal results of $x_{\mathrm{c}} \to 1$ and $y_{\mathrm{c}} \to k$ when k is sufficiently small. That is, when the pinning is not so strong, the critical current density can be simply obtained without considering the effect of kinetic energy as is done usually. Figure 1 shows the theoretical results on the order parameter and the critical current density as a function of the flux pinning strength. The equilibrium value x_{c} increases from 1 with increasing flux pinning strength. That is, $|\Psi|^2$ takes a value larger than $|\Psi_\infty|^2$, which is opposite to the expected behavior of the order parameter in the theoretical model of Tinkham.

On the other hand, Tinkham[21)] discussed the maximum current density in a pin free superconductor in a different way. In this theoretical treatment the current density $\boldsymbol{j}$ is one of the external variables and the Gibbs free energy density is given by

$$\mathcal{g} = \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \frac{m^*}{2}\boldsymbol{v}_{\mathrm{s}}{}^2|\Psi|^2 + \frac{m^*\boldsymbol{v}_{\mathrm{s}}\cdot\boldsymbol{j}}{2e} = \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 - \frac{m^*j^2}{8e^2|\Psi|^2}, \tag{19}$$

where the relationship:

$$\boldsymbol{v}_{\mathrm{s}} = -\frac{m^* \boldsymbol{j}}{2e|\Psi|^2} \tag{20}$$

was used. The free energy is minimized with respect to $|\Psi|$. This leads to

$$\frac{m^*}{8e^2} j^2 = -\alpha|\Psi|^4 - \beta|\Psi|^6. \tag{21}$$

The current density j is maximum when $|\Psi|^2 = -2\alpha/(3\beta) = (2/3)|\Psi_\infty|^2$ and the maximum value is

$$j'_{\mathrm{d}} = \left(\frac{2}{3}\right)^{3/2} \frac{H_{\mathrm{c}}}{\lambda} = \left(\frac{2}{3}\right)^{3/2} j_{\mathrm{d}}. \tag{22}$$

When the flux pinning interaction becomes strong, the critical current density J_{c} increases and has a maximum $0.6712 j_{\mathrm{d}}$ at around $k = 0.9512 \equiv k_{\mathrm{m}}$. It is expected that the critical current density can reach 67% of the depairing current density by making the flux pinning strong sufficiently. This value is even higher than the theoretically estimated value of $j'_{\mathrm{d}} = (2/3)^{3/2} j_{\mathrm{d}} = 0.5443 j_{\mathrm{d}}$ by Tinkham. For a current density above J_{c}, the resistive flux flow state occurs as usually understood for $k < k_{\mathrm{m}}$. Hence, this is quite different from the condition at the depairing current density. Note that, even at the situation where the current with the density above J_{c} flows, the energy density $\mathcal{g}$ is lower than that in the normal state and the superconducting state is kept.

For the case of much stronger flux pinning ($k > k_{\mathrm{m}}$), the above theoretical estimation suggests a decrease in J_{c} value. This decrease is speculated to take place under the assumption that flux pinning interactions fully work. If we remember that the pinned flux lines tend to self-organize themselves to minimize the energy dissipation as predicted by the critical state model,[22)] however, it is considered that the flux lines select a relatively weakly pinned condition to attain a higher J_{c} value. Note that a similar behavior occurs in the formation of flux bundle under the influence of flux creep: It is considered that flux lines select a relatively weakly pinned state to enhance the flux bundle size, which is effective against the strong flux creep.[23)] This comes from the principle of irreversible thermodynamics to minimize the energy dissipation, which is realized by maximizing the critical current density, and explains various experimental results on the film-thickness dependence of irreversibility field, etc.[24] Hence, it is expected that J_c keeps the maximum J_c value even for $k > k_\mathrm{m}$.

