

## State Observer-Based Robust Control Scheme for Electrically Driven Robot Manipulators

Masahiro Oya, Chun-Yi Su, and Toshihiro Kobayashi

**Abstract**—By using a state observer, a new robust trajectory tracking-control scheme is developed in this paper for electrically driven robot manipulators. The role of the observer is to estimate joint angular velocities. The proposed controller does not employ adaptation, but assures robust stability of tracking error between joint angles and desired trajectories. At sacrificing asymptotical stability of the tracking errors, the configuration of the proposed controller becomes very simple, compared with regressor-based adaptive controllers. It is shown in the closed-loop system using the proposed controller that the Euclidian norm of tracking errors arrives at any small closed region with any convergent rate by setting only one design parameter. Especially for the desired trajectories converging to constant ultimate values, it is assured that tracking errors converge to zero.

**Index Terms**—Electrically driven robot, robot manipulators, robust control, state observer, tracking control.

### I. INTRODUCTION

As demonstrated in [1], the actuator dynamics constitute an important component of the complete robot dynamics. If the actuator dynamics is ignored, the designed controller may not yield good system overall performance. In recent years, controls for robot manipulators, including the actuator dynamics, have received considerable attention and several control schemes have been developed [2]–[14]. In the early works [2], [3], the controllers required full knowledge of system dynamics. If there are uncertainties in the dynamics, the controllers proposed in [2] and [3] may give a poor performance, and may even cause instability. To overcome the uncertainties in the dynamics, robust controllers have been proposed in [4]–[15]. However, these controllers normally require full state measurements. In general, full state measurements may not be available, due to cost of sensors, weight limitation, effects of noises, etc. Especially for the velocity measurement of joint angles, the required accuracy may not be achieved in practical applications, due to the existence of noises [16]. Most recently, control schemes without using velocity measurements were proposed in [17] and [18], where regressor-based adaptive controllers are employed. These controllers are effective for uncertainties in robot and actuator dynamics, and guarantee asymptotical stability of the tracking errors. However, the construction of the regressor is not trivial for a general robot, even when only desired values are involved.

In this paper, we will develop a new robust tracking-control scheme with the use of a state observer without involving adaptations, velocity measurements of joint angles, and the regressor. A precompensator is first introduced to obtain a new representation for the electrically driven robot dynamics. Then, a new robust controller is developed, which has the following features: 1) it is assured that the Euclidian norm of tracking errors can reach to any small closed region with any convergent rate by setting only one design parameter; 2) it is assured that the tracking error converges to zero when the desired trajectories  $q_d(t)$

converge to ultimate constant values; and 3) the configuration of the developed robust controller is very simple, if compared with that of the regressor-based adaptive controllers.

### II. NEW REPRESENTATION OF ROBOT MANIPULATORS WITH INTEGRAL PRECOMPENSATOR

Consider an  $n$ -link manipulator with revolute joints driven by armature-controlled dc motors with voltages being inputs to amplifiers. As in [3] and [4], the dynamics are described by

$$\left. \begin{aligned} M_P(q)\ddot{q}(t) + B_P(q, \dot{q})\dot{q}(t) + g_P(q) &= K_{PN}I_P(t) \\ M_P(q) &= D_P(q) + J \end{aligned} \right\} \quad (1)$$

$$L\dot{I}_P(t) + RI_P(t) + K_e\dot{q}(t) = u_P(t) \quad (2)$$

where  $q(t) \in R^n$  is joint angles,  $I_P(t) \in R^n$  is the armature currents, and  $u_P(t) \in R^n$  is armature voltages.  $D_P(q) \in R^{n \times n}$  is a positive definite inertia matrix of the manipulator,  $B_P(q, \dot{q}) \in R^{n \times n}$  is Coriolis and centrifugal torques, and  $g_P(q) \in R^n$  is the gravitational torques.  $J$ ,  $L$ , and  $R$  are the actuator inertia matrix, the actuator inductance matrix, and the actuator resistance matrix, respectively.  $K_e$  is the matrix that characterizes the voltage constant of the actuators, and  $K_{PN}$  is the matrix characterizing the electromechanical conversion between current and torque.  $J$ ,  $L$ ,  $R$ ,  $K_e$ , and  $K_{PN}$  are positive definite constant diagonal matrices.

The control objective pursued here is as follows. For any given desired bounded trajectories  $q_d(t)$ ,  $\dot{q}_d(t)$ , and  $\ddot{q}_d(t)$ , with some or all of the manipulator and actuator parameters unknown, derive a robust controller for the actuator  $u_P$  without using the measurement of joint velocities, such that the manipulator position vector  $q(t)$  tracks  $q_d(t)$ .

In the following development, suppose that the armature current  $I_P(t)$ , joint angles  $q(t)$ , and armature voltage  $u_P(t)$  are measurable, and the joint angles of the robot manipulator (1), (2) are held at some fixed angle by using some simple joint-angle feedback controller. Then,  $u_P(t)$  can be represented as  $u_P(t) = u(t) + \bar{u}$ , where  $\bar{u}$  is a constant voltage to hold the joint angle. In this case, the relations  $\dot{q}(0) = 0$ ,  $g_P(q(0)) = K_{PN}I_P(0)$ , and  $RI_P(0) = \bar{u}$  are satisfied. When the signals  $I(t) = I_P(t) - I_P(0)$ ,  $u(t) = u_P(t) - \bar{u}$  are used to initialize the manipulator (1), (2) in the case of the fixed-angle situation, the electrically driven robot manipulator can be described by

$$\left. \begin{aligned} M(q)\ddot{q}(t) + B(q, \dot{q})\dot{q}(t) + g(q) &= K_N I(t) \\ L\dot{I}(t) + RI(t) + K_e\dot{q}(t) &= u(t) \\ M(q) &= \frac{M_P(q)}{\rho_{Pm}}, \quad B(q, \dot{q}) = \frac{B_P(q, \dot{q})}{\rho_{Pm}} \\ g(q) &= \frac{g_P(q) - g_P(q(0))}{\rho_{Pm}}, \quad K_N = \frac{K_{PN}}{\rho_{Pm}} \end{aligned} \right\} \quad (3)$$

where  $\rho_{Pm} = \lambda_{\min}[M_P(q)]$  is the lower limit of eigenvalues of  $M_P(q)$ , and the symbols  $M(q)$ ,  $B(q, \dot{q})$ ,  $g(q)$ , and  $K_N$  are introduced to normalize the lower-limit eigenvalue of the manipulator inertia matrix. It should be noted that the constant values  $\bar{u}$  and  $I_P(0)$  can be obtained from the measurable signals  $I_P(t)$  and  $u_P(t)$ .

It is well known that manipulators and actuators are characterized by the following properties [19]–[21].

**P1)** The relation  $B(q, x)y = B(q, y)x$  holds, and there exists a bounded positive constant  $\bar{\rho}_b$  such that  $\|B(q, x)y\| \leq \bar{\rho}_b \|x\| \|y\|$  for any two given vectors  $x, y \in R^n$ .

**P2)** The relation  $\dot{M}(q)\dot{q}(t) - B(q, \dot{q})\dot{q}(t) = (1/2)[(\partial/\partial q)[\dot{q}(t)^T M(q)\dot{q}(t)]]^T$  is satisfied.

**P3)** The matrix  $M(q)$  is symmetric positive definite and there exist bounded positive constants  $\rho_m (= 1)$  and  $\bar{\rho}_m$  for any vector  $x$  such that  $\rho_m x^T x \leq x^T M(q)x \leq \bar{\rho}_m x^T x$ .

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**P4)** There exist bounded positive constant values  $\underline{\rho}_{kn}, \bar{\rho}_{kn}$  for any vector  $x$  such that  $\underline{\rho}_{kn} x^T x \leq x^T K_N x \leq \bar{\rho}_{kn} x^T x$ .

**P5)** There exist bounded positive constants  $\bar{\rho}_{m1}, \bar{\rho}_a, \bar{\rho}_{a1}, \bar{\rho}_r, \bar{\rho}_l, \bar{\rho}_{ke}$ , such that  $\|\dot{M}(q)\| \leq \bar{\rho}_{m1} \|\dot{q}(t)\|$ ,  $\|g(q)\| \leq \bar{\rho}_a$ ,  $\|\partial g(q)/\partial q\| \leq \bar{\rho}_{a1}$ ,  $\|R\| \leq \bar{\rho}_r$ ,  $\|L\| \leq \bar{\rho}_l$ ,  $\|K_e\| \leq \bar{\rho}_{ke}$ .

To make  $q(t)$  track a desired trajectory  $q_d(t)$ , let us consider a pre-compensator of the following form:

$$u(t) = \mu(t) + \int_0^t \mu(\tau) d\tau = \frac{p+1}{p} [\mu(t)] \quad (4)$$

where the symbol  $p$  denotes the differential operator, and the signal  $\mu(t)$  is the input to the compensator. Then, to develop a control scheme achieving the control objective, the robot model (3) is rewritten as follows (see Appendix I):

$$M(q)\ddot{q}(t) - M(q)\dot{q}(t) + B(q, \dot{q})\dot{q}(t) + h_q(t) = K_N I_F(t) \quad (5)$$

$$L\dot{I}_F(t) + R I_F(t) + K_e \dot{q}(t) - h_l(t) = \mu(t) \quad (6)$$

and

$$\left. \begin{aligned} h_q(t) &= \int_0^t e^{-(t-\tau)} \left[ M(q)\dot{q}(\tau) + \frac{\partial g(q)}{\partial q} \dot{q}(\tau) \right. \\ &\quad \left. + \frac{1}{2} f(\dot{q}, \dot{q}) \right] d\tau \\ f(\dot{q}, \dot{q}) &= \left[ \frac{\partial}{\partial q} [\dot{q}(t)^T M(q) \dot{q}(t)] \right]^T \\ h_l(t) &= \int_0^t e^{-(t-\tau)} K_e \dot{q}(\tau) d\tau \\ I_F(t) &= I(t) - \int_0^t e^{-(t-\tau)} I(\tau) d\tau = \frac{p}{p+1} [I(t)]. \end{aligned} \right\} \quad (7)$$

In the above expression, the function  $f(a, b)$  is defined as  $f(a, b) = [(\partial/\partial q)[a^T M(q)b]]^T$  for any vector  $a$  and  $b \in R^n$ , where the dependence on  $q(t)$  is dropped for the simplicity in the later development.

