More Accurate Breakdown Voltage Estimation for the New Step-up Test Method in the Weibull Model

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ABSTRACT

The estimation problems for the conventional step-up method (the observed breakdown voltages are not given at all) and the new step-up method (some of the observed breakdown voltages are given) are analyzed when the underlying probability distribution is assumed to be a Weibull model. This paper is a consecutive research of the case that the underlying probability distribution is assumed to be a normal model. Similarly to the normal model, the new step-up test method, in the Weibull model, also has advantages compared to the conventional method: (1) the confidence intervals of the estimates become smaller and (2) the estimates can be obtained with higher probability. The bias observed when sample size is small can be reduced by using the bootstrap method.

Index Terms — **Impulse breakdown voltage, step-up test method, optimal test, electrical insulation, Weibull distribution, bootstrap method.**

INTRODUCTION

To estimate the impulse breakdown voltage (or impulse flashover voltage) for electrical insulation which does not have a self-restoring property, e.g., the insulation will not be able to be used when it is broken, such as epoxy resin, an impulse test by increasing voltage is used (IEC Pub. 60-1 [1], JEC-0202 [2]). The step-up test method is as follows: (1) the initial voltage is set to a sufficiently low stress level (e.g., ν_0) where the insulation would not be broken, and (2) the stress level will be set to a higher level, $\nu_1 = \nu_0 + d$ if the insulation is not broken at stress level ν_0 in m times impulse tests, this procedure continues until the insulation is finally broken. If the breakdown voltage itself is obtained when the insulation is broken, the test method is called *the new step-up method*, while it is not obtained, the test method is called *the (conventional) step-up method* (Hirose [3]).

When the impulse breakdown voltage follows a normal distribution, $N(\mu,\sigma^2)$, with mean, μ, and standard deviation, σ, Hirose [3] first recommends the use of the parameters of the underlying probability distribution rather than the use of the nominal breakdown voltage, and second to use the new step-up method if the observed breakdown voltage itself rather than the two-valued information of breakdown and non-breakdown is available, from a viewpoint of stable and accurate parameter estimation. This paper deals with a very similar problem to [3], but the underlying distribution is assumed to be a two-parameter Weibull model, which is much more realistic in impulse breakdown of electrical insulation which does not have a self-restoring property. Regarding the conventional step-up test with the Weibull model, Hirose [4] is referred to; this paper describes the comparison between the conventional and new step-up methods in the Weibull model.

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2 STEP-UP TEST WITH WEIBULL MODEL

We assume that the underlying probability distribution for the breakdown voltage follows a two-parameter Weibull distribution

$$p = P(V \le \nu) = F(\nu;\eta,\beta) = 1 - \exp\left\{-\left(\frac{\nu}{\eta}\right)^{\beta}\right\} \quad (1)$$

where η and β are scale and shape parameters, respectively. Then, mean, μ, and standard deviation, σ, are

$$\mu = \eta\Gamma(1+1/\beta),\ \sigma = \eta\sqrt{\Gamma(1+2/\beta) - \{\Gamma(1+1/\beta)\}^2} \quad (2)$$

The impulse breakdown test by the step-up method starts at a very low stress level ν_0 and continues until the insulation is broken at some stress level $\nu_i = \nu_0 + id$. If each test piece is numbered as 1, 2, ..., n, we obtain n sampled values of $\nu_i(k)$, $(k = 1, \ldots, n)$.

By experience, we may assume that the coefficient of variation (ratio of standard deviation to mean), $c\nu$, is located to be $0.03 \le c\nu \le 0.20$ for non-self-restoring insulation. This range corresponds approximately to $6 \le \beta \le 40$. Thus, we deal with the cases here that $\beta = 6$, 8,12, 25, 40 which corresponds approximately to $c\nu = 0.20$, 0.15, 0.10, 0.05, 0.03, respectively.

3 ESTIMATION METHOD

Suppose first that the breakdown voltage test is done by the conventional step-up method. Then, the likelihood function for the test sequence is denoted as

$$L^F = \prod_{k=1}^{n} l_k^F \tag{3}$$

and

$$l_k^F = F(\nu_{i(k)})\{1 - F(\nu_{i(k)})\}^{m(k)-1} \prod_{j=0}^{i(k)-1} \{1 - F(\nu_j)\}^m \tag{4}$$

where $m(k)$ denotes the number of strikes until the insulation is broken at the final stage $i(k)$ for sample k. The expression $l_{i(k)}^F$ can be considered as the probability of an extended geometric distribution that the insulation is first broken at stress level $\nu_{i(k)}$.