Here we discuss how the order parameter changes under the flux pinning interaction. Figure 1 shows that $|\Psi|^2$ increases when the flux pinning strength increases. This is caused by the kinetic energy density given by the third term in Eq. (15). The relationship between $\mathcal{F}$ and $|\Psi|^2$ is shown in Fig. 2 for $k = 0,\ 0.5$ and $k_\mathrm{m} = 0.9152$. When k becomes large, the minimum free energy density is attained at a larger value of the order parameter with the decrease in the condensation energy density. On the other hand, Tinkham predicted that $|\Psi|^2$ decreases with increasing current density. Here we discuss which behavior is preferable from the viewpoint of energy. The kinetic energy density is given by:

$$\left(\frac{1}{2}\mu_0 H_\mathrm{c}{}^2\right)^{-1} \mathcal{F}_\mathrm{k} \equiv \mathcal{F}_\mathrm{k}{}' = \frac{y^2}{x}. \tag{23}$$

It is preferable to increase the order parameter x to reduce the kinetic energy under the condition of large normalized current density y. This is the reason why $|\Psi|^2$ takes a larger value than $|\Psi_\infty|^2$. This may also be effective to reach the limit higher than the critical current density predicted by Tinkham. The conditions are compared in more detail between the two theoretical approaches in Table 1. The free energy density in our critical state is lower than that in the critical state of Tinkham.

The above theoretical result suggests that we can hopefully introduce strong pinning centers to superconductors to improve the critical current density. Then, our interest is how close we can increase the critical current density to the theoretical limit. We assume the case of strong flux pinning by nano-rods at 4.2 K and at low magnetic fields along the c-axis. Assumed superconducting parameters are $\lambda_{ab} = 121$ nm ($\xi_{ab} = 1.8$ nm and $\kappa_{ab} = 67$)[25] and $\mu_0 H_\mathrm{c} = 2.5$ T.[26] The pinning efficiency is not a constant in a strict sense but increases with flux pinning strength.[27] Then, we assume $\eta = 1$ in the limit of strong pinning. In this case the parameter k is estimated as 0.207, which leads to $y_\mathrm{c} = 0.207$. As a result, the depairing current density is $j_\mathrm{d} = 1.64 \times 10^{13}\ \mathrm{Am^{-2}}$ and the critical current density is $J_\mathrm{c} = 3.39 \times 10^{12}\ \mathrm{Am^{-2}}$. This value is 30.8% of the theoretical limit. Since the optimum situation for flux pinning is treated here, it is speculated to be difficult to attain a more strongly pinned case so far as the condensation energy interaction with normal precipitates is involved. There is sufficient room for improvement in the future, however, since practically obtained critical current densities are fairly lower than the present estimation.

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Figure Captions

Fig. 1. Normalized order parameter (solid symbols) and critical current density (open symbols) as a function of flux pinning strength.

Fig. 2. Free energy density as a function of normalized order parameter for $k = 0, 0.5$ and $k_{\mathrm{m}} = 0.9152$.

Table I. Comparison on the conditions in the critical state between the two theoretical approaches: x_{c} is the order parameter, y_{c} is the critical current density, $\mathcal{F}_{\mathrm{c}}{}'$ is the condensation energy density, $\mathcal{F}_{\mathrm{k}}{}'$ is the kinetic energy density and $-\mathcal{F}_{\mathrm{c}}' + \mathcal{F}_{\mathrm{k}}{}'$ is the total free energy density. All quantities are normalized.

	x_{c}	y_{c}	$\mathcal{F}_{\mathrm{c}}' = 2x_{\mathrm{c}} - {x_{\mathrm{c}}}^2$	$\mathcal{F}_{\mathrm{k}}' = {y_{\mathrm{c}}}^2/x_{\mathrm{c}}$	$-\mathcal{F}_{\mathrm{c}}' + \mathcal{F}_{\mathrm{k}}'$
Present	1.5163	0.6712	0.7334	0.2971	−0.4363
Tinkham[21)]	0.6667	0.5443	0.6667	0.4444	−0.2222