It should be noted that property **P2** has been used to derive the new representation (5)–(7). Also, the gravitational torques  $g(q)$  are represented by the term  $(\partial_a(q)/\partial q)\dot{q}(t)$ .

### III. CONTROLLER DEVELOPMENT

In the following development, the design procedure will be described as a two-step process. Firstly, the signal  $I_F(t)$  in (5) is regarded as the input signal. An embedded control input  $I_{Fd}(t)$  is designed so that the desired tracking can be achieved. Secondly,  $\mu(t)$  is designed so that  $I_F(t)$  tracks  $I_{Fd}(t)$  without using the signal  $\dot{q}(t)$ . In turn, this allows  $q(t)$  to track the desired trajectory  $q_d(t)$ .

#### A. Synthesis of Embedded Signal $I_{Fd}(t)$

First, let us suppose that the signal  $\dot{q}(t)$  is available and the signal  $I_F(t)$  can be treated as an input signal. A robust controller will be synthesized, so that  $q(t)$  tracks the desired trajectory  $q_d(t)$  for the subsystem (5). To achieve such an objective, an additional property is exploited.

**P6)** There exists a bounded positive constant value  $\bar{\rho}_f$  such that  $\|f(a, b)\| \leq \bar{\rho}_f \|a\| \|b\|$  for any  $a, b \neq q(t)$ .

For the development of robust controller, the following standard assumptions are required for the system (5)–(7).

**A1)** There exist known diagonal constant matrices  $\hat{K}_N, \hat{R}, \hat{K}_e$ , and there exist bounded positive constant values  $\bar{\rho}_{kk}, \underline{\rho}_{kk}, \bar{\rho}_r, \bar{\rho}_{ke}$  such that  $\underline{\rho}_{kk} x^T x \leq x^T K_N \hat{K}_N^{-1} x \leq \bar{\rho}_{kk} x^T x$  for all  $x$ ,  $\|\hat{R}\| = \|R - \hat{R}\| \leq \bar{\rho}_r$ ,  $\|\hat{K}_e\| = \|K_e - \hat{K}_e\| \leq \bar{\rho}_{ke}$ .

**A2)** There exists a known bounded function  $\hat{g}(q)$ , and there exist bounded positive constant values  $\bar{\rho}_{q1}, \bar{\rho}_{a1}$  such that  $\|(\partial \hat{g}(q)/\partial q)\| \leq \bar{\rho}_{q1}$ ,  $\|(\partial \hat{g}(q)/\partial q)\| = \|(\partial g(q)/\partial q) - K_N \hat{K}_N^{-1} (\partial \hat{g}(q)/\partial q)\| \leq \bar{\rho}_{a1}$ .

**A3)** For given desired trajectories  $q_d(t)$ , there exist bounded positive constant values  $\bar{\rho}_{d1}, \bar{\rho}_{d2}$ ,  $i = 1, 2$ , such that  $\|q_d(t)\| \leq \bar{\rho}_{d1}$ ,  $\|\dot{q}_d(t)\| \leq \bar{\rho}_{d2}$ .

**A4)** The initial value  $q(0)$  is bounded.

It is noted that  $\hat{K}_N, \hat{R}$ , and  $\hat{K}_e$  are estimates for  $K_N, R$ , and  $K_e$ , respectively,  $\hat{g}(q)$  is an estimate of  $g(q)$ , and  $\hat{\bullet}$  denotes estimated error.

To design an embedded control input so that  $q(t)$  in the subsystems (5) tracks  $q_d(t)$ , in the following, we define:

$$s(t) = \beta^{-1} \dot{\tilde{q}}(t) + \tilde{q}(t), \quad \tilde{q}(t) = q(t) - q_d(t) \quad (8)$$

where  $\beta$  is a positive design parameter introduced to improve tracking performance. It is noted that the norm of the initial tracking error  $s(0)$  does not increase with respect to the design parameter  $\beta$ . Using the relation

$$\hat{g}(q) - \int_0^t e^{-(t-\tau)} \hat{g}(q) d\tau = \int_0^t e^{-(t-\tau)} \frac{\partial \hat{g}}{\partial q} \dot{q}(\tau) d\tau + e^{-t} \hat{g}(q(0)) \quad (9)$$

and the property **P1**, the error system can be obtained as

$$\begin{aligned} M(q)\dot{s}(t) &= -(\beta B(q, s - \tilde{q}) - (\beta + 1)M(q) + 2B(q, \dot{q}_d)) \\ &\quad \times (s(t) - \tilde{q}(t)) - \beta^{-1} \omega_{sd}(q, \dot{q}_d, \ddot{q}_d) - \beta^{-1} h(t) \\ &\quad + \beta^{-1} K_N I_F(t) - \beta^{-1} K_N \hat{K}_N^{-1} \\ &\quad \times \left( \hat{g}(q) - \int_0^t e^{-(t-\tau)} \hat{g}(q) d\tau \right) \end{aligned} \quad (10)$$

$$\begin{aligned} h(t) &= h_q(t) - \int_0^t e^{-(t-\tau)} K_N \hat{K}_N^{-1} \frac{\partial \hat{g}}{\partial q} \dot{q}(\tau) d\tau \\ &= \int_0^t e^{-(t-\tau)} \left[ \frac{\beta^2}{2} f(s - \tilde{q}, s - \tilde{q}) + \omega_{hd}(q, \dot{q}_d) \right. \\ &\quad \left. + \beta (M(q)(s(t) - \tilde{q}(t)) \right. \\ &\quad \left. + \frac{\partial \hat{g}}{\partial q} (s(t) - \tilde{q}(t)) \right. \\ &\quad \left. + f(\dot{q}_d, s - \tilde{q}) \right] d\tau \end{aligned} \quad (11)$$

where  $\omega_{sd}(q, \dot{q}, \ddot{q}_d)$ ,  $\omega_{hd}(q, \dot{q}_d)$  are given by

$$\left. \begin{aligned} \omega_{sd}(q, \dot{q}_d, \ddot{q}_d) &= M(q)(\ddot{q}_d(t) - \dot{q}_d(t)) + B(q, \dot{q}_d)\dot{q}_d(t) \\ &\quad - K_N \hat{K}_N^{-1} e^{-t} \hat{g}(q(0)) \\ \omega_{hd}(q, \dot{q}_d) &= M(q)\dot{q}_d(t) + \frac{1}{2} f(\dot{q}_d, \dot{q}_d) + \frac{\partial \hat{g}}{\partial q} \dot{q}_d(t) \end{aligned} \right\} \quad (12)$$

It should be emphasized that the notion for  $f(s - \tilde{q}, s - \tilde{q})$  and  $f(\dot{q}_d, s - \tilde{q})$  should follow the definition of  $f(a, b)$ . For example,  $f(\dot{q}_d, s - \tilde{q}) = [(\partial/\partial q)[\dot{q}_d(t)^T M(q)(s - \tilde{q})]]^T$ .

Based on the error system (10), if the signal  $I_F(t)$  can be treated as a control input signal, the signal  $I_F(t)$  can be synthesized as

$$I_F(t) = -\beta \gamma_s \hat{K}_N^{-1} s(t) + \hat{K}_N^{-1} \left( \hat{g}(q) - \int_0^t e^{-(t-\tau)} \hat{g}(q) d\tau \right) \quad (13)$$

for any  $\beta \geq 2$ . Then, it can be proved that this robust control law can stabilize the error system (10) with proper selection of the scalar positive gain  $\gamma_s$ .

*Remark:* As a matter of fact, many robust control laws have been proposed in the literature and can be directly applied. However, the merit of this new control law is that only one design parameter  $\beta$  needs to be adjusted, and there is no regressor involved, which makes the controller design simple.

In the above control law, the measurement of joint velocities  $\dot{q}(t)$  is required. However, as assumed in the paper, the signal  $\dot{q}(t)$  is not available. To remove such a requirement, instead of using  $I_F(t)$  in (13), an embedded signal  $I_{F_d}(t)$  is defined as

$$I_{F_d}(t) = -\beta\gamma_s \widehat{K}_N^{-1} \widehat{s}(t) + \widehat{K}_N^{-1} \left( \widehat{g}(q) - \int_0^t e^{-(t-\tau)} \widehat{g}(q) d\tau \right) \quad (14)$$

where the signal  $\widehat{s}(t)$  denotes the estimate of the tracking error signal  $s(t)$  and is generated by

$$\left. \begin{aligned} \widehat{s}(t) &= (\gamma_e \beta^{-1} + 1) \widetilde{q}(t) + \beta^{-1} \widetilde{\dot{q}}(t) + s_e(t) \\ \dot{s}_e(t) &= \gamma_e \widetilde{\dot{q}}(t) - \gamma_e \widehat{s}(t) \\ s_e(0) &= -(\gamma_e + 1) \beta^{-1} \widetilde{q}(0) \end{aligned} \right\} \quad (15)$$

where the scalar constant  $\gamma_e$  is a positive design parameter and will be specified later.

It can be proved that the embedded signal  $I_{F_d}(t)$  in (14) can still guarantee the tracking of the subsystem (5) if  $I_{F_d}(t)$  is treated as an input signal  $I_F(t)$ . The proof is omitted, since the conclusion can easily be drawn for the proof of the main result (*Theorem 2*) in Section III-C.