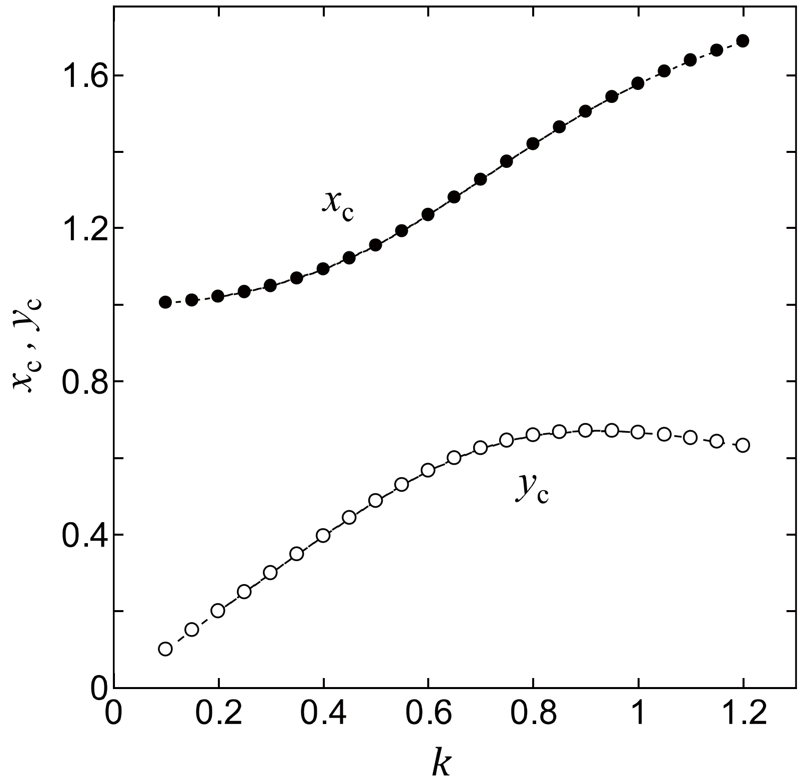

Fig. 1

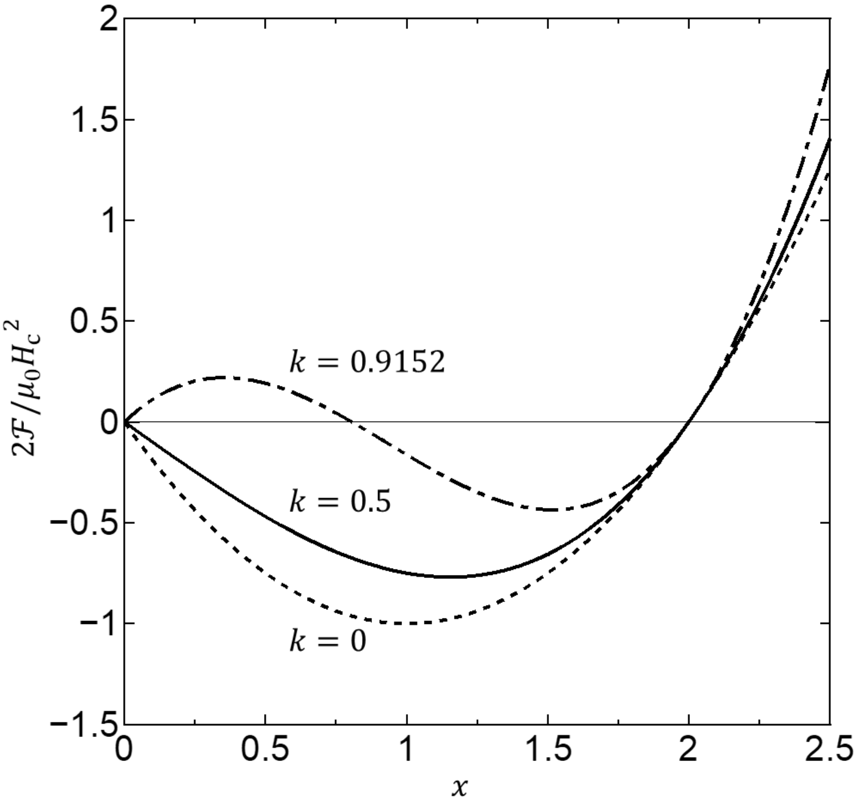

Fig. 2