### B. Synthesis of Control Signal $\mu$

From the dynamic (5), (6), it is clear that the signal  $I_F(t)$  cannot be used as an input signal. It is the control signal  $\mu$  that generates the  $I_F(t)$ . The design task turns to the development of the control signal  $\mu$ , which forces  $I_F(t)$  to track  $I_{F_d}(t)$ .

To develop such a controller, the error signals  $\widetilde{I}_F(t)$ ,  $\widetilde{s}(t)$  are, respectively, defined as

$$\widetilde{I}_F(t) = \beta^{-1} \gamma_s^{-1} \widehat{K}_N (I_F(t) - I_{F_d}(t)), \quad \widetilde{s}(t) = s(t) - \widehat{s}(t). \quad (16)$$

The error signal  $\widetilde{I}_F(t)$  is defined so that  $\|\widetilde{I}_F(0)\|$  does not increase with respect to  $\beta$  and  $\gamma_s$ . Then, it is seen from (10), (11), (15), (6), and (7) that the tracking error system can be described by

$$\begin{aligned} M(q) \dot{s}(t) &= -(\beta B(q, s - \widetilde{q}) - (\beta + 1)M(q) + 2B(q, \dot{q}_d)) \\ &\quad \times (s(t) - \widetilde{q}(t)) - \beta^{-1} h_{q_a}(t) - \beta^{-1} h_{q_d}(t) \\ &\quad + \gamma_s K_N \widehat{K}_N^{-1} \widetilde{I}_F(t) \\ &\quad - \gamma_s K_N \widehat{K}_N^{-1} (s(t) - \widehat{s}(t)) \end{aligned} \quad (17)$$

$$\begin{aligned} M(q) \dot{\widetilde{s}}(t) &= -(\beta B(q, s - \widetilde{q}) + 2B(q, \dot{q}_d)) (s(t) - \widetilde{q}(t)) \\ &\quad - \beta^{-1} h_{q_a}(t) - \beta^{-1} h_{q_d}(t) + \gamma_s K_N \widehat{K}_N^{-1} \widetilde{I}_F(t) \\ &\quad - \gamma_s K_N \widehat{K}_N^{-1} (s(t) - \widehat{s}(t)) - \gamma_e M(q) \widetilde{s}(t) \end{aligned} \quad (18)$$

$$\left. \begin{aligned} h_{q_a}(t) &= \int_0^t e^{-(t-\tau)} \left[ \frac{\beta^2}{2} f(s - \widetilde{q}, s - \widetilde{q}) \right. \\ &\quad \left. + \beta(M(q)(s(t) - \widetilde{q}(t)) \right. \\ &\quad \left. + \frac{\partial \widehat{g}}{\partial q}(s(t) - \widetilde{q}(t)) \right. \\ &\quad \left. + f(\dot{q}_d, s - \widetilde{q}) \right] d\tau \\ h_{q_d}(t) &= \omega_{sd}(q, \dot{q}_d, \ddot{q}_d) + \int_0^t e^{-(t-\tau)} \omega_{hd}(q, \dot{q}_d) d\tau \end{aligned} \right\} \quad (19)$$

$$\begin{aligned} L \widetilde{I}_F(t) &= -R \widetilde{I}_F(t) - \frac{\widehat{K}_N}{\beta \gamma_s} \\ &\quad \times \left( \widehat{R} I_{F_d}(t) + \widehat{K}_e \dot{q}_d(t) - \int_0^t e^{-(t-\tau)} \widehat{K}_e \dot{q}_d(\tau) d\tau \right) \\ &\quad + \frac{\widehat{K}_N}{\beta \gamma_s} \mu(t) + \omega_{Is}(t) s(t) + \omega_{I\widetilde{s}}(t) \widetilde{s}(t) \\ &\quad + \omega_{I\widetilde{q}}(t) \widetilde{q}(t) + \frac{1}{\gamma_s} h_{I_a}(t) + \frac{1}{\beta \gamma_s} h_{I_d}(t) \end{aligned} \quad (20)$$

$$\left. \begin{aligned} \omega_{Is}(t) &= \widetilde{R} - \frac{1}{\gamma_s} \widehat{K}_N K_e + (\beta + 1)L - \frac{L}{\gamma_s} \frac{\partial \widehat{g}}{\partial q} \\ \omega_{I\widetilde{s}}(t) &= -\widetilde{R} + \gamma_e L \\ \omega_{I\widetilde{q}}(t) &= \frac{1}{\gamma_s} \widehat{K}_N K_e - (\beta + 1)L + \frac{L}{\gamma_s} \frac{\partial \widehat{g}}{\partial q} \end{aligned} \right\} \quad (21)$$

$$\left. \begin{aligned} h_{I_a}(t) &= (L - \widetilde{R}) \int_0^t e^{-(t-\tau)} \frac{\partial \widehat{g}}{\partial q}(s(\tau) - \widetilde{q}(\tau)) d\tau \\ &\quad + \widehat{K}_N K_e \int_0^t e^{-(t-\tau)} (s(\tau) - \widetilde{q}(\tau)) d\tau \\ h_{I_d}(t) &= -\widehat{K}_N \widetilde{K}_e \dot{q}_d(t) + \int_0^t e^{-(t-\tau)} \widehat{K}_N \widetilde{K}_e \dot{q}_d(\tau) d\tau \\ &\quad - L \frac{\partial \widehat{g}}{\partial q} \dot{q}_d(t) + (L - \widetilde{R}) \int_0^t e^{-(t-\tau)} \frac{\partial \widehat{g}}{\partial q} \dot{q}_d(\tau) d\tau \\ &\quad + (L - \widetilde{R}) e^{-t} \widehat{g}(q(0)). \end{aligned} \right\} \quad (22)$$

In the above error system, it is obvious from the properties **P1**, **P3-P6**, and **A1-A4** that there exist bounded positive constants  $\bar{\rho}_{q_d}$ ,  $\bar{\rho}_{I_d}$  such that

$$\|h_{q_d}(t)\| \leq \bar{\rho}_{q_d}, \quad \|h_{I_d}(t)\| \leq \bar{\rho}_{I_d} \quad (23)$$

and there exist bounded positive constants  $\bar{\rho}_{q_i}$ ,  $i = 1 \dots 4$ ,  $\bar{\rho}_{I_1}$  and scalar positive signals  $h_{q_i}(t)$ ,  $i = 1 \dots 4$ ,  $h_{I_1}(t)$  such that

$$\begin{aligned} \beta^{-1} \|h_{q_a}(t)\| &\leq \frac{1}{2} \sqrt{\beta \gamma_0 \bar{\rho}_{q_d} \bar{\rho}_{q_1}} h_{q_1}(t) + \frac{1}{2} \sqrt{\beta \gamma_0 \bar{\rho}_{q_d} \bar{\rho}_{q_2}} h_{q_2}(t) \\ &\quad + \frac{1}{2} \sqrt{\beta \gamma_0 \bar{\rho}_{q_d} \bar{\rho}_{q_3}} h_{q_3}(t) + \bar{\rho}_{q_4} h_{q_4}(t) \end{aligned} \quad (24)$$

$$\|h_{I_a}(t)\| \leq \bar{\rho}_{I_1} h_{I_1}(t) \quad (25)$$

$$\left. \begin{aligned} \bar{\rho}_{q_1} &= \sqrt{\bar{\rho}_f}, \quad \bar{\rho}_{q_2} = \sqrt{\frac{2\bar{\rho}_f}{\varepsilon_q}} \\ \bar{\rho}_{q_3} &= \sqrt{\frac{\bar{\rho}_f}{\varepsilon_q}}, \quad \bar{\rho}_{q_4} = \sqrt{\bar{\rho}_m + \bar{\rho}_{a_1} + \bar{\rho}_f \bar{\rho}_{d_1}} \\ \bar{\rho}_{I_1} &= (\bar{\rho}_\varepsilon + \bar{\rho}_r) \bar{\rho}_{q_1} + \|\widehat{K}_N\| \bar{\rho}_{k_e} \\ h_{q_1}(t) &= -h_{q_1}(t) + \sqrt{\beta \bar{\rho}_f} \sqrt{\frac{1}{\gamma_0 \bar{\rho}_{q_d}}} \|s(t)\|^2 \\ h_{q_2}(t) &= -h_{q_2}(t) + \sqrt{2\beta \bar{\rho}_f} \sqrt{\frac{\varepsilon_q}{\gamma_0 \bar{\rho}_{q_d}}} \|s(t)\| \|\widetilde{q}(t)\| \\ h_{q_3}(t) &= -h_{q_3}(t) + \sqrt{\beta \bar{\rho}_f} \sqrt{\frac{\varepsilon_q}{\gamma_0 \bar{\rho}_{q_d}}} \|\widetilde{q}(t)\|^2 \\ h_{q_4}(t) &= -h_{q_4}(t) + \bar{\rho}_{q_4} \|s(t) - \widetilde{q}(t)\| \\ h_{I_1}(t) &= -h_{I_1}(t) + \|s(t)\| + \|\widetilde{q}(t)\| \\ h_{q_i} &= 0, \quad i = 1, 2, 3, \quad h_{I_1}(0) = 0 \end{aligned} \right\} \quad (26)$$

where  $\gamma_0$  is a positive design parameter,  $\varepsilon_q$  is a positive constant, and these parameters are specified later.

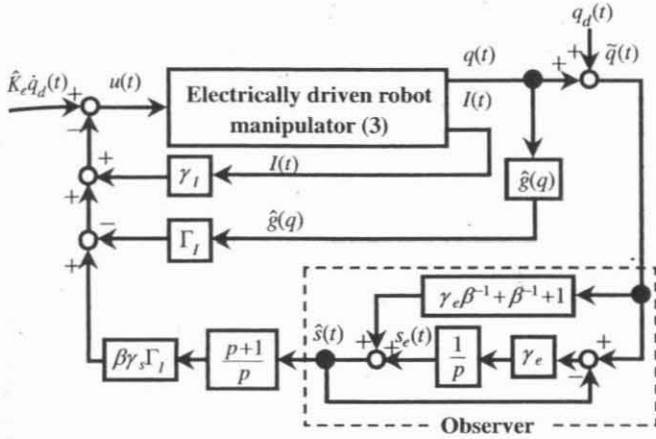


Fig. 1. Configuration of the closed-loop system using the proposed controller.

Based on the above error dynamics, the robust control law is then synthesized by

$$\mu(t) = -\beta\gamma_s\gamma_i\widehat{K}_N^{-1}\tilde{I}_F + \widehat{R}I_{Fd}(t) + \widehat{K}_e\dot{q}_d(t) - \int_0^t e^{-(t-\tau)}\widehat{K}_e\dot{q}_d(\tau)d\tau \quad (28)$$

and the design parameters are given by the following form:

$$\left. \begin{aligned} \gamma_s &= \gamma_{s1}\gamma_0\beta^2 + \gamma_{s2}\gamma_0\beta + \gamma_{s3}\beta + \gamma_{s4} \\ \gamma_e &= \gamma_{e1}\gamma_s + \gamma_{e2}\gamma_0\beta^2 + \gamma_{e3}\gamma_0\beta + \gamma_{e4}\beta + \gamma_{e5} \\ \gamma_I &= \gamma_{I1}\gamma_s + \gamma_{I2}\gamma_e + \gamma_{I3}\beta^2 + \frac{1}{4}\bar{\rho}_\ell\beta + \gamma_{I4} + \gamma_{I5}\frac{1}{\gamma_s^2} \end{aligned} \right\} \quad (29)$$

where  $\gamma_{si}, i = 1 \dots 4$ ;  $\gamma_{ei}, i = 1 \dots 5$ ;  $\gamma_{Ii}, i = 1 \dots 5$  are positive design parameters.

**Theorem 1:** In (28), the control law is expressed as the input to the compensator. Based on the definition of  $\mu(t)$  in (4), the control law (28) can be expressed as

$$\begin{aligned} u(t) &= -\beta\gamma_s\Gamma_I \left( \hat{s}(t) + \int_0^t \hat{s}(\tau)d\tau \right) - \gamma_I I(t) + \Gamma_I \hat{g}(q) + \widehat{K}_e \dot{q}_d(t) \\ \Gamma_I &= \gamma_I \widehat{K}_N^{-1} + \widehat{R} \widehat{K}_N^{-1}. \end{aligned} \quad (30)$$

*Proof:* See Appendix II.

The configuration of the closed-loop system using the proposed controller is shown in Fig. 1. The symbol  $p$  in Fig. 1 denotes the differential operator. In Fig. 1, the gains  $\gamma_s, \gamma_e, \gamma_I$ , and  $\beta$  are scalar, and matrices  $\widehat{K}_e$  and  $\Gamma_I$  are constant diagonal matrices. The configuration of the controller is very simple, as compared with the regressor-based adaptive controllers.

### C. Stability Analysis

Before describing the stability analysis of the closed-loop system, the following lemma is required.

**Lemma 1:** Let us consider a nonnegative function  $V(t)$ . It is assumed that  $\gamma_0$  is a fixed constant value, such that  $\gamma_0\bar{\rho}_{qd} \geq 2V(0)$ . If the derivative of  $V(t)$  satisfies

$$\dot{V}(t) \leq -V(t) + \frac{1}{\gamma_0\bar{\rho}_{qd}}V(t)^2 + \frac{5\bar{\rho}_{qd}}{4\beta^2} \quad (31)$$

then  $V(t)$  is uniformly bounded for any  $\beta > \sqrt{5\gamma_0^{-1}}$ , and satisfies the relation

$$V(t) \leq \frac{\gamma_0\bar{\rho}_{qd}}{2}. \quad (32)$$

*Proof:* See Appendix III.

For stability analysis, we also need the following definitions:

$$\varepsilon_q = 3\bar{\rho}_m + \bar{\rho}_f + 4\bar{\rho}_b\bar{\rho}_{d1} + 5 + \bar{\rho}_{q4}^2. \quad (33)$$

$$\begin{aligned} \bar{\rho}_{\omega 11} &= \|\widehat{K}_N\|\bar{\rho}_{ke} + \bar{\rho}_{\omega 1}\bar{\rho}_\ell \\ \bar{\rho}_{vs1} &= \left( \frac{(\bar{\rho}_{m1} + 2\bar{\rho}_b)^2 + \bar{\rho}_{m1}^2 + 2\bar{\rho}_b^2}{4} \right. \\ &\quad \left. + \frac{(\bar{\rho}_{m1} + 4\bar{\rho}_b)^2 + 32\bar{\rho}_b^2}{8\varepsilon_q} + \frac{2\bar{\rho}_b^2}{\varepsilon_q^2} \right) \bar{\rho}_{qd} \end{aligned} \quad (34)$$

$$\bar{\rho}_{vs2} = \sum_{i=1}^3 \frac{\bar{\rho}_{qi}^2\bar{\rho}_{qd}}{2}, \quad \bar{\rho}_{vs3} = 3\bar{\rho}_m$$

$$\bar{\rho}_{vs4} = \bar{\rho}_{m1}\bar{\rho}_{d1} + 3\bar{\rho}_m + 8\bar{\rho}_b\bar{\rho}_{d1} + 4\bar{\rho}_{q4}^2 + 2\bar{\rho}_{qd}$$

$$\bar{\rho}_{vs5} = \left( \frac{\bar{\rho}_{m1}^2}{4\varepsilon_q} + \frac{2\bar{\rho}_b^2}{\varepsilon_q^2} \right) \bar{\rho}_{qd}$$

$$\bar{\rho}_{vs6} = \bar{\rho}_{m1}\bar{\rho}_{d1} + 4\bar{\rho}_b\bar{\rho}_{d1} + 4\bar{\rho}_{q4}^2 + 2\bar{\rho}_{qd}.$$

The stability of the closed-loop system described by (5), (6), (14), (15), (28), and (29) is now stated by the following theorem.

**Theorem 2:** Let us consider the controller (15), (28), and (29) for the robot manipulator (3) with initial values  $\dot{q}(0) = 0, I(0) = 0$ , and  $g(q(0)) = 0$ . If the positive design parameters  $\gamma_0$  and  $\gamma_{si}, i = 1 \dots 4; \gamma_{ei}, i = 1 \dots 5; \gamma_{Ii}, i = 1 \dots 5$  are fixed so that the following inequalities are satisfied:

$$\left. \begin{aligned} \gamma_{s1} &\geq \frac{1}{\bar{\rho}_{kk}}\bar{\rho}_{vs1}, \quad \gamma_{s2} \geq \frac{1}{\bar{\rho}_{kk}}\bar{\rho}_{vs2} \\ \gamma_{s3} &\geq \frac{1}{\bar{\rho}_{kk}}(\bar{\rho}_{vs3} + \bar{\rho}_f + \varepsilon_q + 0.5\bar{\rho}_m) \\ \gamma_{s4} &\geq \frac{1}{\bar{\rho}_{kk}}(5 + \bar{\rho}_{vs4}) \end{aligned} \right\} \quad (35)$$

$$\left. \begin{aligned} \gamma_{e1} &\geq 1 + 2\bar{\rho}_{kk}, \quad \gamma_{e2} \geq \bar{\rho}_{vs5}, \quad \gamma_{e3} \geq \bar{\rho}_{vs2} \\ \gamma_{e4} &\geq 0.5\bar{\rho}_m, \quad \gamma_{e5} \geq \bar{\rho}_{vs6} + 1 \end{aligned} \right\} \quad (36)$$

$$\left. \begin{aligned} \gamma_{I1} &\geq \frac{\bar{\rho}_{kk}^2}{2\bar{\rho}_{kk}} + \frac{\bar{\rho}_{kk}^2}{2}, \quad \gamma_{I2} \geq \frac{\bar{\rho}_\ell^2}{2}, \quad \gamma_{I3} \geq 3\bar{\rho}_\ell^2 \\ \gamma_{I4} &\geq \frac{3}{2}(\bar{\rho}_\tau + \bar{\rho}_\ell)^2 + \frac{3}{2}\bar{\rho}_\ell^2 + \frac{\bar{\rho}_\tau^2}{2} \\ \gamma_{I5} &\geq 3\bar{\rho}_{\omega 11}^2 + \bar{\rho}_{11}^2 + 2\frac{\bar{\rho}_{1d}^2}{\bar{\rho}_{qd}} \end{aligned} \right\} \quad (37)$$

$$\begin{aligned} \gamma_0 &\geq \frac{2}{\bar{\rho}_{qd}} \left[ \bar{\rho}_m \left( \|\tilde{q}(0)\| + \frac{1}{2}\|\dot{\tilde{q}}(0)\| \right)^2 \right. \\ &\quad \left. + \frac{\bar{\rho}_m}{4}\|\dot{\tilde{q}}(0)\|^2 + \varepsilon_q\|\tilde{q}(0)\|^2 \right. \\ &\quad \left. + \bar{\rho}_\ell \left( \|\tilde{q}(0)\| + \frac{\bar{\rho}_{kk}}{10}\|\hat{g}(q(0))\| \right)^2 \right]. \end{aligned} \quad (38)$$

Then, the closed-loop system using the controller (15), (28), and (29) becomes stable for any  $\beta$  such that

$$\beta \geq 2 \text{ and } \beta > \sqrt{5\gamma_0^{-1}}. \quad (39)$$

Moreover, there exists a positive constant  $\delta$  independent of the design parameter  $\beta$  such that

$$\|\tilde{q}(t)\|^2 \leq \varepsilon_q^{-1}e^{-\frac{\beta}{2}t}V(0) + \frac{\delta}{\varepsilon_q\beta}. \quad (40)$$

*Proof:* Consider a positive definite function

$$\begin{aligned} V(t) &= V_s(t) + V_{\tilde{s}}(t) + V_q(t) + V_I(t) + V_h(t) \\ V_s(t) &= s(t)^T M(q)s(t), \quad V_{\tilde{s}}(t) = \tilde{s}(t)^T M(q)\tilde{s}(t) \\ V_q(t) &= \varepsilon_q \tilde{q}(t)^T \tilde{q}(t), \quad V_I(t) = \tilde{I}_F(t)^T L \tilde{I}_F(t) \\ V_h(t) &= \sum_{i=1}^4 h_{qi}(t)^2 + h_{I1}(t)^2 \end{aligned} \quad (41)$$

where  $\varepsilon_q$  is defined in (33).

It can be proved that the derivatives of  $V_s(t)$ ,  $V_{\tilde{s}}(t)$ ,  $V_q(t)$ ,  $V_I(t)$ , and  $V_h(t)$  satisfy the following inequalities (see Appendix IV), where relation  $\rho_m = 1$  has been used in the derivation of the following inequalities (43) and (46), and the relation  $\beta \varepsilon_q - 2\bar{\rho}_{q4}^2 \geq 0$  has been used in the derivation of the following inequality (45):

$$\begin{aligned} \dot{V}_s(t) + \dot{V}_{\tilde{s}}(t) &\leq -\underline{\rho}_{kk} \widehat{\gamma}_s \|s(t)\|^2 - (2\gamma_c - \gamma_s - 2\bar{\rho}_{kk} \widehat{\gamma}_s) \|\tilde{s}(t)\|^2 \\ &\quad + (\bar{\rho}_{vs1} \gamma_0 \beta^2 + \bar{\rho}_{vs2} \gamma_0 \beta + \bar{\rho}_{vs3} \beta + \bar{\rho}_{vs4}) \|s(t)\|^2 \\ &\quad + (\bar{\rho}_{vs5} \gamma_0 \beta^2 + \bar{\rho}_{vs2} \gamma_0 \beta + \bar{\rho}_{vs6}) \|\tilde{s}(t)\|^2 \\ &\quad + (\bar{\rho}_m \beta + \bar{\rho}_m + 4\bar{\rho}_b \bar{\rho}_{d1}) \|\tilde{q}(t)\|^2 \\ &\quad + \frac{1}{\gamma_0 \bar{\rho}_{qd}} (V_s(t) + V_{\tilde{s}}(t) + V_q(t))^2 + \sum_{i=1}^3 h_{qi}(t)^2 \\ &\quad + \frac{1}{2} h_{q4}(t)^2 + \left( \frac{\bar{\rho}_{kk}^2}{\underline{\rho}_{kk}} + \bar{\rho}_{kk}^2 \right) \gamma_s \|\tilde{I}_F(t)\|^2 \\ &\quad + \frac{1}{\beta^2} \frac{\|h_{qd}(t)\|^2}{\bar{\rho}_{qd}} \end{aligned} \quad (43)$$

$$\begin{aligned} \dot{V}_I(t) &\leq -2\gamma_I \|\tilde{I}_F(t)\|^2 + \left[ \left( 6\bar{\rho}_{\omega 11}^2 + 2\bar{\rho}_{11}^2 + 4\frac{\bar{\rho}_{1d}^2}{\bar{\rho}_{qd}} \right) \frac{1}{\gamma_s^2} \right. \\ &\quad \left. + 3(\bar{\rho}_r + \bar{\rho}_\ell)^2 + 3\bar{\rho}_\ell^2 + \bar{\rho}_r^2 \right. \\ &\quad \left. + 6\bar{\rho}_\ell^2 \beta^2 + \bar{\rho}_\ell^2 \gamma_c \right] \|\tilde{I}_F(t)\|^2 \\ &\quad + \|s(t)\|^2 + \|\tilde{q}(t)\|^2 + (\gamma_c + 1) \|\tilde{s}(t)\|^2 \\ &\quad + \frac{1}{2} h_{I1}(t)^2 + \frac{1}{4\beta^2} \frac{\|h_{Id}(t)\|^2 \bar{\rho}_{qd}}{\bar{\rho}_{1d}^2} \end{aligned} \quad (44)$$

$$\begin{aligned} \dot{V}_q(t) &\leq -(\varepsilon_q \beta + 2\bar{\rho}_{q4}^2) \|\tilde{q}(t)\|^2 + (\varepsilon_q \beta - 2\bar{\rho}_{q4}^2) \|s(t)\|^2 \\ &\quad + 4\bar{\rho}_{q4}^2 \tilde{q}(t)^T s(t) \end{aligned} \quad (45)$$

$$\begin{aligned} \dot{V}_h(t) &\leq -V_h(t) - \sum_{i=1}^3 h_{qi}(t)^2 - \frac{1}{2} h_{q4}(t)^2 - \frac{1}{2} h_{I1}(t)^2 \\ &\quad + (4 + 2\bar{\rho}_{q4}^2 + \bar{\rho}_f \beta) \|s(t)\|^2 \\ &\quad + \left( 4 + 2\bar{\rho}_{q4}^2 + \frac{\bar{\rho}_f \beta}{2} \right) \|\tilde{q}(t)\|^2 - 4\bar{\rho}_{q4}^2 \tilde{q}(t)^T s(t) \\ &\quad + \frac{2}{\gamma_0 \bar{\rho}_{qd}} (V_s(t) + V_q(t)) V_h(t). \end{aligned} \quad (46)$$

Since  $\beta \geq 2$ , it follows from (33) that

$$\begin{aligned} -\beta \varepsilon_q &= -\frac{\beta}{2} \varepsilon_q - \frac{\beta}{2} \varepsilon_q \\ &\leq -\frac{\beta}{2} \varepsilon_q - \left( \bar{\rho}_m + \frac{\bar{\rho}_f}{2} \right) \beta - \bar{\rho}_m - 4\bar{\rho}_b \bar{\rho}_{d1} - 5 - \bar{\rho}_{q4}^2. \end{aligned} \quad (47)$$

Considering inequalities (35)–(37) are satisfied, it is seen from (43)–(47) that the following relation can be obtained:

$$\begin{aligned} \dot{V}(t) &\leq -\frac{\beta}{2} (V_s(t) + V_{\tilde{s}}(t) + V_q(t) + V_I(t)) - V_h(t) \\ &\quad + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V(t)^2 + \frac{1}{\beta^2} \frac{\|h_{qd}(t)\|^2}{\bar{\rho}_{qd}} + \frac{1}{4\beta^2} \frac{\|h_{Id}(t)\|^2 \bar{\rho}_{qd}}{\bar{\rho}_{1d}^2} \\ &\leq -V(t) + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V(t)^2 + \frac{1}{\beta^2} \frac{\|h_{qd}(t)\|^2}{\bar{\rho}_{qd}} \\ &\quad + \frac{1}{4\beta^2} \frac{\|h_{Id}(t)\|^2 \bar{\rho}_{qd}}{\bar{\rho}_{1d}^2} \\ &\leq -V(t) + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V(t)^2 + \frac{5\bar{\rho}_{qd}}{4\beta^2}. \end{aligned} \quad (48)$$

The initial value  $V(0)$  does not increase with respect to  $\beta$  and  $\gamma_s$ . Considering this fact, it is seen from (38) and (41) that  $2V(0) \leq \gamma_0 \bar{\rho}_{qd}$  is assured for any  $\beta \geq 2$  and  $\gamma_s \geq (5/\underline{\rho}_{kk})$ . According to Lemma 1, it is concluded from (48) and (39) that the closed-loop system is stable, and  $2V(t) \geq \gamma_0 \bar{\rho}_{qd}$ . Analyzing the derivative of the positive definite function  $V_c(t) = V_s(t) + V_{\tilde{s}}(t) + V_q(t) + V_I(t)$  by using the fact  $2V(t) \leq \gamma_0 \bar{\rho}_{qd}$ , it is easy to ascertain from (43)–(45) that there exists a positive constant  $\delta_1$  independent of the design parameter  $\beta$

$$\dot{V}_c(t) \leq -\frac{\beta}{2} V_c(t) + \delta_1. \quad (49)$$

The relation (40) can be obtained from (49).

*Remarks:*

- 1) From Theorem 2, it can be concluded that the closed-loop system using the proposed controller is robust stable while the inequalities (35)–(39) are satisfied, and we can make  $\|\tilde{q}(t)\|^2$  arrive at any small closed region with any convergent rate by setting the design parameter  $\beta$ .
- 2) Especially in the case of  $V(0) = 0$ , from (40), it is seen that the maximum value of  $\|\tilde{q}(t)\|$  can be arbitrarily reduced by increasing the value of the design parameter  $\beta$ .
- 3) If a bounded disturbance  $c(t)$  exists in robot dynamics (3) as  $M(q)\ddot{q}(t) + B(q, \dot{q})\dot{q}(t) + g(q) = K_N I(t) + c(t)$ . The changes appear only in  $h_{qd}(t)$  (19) as

$$\begin{aligned} h_{qd}(t) &= \omega_{sd}(q, \dot{q}, \ddot{q}) + \int_0^t e^{-(t-\tau)} \omega_{hd}(q, \dot{q}, \ddot{q}) d\tau \\ &\quad + c(t) - \int_0^t e^{-(t-\tau)} c(\tau) d\tau. \end{aligned}$$

In the closed-loop system using the proposed controller, it can also be shown that the tracking error  $\tilde{q}(t)$  converges to zero if the desired trajectories  $q_d(t)$  converge to constant ultimate values. To show this, let us consider the new signal

$$h_d(t) = \frac{\|h_{qd}(t)\|^2}{\bar{\rho}_{qd}} + \frac{\|h_{Id}(t)\|^2 \bar{\rho}_{qd}}{4\bar{\rho}_{1d}^2}. \quad (50)$$

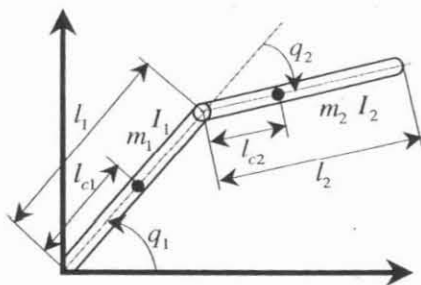


Fig. 2. Two-link manipulator.

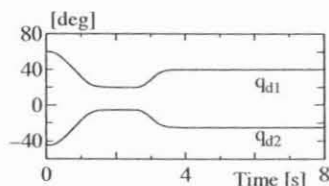


Fig. 3. Desired trajectory.

In the case of  $q_d(t)$  converging to a constant vector, there exist bounded positive constants  $d_1, d_2$  such that

$$h_d(t) \leq d_2 e^{-d_1 t}. \quad (51)$$

Using the relation above, it can be seen from *Theorem 1* that the following corollary holds.

*Corollary 1:* Suppose  $q_d(t)$  converges to a constant vector exponentially. If the fixed design parameters  $\gamma_0, \gamma_{si}, i = 1 \dots 4; \gamma_{ei}, i = 1 \dots 5; \gamma_{li}, i = 1 \dots 5$  satisfy (35)–(38), then the tracking error converges to zero for any  $\beta$  satisfying (39).

*Proof:* It is obvious that *Theorem 2* holds. Then, as stated in the proof of *Theorem 2*,  $V(t)$  satisfies the relation  $V(t) \leq (1/2)\gamma_0 \bar{p}_{q_d}$ . According to the second inequality in (48), it can be seen that the following relation is satisfied:

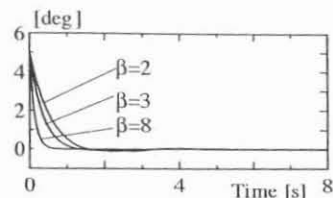
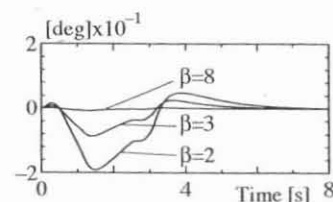
$$\dot{V}(t) \leq -\frac{1}{2}V(t) + \frac{h_d(t)}{\beta^2} \leq -\frac{1}{2}V(t) + \frac{d_2}{\beta^2} e^{-d_1 t}. \quad (52)$$

It is concluded immediately from the equation above that  $V(t)$  converges to zero and the tracking error  $\tilde{q}(t)$  also converges to zero.

#### IV. SIMULATION EXAMPLE

In this simulation, the controller is designed for a two-link robot manipulator, shown in Fig. 2. The nominal values of the manipulator and actuator parameters are given as [22]  $l_1 = 0.6$  m,  $m_1 = 18.3$  kg,  $l_{c1} = 0.37$  m,  $I_1 = 0.892$  kg·m<sup>2</sup>,  $l_2 = 1.02$  m,  $m_2 = 28.5$  kg,  $l_{c2} = 0.0234$  m,  $I_2 = 3.29$  kg·m<sup>2</sup>,  $m_e = 2$  kg,  $J_1 = 7.91$  kg·m<sup>2</sup>,  $J_2 = 7.91$  kg·m<sup>2</sup>,  $L_1 = 5.2 \times 10^3$  V·s/A,  $L_2 = 5.2 \times 10^3$  V·s/A,  $R_1 = 2$   $\Omega$ ,  $R_2 = 2$   $\Omega$ ,  $K_{e1} = 21$  V·s,  $K_{e2} = 21$  V·s,  $K_{N1} = 28.8$  V·s,  $K_{N2} = 28.8$  V·s, where  $m_e$  denotes the weight of the end-effector.

Let the uncertainty in robot dynamics be originated by the weight of the end-effector varying in the range of 1–3 kg, and electrical parameters be assumed to have  $\pm 20\%$  uncertainty. The controller shown in Fig. 1 is applied to the electrically driven robot manipulator with the true parameters that are given by  $m_e = 3$  kg,

Fig. 4. Tracking error responses for  $\beta = 2, 4, 10$  in the case of  $\tilde{q}(0) = [5, -5]^T$  deg.Fig. 5. Tracking error responses for  $\beta = 2, 4, 10$  in the case of  $\tilde{q}(0) = [0, 0]^T$  deg.

$J_1 = 9.492$  kg·m<sup>2</sup>,  $J_2 = 9.492$  kg·m<sup>2</sup>,  $L_1 = 6.24 \times 10^{-3}$  V·s/A,  $L_2 = 6.24 \times 10^{-3}$  V·s/A,  $R_1 = 2.4$   $\Omega$ ,  $R_2 = 2.4$   $\Omega$ ,  $K_{e1} = 25.2$  V·s,  $K_{e2} = 25.2$  V·s,  $K_{N1} = 34.56$  V·s,  $K_{N2} = 34.56$  V·s. The initial values of the manipulator are  $\dot{q}_1(0) = \dot{q}_2(0) = 0$ ,  $q_1(0) = 65$  deg,  $q_2(0) = -40$  deg, and the initial value of the desired trajectories are set as  $\dot{q}_{d1}(0) = \dot{q}_{d2}(0) = 0$ ,  $q_{d1}(0) = 60$  deg,  $q_{d2}(0) = -45$  deg. The desired trajectories of joint 1 and joint 2 are shown in Fig. 3. It is noted that the desired trajectories converge to constant ultimate values. The design parameters are set so as to satisfy the inequalities in (35)–(38) and are given by  $\gamma_0 = 1.3$ ,  $\gamma_{s1} = 6.3 \times 10$ ,  $\gamma_{s2} = 1.5 \times 10$ ,  $\gamma_{s3} = 8.5 \times 10$ ,  $\gamma_{s4} = 2.1 \times 10^2$ ,  $\gamma_{e1} = 3.4$ ,  $\gamma_{e2} = 1.1 \times 10^{-1}$ ,  $\gamma_{e3} = 1.2 \times 10$ ,  $\gamma_{e4} = 3.1$ ,  $\gamma_{e5} = 1.5 \times 10^2$ ,  $\gamma_{l1} = 1.7$ ,  $\gamma_{l2} = 1.3 \times 10^{-3}$ ,  $\gamma_{l3} = 7.6 \times 10^{-3}$ ,  $\gamma_{l4} = 2.5 \times 10^{-2}$ ,  $\gamma_{l5} = 3.9 \times 10^{-1}$ ,  $\gamma_{l6} = 2.1 \times 10^4$ .

Fig. 4 shows trajectory tracking errors for joint 1. As shown in Fig. 4, the tracking error converges to zero, and the convergent rate becomes more rapid as the design parameter  $\beta$  becomes larger. To show that  $\|\tilde{q}(t)\|^2$  can be arbitrarily reduced in the case of  $V(0) = 0$  in (40), tracking errors for joint 1 in the case of the initial tracking error  $\tilde{q}(0) = 0$  are shown in Fig. 5. It can be seen that the maximum value of  $\|\tilde{q}(t)\|^2$  decreases as the design parameter  $\beta$  increases. As shown in Figs. 4 and 5, it is concluded that the tracking performance can be easily improved by using design parameter  $\beta$ . We should mention that only joint 1 is illustrated in the above and that the joint 2 is omitted to save space.

#### V. CONCLUSIONS

In this paper, a novel robust tracking controller is developed for electrically driven robot manipulators. The main feature of the controller is that the measurements of joint velocities and calculation of the robot regressor are not required. Its configuration is very simple. Moreover, by theoretical analysis and numerical simulations, the proposed controller has the following properties. Tracking performance can be easily improved by setting only one design parameter. Especially in the case of  $V(0) = 0$ , the maximum value of  $\|\tilde{q}(t)\|$  can be arbitrarily reduced by increasing the value of the design parameter  $\beta$ . Even if unmodeled bounded disturbances appear in robot dynamics, this property is still assured. If desired trajectories converge to constant ultimate values, the asymptotic stability of the tracking errors is assured.

APPENDIX I  
DERIVATION OF NEW REPRESENTATION

Substituting the symbol  $\tau$  for the symbol  $t$  in the first and second equation of (3), and multiplying both sides by  $e^{-(t-\tau)}$ , and then integrating from 0 to  $t$  with respect to  $\tau$ , we obtain

$$\left. \begin{aligned} \int_0^t e^{-(t-\tau)} \xi(\tau) d\tau + \int_0^t e^{-(t-\tau)} g(\tau) d\tau \\ = K_N \int_0^t e^{-(t-\tau)} I(\tau) d\tau \\ \int_0^t e^{-(t-\tau)} (LI(\tau) + RI(\tau) + K_e \dot{q}(\tau)) d\tau \\ = \int_0^t e^{-(t-\tau)} u(\tau) d\tau \end{aligned} \right\} \quad (53)$$

where

$$\xi(t) = M(q)\ddot{q}(t) + B(q, \dot{q})\dot{q}(t). \quad (54)$$

Differentiating both sides of (53), we have the new representation of electrically driven robot manipulators as (5)–(7). Here, the relations  $\dot{q}(0) = 0$ ,  $I(0) = 0$ ,  $g(q(0)) = 0$ , and the following relations are used to derive the new representation:

$$\begin{aligned} \frac{d}{dt} \left[ \int_0^t e^{-(t-\tau)} g(q(\tau)) d\tau \right] \\ = \frac{d}{dt} \left[ g(q) - e^{-t} g(q(0)) - \int_0^t e^{-(t-\tau)} \frac{\partial g}{\partial q} \dot{q}(\tau) d\tau \right] \\ = e^{-t} g(q(0)) + \int_0^t e^{-(t-\tau)} \frac{\partial g}{\partial q} \dot{q}(\tau) d\tau \end{aligned} \quad (55)$$

$$\begin{aligned} \int_0^t e^{-(t-\tau)} M(q(\tau)) \ddot{q}(\tau) d\tau \\ = M(q)\ddot{q}(t) - e^{-t} M(q(0))\dot{q}(0) \\ - \int_0^t e^{-(t-\tau)} (M(q(\tau))\dot{q}(\tau) + \dot{M}(q(\tau))\dot{q}(\tau)) d\tau \end{aligned} \quad (56)$$

$$\begin{aligned} \frac{d}{dt} \left[ \int_0^t e^{-(t-\tau)} I(\tau) d\tau \right] \\ = I_F(t) \end{aligned} \quad (57)$$

$$\frac{d}{dt} \left[ \int_0^t e^{-(t-\tau)} \dot{I}(\tau) d\tau \right] = \frac{d}{dt} [I_F(t) - e^{-t} I(0)] = \dot{I}_F(t) + e^{-t} I(0) \quad (58)$$

$$\begin{aligned} \frac{d}{dt} \left[ \int_0^t e^{-(t-\tau)} u(\tau) d\tau \right] \\ = \frac{d}{dt} \left[ \int_0^t e^{-(t-\tau)} \mu(\tau) d\tau + \int_0^t e^{-(t-\tau)} \int_0^\tau \mu(\sigma) d\sigma d\tau \right] \\ = \frac{d}{dt} \left[ \int_0^t e^{-(t-\tau)} \mu(\tau) d\tau + \int_0^t \mu(\tau) d\tau \right. \\ \left. - \int_0^t e^{-(t-\tau)} \mu(\tau) d\tau \right] = \mu(t). \end{aligned} \quad (59)$$

APPENDIX II  
PROOF OF THEOREM 1

Let the signal  $\xi(t)$  be given by

$$\xi(t) = \eta(t) - \int_0^t e^{-(t-\tau)} \eta(\tau) d\tau. \quad (60)$$

Using the integration by parts, it is easily ascertain that the following relation holds:

$$\int_0^t e^{-\tau} \int_0^\tau e^{\sigma} \eta(\sigma) d\sigma d\tau = - \int_0^t e^{-(t-\tau)} \eta(\tau) d\tau + \int_0^t \eta(\tau) d\tau. \quad (61)$$

From (60) and (61), it follows that:

$$\begin{aligned} \xi(t) + \int_0^t \xi(\tau) d\tau \\ = \eta(t) - \int_0^t e^{-(t-\tau)} \eta(\tau) d\tau + \int_0^t \eta(\tau) d\tau - \int_0^t e^{-\tau} \int_0^\tau e^{\sigma} \eta(\sigma) d\sigma d\tau \\ = \eta(t). \end{aligned} \quad (62)$$

Using the fact that the signal  $\xi(t)$  given by (60) satisfies (62), it can be seen from (4), the third equation in (7), (14), (16), and (28) that the expression (30) is derived.

APPENDIX III  
PROOF OF LEMMA 1

The roots of the equation

$$-V(t) + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V(t)^2 + \frac{5\bar{\rho}_{qd}}{4\beta^2} = 0 \quad (63)$$

are given by

$$\left. \begin{aligned} D_- &= \frac{\gamma_0 \bar{\rho}_{qd}}{2} \left( 1 - \sqrt{1 - 5\beta^{-2} \gamma_0^{-1}} \right) \\ D_+ &= \frac{\gamma_0 \bar{\rho}_{qd}}{2} \left( 1 + \sqrt{1 - 5\beta^{-2} \gamma_0^{-1}} \right) \end{aligned} \right\}. \quad (64)$$

It can be seen that there are two different real roots for any  $\beta > \sqrt{5\gamma_0^{-1}}$ . From this fact, it follows that the following inequality holds:

$$-V(t) + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V(t)^2 + \frac{5\bar{\rho}_{qd}}{4\beta^2} \leq 0, \quad \text{for } V(t) \in [D_-, D_+]. \quad (65)$$

Using the fact, it can be proved as stated below that the following properties hold:

- 1) in the case where  $V(0) \leq D_-$ ,  $V(t)$  remains in the region  $V(t) \leq D_-$ ;
- 2) in the case where  $D_- < V(0) < D_+$ ,  $V(t)$  remains in the region  $V(t) \leq V(0)$ .

It is assumed in Lemma 1 that  $V(0) \leq (1/2)\gamma_0 \bar{\rho}_{qd} < D_+$ . If  $D_- < V(0) < D_+$ , from 2) it follows that  $V(t) \leq V(0) \leq (1/2)\gamma_0 \bar{\rho}_{qd}$ . If  $V(0) \leq D_-$ , from 1) it follows that  $V(t) \leq D_- < (1/2)\gamma_0 \bar{\rho}_{qd}$ . From the facts, it immediately follows that Lemma 1 holds.

1) Now, let us suppose that there is  $t_2 > 0$ , such that  $V(t_2) = \rho_D > D_-$ . Since  $V(0) \leq D_-$ , there is a  $t_1 \in [0, t_2)$  such that

$$V(t_1) = D_-, D_- < V(t) \leq \rho_D, \quad \text{for } t \in (t_1, t_2]. \quad (66)$$

However, integrating both sides of (31) from  $t_1$  to  $t_2$ , it is seen from (65) that the following inequality is satisfied:

$$V(t_2) \leq V(t_1) + \int_{t_1}^{t_2} \left( -V(t) + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V(t)^2 + \frac{4\bar{\rho}_{qd}}{5\beta^2} \right) dt \leq D_- \quad (67)$$

The relation above contradicts supposition (66). Consequently,  $V(t) \leq D_-$  holds.

2) From (65) it can be seen that  $V(t)$  does not increase while  $V(t) \in [D_-, D_+]$ . Additionally, from the fact that if property 1) holds, it is obvious that property 2) also holds.

#### APPENDIX IV DERIVATION OF (43)–(46)

Based on the definition of  $\rho_m$ , we can simply take it as  $\rho_m = 1$ . In this case, the relations  $2\|a(t)\|\|s(t)\| \leq \rho_m \|a(t)\|^2 + \|s(t)\|^2$ ,  $\rho_m \leq \|a(t)\|^2 + V_s(t)$ , and  $2\|a(t)\|\|\tilde{s}(t)\| \leq \rho_m \|a(t)\|^2 + \|\tilde{s}(t)\|^2$ ,  $\rho_m \leq \|a(t)\|^2 + V_s(t)$  can be established for any signal  $a(t)$ . The following analysis is performed by using the above relations.

1) It can be seen from (42), (23), and (24) that the following inequalities hold:

$$\begin{aligned} & (\bar{\rho}_{m1} + 2\bar{\rho}_b)\beta \|s(t)\|^3 \\ & \leq \frac{(\bar{\rho}_{m1} + 2\bar{\rho}_b)^2 \bar{\rho}_{qd}}{4} \gamma_0 \beta^2 \|s(t)\|^2 + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V_s(t)^2 \\ & (\bar{\rho}_{m1} + 4\bar{\rho}_b)\beta \|s(t)\|^2 \|\tilde{q}(t)\| \\ & \leq \frac{(\bar{\rho}_{m1} + 4\bar{\rho}_b)^2 \bar{\rho}_{qd}}{8\varepsilon_q} \gamma_0 \beta^2 \|s(t)\|^2 + \frac{2}{\gamma_0 \bar{\rho}_{qd}} V_s(t) V_q(t) \\ & 2\bar{\rho}_b \beta \|s(t)\| \|\tilde{q}(t)\|^2 \\ & \leq \frac{2\bar{\rho}_b^2 \bar{\rho}_{qd}}{\varepsilon_q^2} \gamma_0 \beta^2 \|s(t)\|^2 + \frac{1}{2\gamma_0 \bar{\rho}_{qd}} V_q(t)^2 \\ & 2(\bar{\rho}_m \beta + \bar{\rho}_m + 2\bar{\rho}_b \bar{\rho}_{d1}) \|s(t)\| \|\tilde{q}(t)\| \\ & \leq (\bar{\rho}_m \beta + \bar{\rho}_m + 2\bar{\rho}_b \bar{\rho}_{d1}) (\|s(t)\|^2 + \|\tilde{q}(t)\|^2) \\ & 2\beta^{-1} \|h_{qd}(t)\| \|s(t)\| \\ & \leq 2\bar{\rho}_{qd} \|s(t)\|^2 + \frac{1}{2\beta^2} \frac{\|h_{qd}(t)\|^2}{\bar{\rho}_{qd}} \\ & 2\bar{\rho}_{kk} \hat{\gamma}_s \|\tilde{I}_F(t)\| \|s(t)\| \\ & \leq \underline{\rho}_{kk} \hat{\gamma}_s \|s(t)\|^2 + \frac{\bar{\rho}_{kk}^2}{\underline{\rho}_{kk}} \hat{\gamma}_s \|\tilde{I}_F(t)\|^2 \\ & 2\beta^{-1} \|h_{qa}(t)\| \|s(t)\| \\ & \leq \left( \sum_{i=1}^3 \frac{\bar{\rho}_{qi}^2 \bar{\rho}_{qd}}{2} \gamma_0 \beta + 4\bar{\rho}_{q4}^2 \right) \|s(t)\|^2 \\ & \quad + \frac{1}{2} \sum_{i=1}^3 h_{qi}(t)^2 + \frac{1}{4} h_{q4}(t)^2 \end{aligned} \quad (68)$$

$$\begin{aligned} & \bar{\rho}_{m1} \beta \|s(t)\| \|\tilde{s}(t)\|^2 \\ & \leq \frac{\bar{\rho}_{m1}^2 \bar{\rho}_{qd}}{4} \gamma_0 \beta^2 \|s(t)\|^2 + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V_s(t)^2 \\ & \bar{\rho}_{m1} \beta \|\tilde{q}(t)\| \|\tilde{s}(t)\|^2 \\ & \leq \frac{\bar{\rho}_{m1}^2 \bar{\rho}_{qd}}{4\varepsilon_q} \gamma_0 \beta^2 \|\tilde{s}(t)\|^2 + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V_s(t) V_q(t) \\ & 2\bar{\rho}_b \beta \|s(t)\|^2 \|\tilde{s}(t)\| \\ & \leq \frac{2\bar{\rho}_b^2 \bar{\rho}_{qd}}{2} \gamma_0 \beta^2 \|s(t)\|^2 + \frac{2}{\gamma_0 \bar{\rho}_{qd}} V_s(t) V_s(t) \\ & 4\bar{\rho}_b \beta \|s(t)\| \|\tilde{s}(t)\| \|\tilde{q}(t)\| \\ & \leq \frac{4\bar{\rho}_b^2 \bar{\rho}_{qd}}{\varepsilon_q} \gamma_0 \beta^2 \|s(t)\|^2 + \frac{1}{\gamma_0 \bar{\rho}_{qd}} V_s(t) V_q(t) \\ & 2\bar{\rho}_b \beta \|\tilde{s}(t)\| \|\tilde{q}(t)\|^2 \\ & \leq \frac{2\bar{\rho}_b^2 \bar{\rho}_{qd}}{\varepsilon_q^2} \gamma_0 \beta^2 \|\tilde{s}(t)\|^2 + \frac{1}{2\gamma_0 \bar{\rho}_{qd}} V_q(t)^2 \\ & 4\bar{\rho}_b \bar{\rho}_{d1} \|\tilde{s}(t)\| (\|s(t)\| + \|\tilde{q}(t)\|) \\ & \leq 4\bar{\rho}_b \bar{\rho}_{d1} \|\tilde{s}(t)\|^2 + 2\bar{\rho}_b \bar{\rho}_{d1} \|s(t)\|^2 \\ & \quad + 2\bar{\rho}_b \bar{\rho}_{d1} \|\tilde{q}(t)\|^2 \\ & 2\bar{\rho}_{kk} \hat{\gamma}_s \|\tilde{I}_F(t)\| \|\tilde{s}(t)\| \\ & \leq \gamma_s \|\tilde{s}(t)\|^2 + \bar{\rho}_{kk}^2 \hat{\gamma}_s \|\tilde{I}_F(t)\|^2 \\ & 2\beta^{-1} \|h_{qd}(t)\| \|\tilde{s}(t)\| \\ & \leq 2\bar{\rho}_{qd} \|\tilde{s}(t)\|^2 + \frac{1}{2\beta^2} \frac{\|h_{qd}(t)\|^2}{\bar{\rho}_{qd}} \\ & 2\beta^{-1} \|h_{qa}(t)\| \|\tilde{s}(t)\| \\ & \leq \left( \sum_{i=1}^3 \frac{\bar{\rho}_{qi}^2 \bar{\rho}_{qd}}{2} \gamma_0 \beta + 4\bar{\rho}_{q4}^2 \right) \|\tilde{s}(t)\|^2 \\ & \quad + \frac{1}{2} \sum_{i=1}^3 h_{qi}(t)^2 + \frac{1}{4} h_{q4}(t)^2 \end{aligned} \quad (69)$$

It can be seen from (68), (69), (17), and (18) that  $V_s(t) + V_s^-(t)$  satisfies (43).

2) There exist upper bounds with respect to signals  $\omega_1(t)$ ,  $\omega_{I_s}^-(t)$ , and  $\omega_{I_q}^-(t)$  in (21) as follows:

$$\left. \begin{aligned} \|\omega_{I_s}(t)\| & \leq \frac{\bar{\omega}_{I1}}{\gamma_s} + \bar{\rho}_\ell \beta + \bar{\rho}_r + \bar{\rho}_\ell \\ \|\omega_{I_s}^-(t)\| & \leq \gamma_e \bar{\rho}_\ell + \bar{\rho}_r \\ \|\omega_{I_q}^-(t)\| & \leq \frac{\bar{\omega}_{I1}}{\gamma_s} + \bar{\rho}_\ell \beta + \bar{\rho}_\ell \end{aligned} \right\} \quad (70)$$



where  $\bar{\rho}_{\omega_{I1}} = \|\tilde{K}_N\| \bar{\rho}_{k_e} + \bar{\rho}_{\gamma_1} \bar{\rho}_\ell$ . It is seen from (23), (25), and (70) that the following inequalities hold:

$$\left. \begin{aligned}
 & 2 \left\| \tilde{I}_F(t) \right\| \left\| \omega_{Is}(t) \right\| \left\| s(t) \right\| \\
 & \leq \left\| s(t) \right\|^2 \\
 & \quad + \left( \frac{3}{\gamma_s^2} \bar{\rho}_{\omega_{I1}}^2 + 3\bar{\rho}_\ell^2 \beta^2 + 3(\bar{\rho}_\tau + \bar{\rho}_\ell)^2 \right) \left\| \tilde{I}_F(t) \right\|^2 \\
 & 2 \left\| \tilde{I}_F(t) \right\| \left\| \omega_{Is}(t) \right\| \left\| \tilde{s}(t) \right\| \\
 & \leq (\gamma_e + 1) \left\| \tilde{s}(t) \right\|^2 + (\bar{\rho}_\ell^2 \gamma_e + \bar{\rho}_\tau^2) \left\| \tilde{I}_F(t) \right\|^2 \\
 & 2 \left\| \tilde{I}_F(t) \right\| \left\| \omega_{Iq}(t) \right\| \left\| \tilde{q}(t) \right\| \\
 & \leq \left\| \tilde{q}(t) \right\|^2 + \left( \frac{3}{\gamma_s^2} \bar{\rho}_{\omega_{I1}}^2 + 3\bar{\rho}_\ell^2 \beta^2 + 3\bar{\rho}_\ell^2 \right) \left\| \tilde{I}_F(t) \right\|^2 \\
 & \frac{2}{\gamma_s} \left\| h_{Iq}(t) \right\| \left\| \tilde{I}_F(t) \right\| \\
 & \leq \frac{1}{2} h_{I1}(t)^2 + \frac{2}{\gamma_s^2} \bar{\rho}_{I1}^2 \left\| \tilde{I}_F(t) \right\|^2 \\
 & \frac{2}{\beta \gamma_s} \left\| h_{Id}(t) \right\| \left\| \tilde{I}_F(t) \right\| \\
 & \leq \frac{4\bar{\rho}_{Id}^2}{\bar{\rho}_{qd} \gamma_s^2} \left\| \tilde{I}_F(t) \right\|^2 + \frac{1}{4\beta^2} \frac{\left\| h_{Id}(t) \right\|^2 \bar{\rho}_{qd}}{\bar{\rho}_{Id}^2}
 \end{aligned} \right\} \quad (71)$$

It can be seen from (20), (28), and (71) that  $V_I(t)$  satisfies the inequality (44).

3) Since  $\beta \varepsilon_q - 2\bar{\rho}_{q4}^2 \geq 0$ , the following relation holds:

$$2(\beta \varepsilon_q - 2\bar{\rho}_{q4}^2) s(t)^T \tilde{q}(t) \leq (\beta \varepsilon_q - 2\bar{\rho}_{q4}^2) (\left\| s(t) \right\|^2 + \left\| \tilde{q}(t) \right\|^2). \quad (72)$$

Using the relation above, the inequality (45) is obtained.

4) Using the relations

$$\left. \begin{aligned}
 & 2\sqrt{\beta \bar{\rho}_f} \sqrt{\frac{1}{\gamma_0 \bar{\rho}_{qd}}} \left\| s(t) \right\|^2 h_{q1}(t) \\
 & \leq \frac{\bar{\rho}_f}{2} \beta \left\| s(t) \right\|^2 + \frac{2}{\gamma_0 \bar{\rho}_{qd}} V_s(t) h_{q1}(t)^2 \\
 & 2\sqrt{\beta \bar{\rho}_f} \sqrt{\frac{\varepsilon_q}{\gamma_0 \bar{\rho}_{qd}}} \left\| s(t) \right\| \left\| \tilde{q}(t) \right\| h_{q2}(t) \\
 & \leq \frac{\bar{\rho}_f}{2} \beta \left\| s(t) \right\|^2 + \frac{2}{\gamma_0 \bar{\rho}_{qd}} V_q(t) h_{q2}(t)^2 \\
 & 2\sqrt{\beta \bar{\rho}_f} \sqrt{\frac{\varepsilon_q}{\gamma_0 \bar{\rho}_{qd}}} \left\| \tilde{q}(t) \right\|^2 h_{q3}(t) \\
 & \leq \frac{\bar{\rho}_f}{2} \beta \left\| \tilde{q}(t) \right\|^2 + \frac{2}{\gamma_0 \bar{\rho}_{qd}} V_q(t) h_{q3}(t)^2 \\
 & 2\bar{\rho}_{q4} \left\| s(t) - \tilde{q}(t) \right\| h_{q4}(t) \\
 & \leq \frac{1}{2} h_{q4}(t)^2 + 2\bar{\rho}_{q4}^2 \left( \left\| s(t) \right\|^2 - 2s(t)^T \tilde{q}(t) + \left\| \tilde{q}(t) \right\|^2 \right) \\
 & 2(\left\| s(t) \right\| + \left\| \tilde{q}(t) \right\|) h_{I1}(t) \\
 & \leq \frac{1}{2} h_{I1}(t)^2 + 4\left\| s(t) \right\|^2 + 4\left\| \tilde{q}(t) \right\|^2
 \end{aligned} \right\} \quad (73)$$

it can be seen from (27) that  $V_h(t)$  satisfies the inequality (46).

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