Suppose next that the breakdown voltage test is done by the new step-up method. Then, the likelihood function for the test sequence is denoted as

$$L^f = \prod_{k=1}^{n} l_k^f \tag{5}$$

where

$$l_k^f = f(V_{i(k)})\{1 - F(\nu_{i(k)})\}^{m(k)-1} \prod_{j=0}^{i(k)-1} \{1 - F(\nu_j)\}^m \tag{6}$$

and

$$f(V) = \left(\frac{\beta}{\eta}\right)\left(\frac{V}{\eta}\right)^{\beta-1} \exp\left\{-\left(\frac{V}{\eta}\right)^{\beta}\right\} \tag{7}$$

The random variable $V_{i(k)}$ is obtained under the condition that $V_{i(k)} \le \nu_{i(k)}$.

The estimates, $\hat{\eta}$ and $\hat{\beta}$ for the conventional step-up method, can be obtained by solving the log-likelihood equations

$$\partial \log L^F/\partial\eta = 0, \qquad \partial \log L^F/\partial\beta = 0 \tag{8}$$

Some iterative methods, e.g., the Newton method, can be used to obtain the estimates of the parameters. Their confidence intervals are computed using the observed Fisher information matrix. However, equations (8) may not have solutions in a mathematical sense when *B-level* < 2 (see [3]).

For the new step-up method, the solution can be obtained by solving the log-likelihood equations

$$\partial \log L^f/\partial\eta = 0, \qquad \partial \log L^f/\partial\beta = 0 \tag{9}$$

It should be noted that equation (9) has the solutions with probability 1, unlike the log-likelihood equations in the conventional step-up method. This beneficial property in the new step-up test procedure is also true, similarly to the case that the underlying distribution is a normal type.

3.1 EXAMPLE

Suppose that the breakdown voltages obtained are exactly the same as in Table 1 in [3]. The starting stress level is 500, the step-up stress is 50, and $m = 1$. Then, the maximum likelihood estimates for the conventional step-up

Table 1. Bias $(\hat{\theta})$ and $sue(\hat{\theta})$ of the estimates in the conventional step-up method.

d/σ	n	M	bias $(\hat{\eta})$	$sue(\hat{\eta})$	bias $(\hat{\beta})$	$sue(\hat{\beta})$
0.1	∞			2.30091		0.84631
0.1	100	1000	−0.00165	2.23232	0.14975	0.85135
0.1	50	1000	−0.00354	2.24003	0.38076	0.88159
0.1	20	1000	−0.00839	2.29782	1.04632	1.11640
0.1	10	1000	−0.01870	2.37595	2.23754	1.31754
0.2	∞			1.84332		0.85044
0.2	100	1000	−0.00178	1.81628	0.18047	0.89122
0.2	50	1000	−0.00410	1.78672	0.45964	0.93140
0.2	20	1000	−0.00766	1.86983	1.05378	1.05423
0.2	10	1000	−0.01331	1.92336	2.22566	1.40514
0.5	∞			1.36152		0.87402
0.5	100	1000	−0.00090	1.39111	0.25273	0.94026
0.5	50	1000	−0.00152	1.41080	0.32548	0.97898
0.5	20	1000	−0.00424	1.33710	0.98870	1.07432
0.5	10	999	−0.01057	1.84750	2.35929	1.40251
1.0	∞			1.18999		0.93580
1.0	100	1000	0.00027	1.17584	0.18874	0.99898
1.0	50	1000	−0.00127	1.21979	0.45542	1.08086
1.0	20	999	−0.00272	2.10092	1.21335	1.29269
1.0	10	951	−0.00468	8.70250	2.03163	1.41997

$\exists j$, s.t. $\nu_j = \mu$, M: number of estimates successfully computed.

method are such that $\hat{\eta}=3290$ and $\hat{\beta}=7.523$. The approximate standard errors for $\hat{\eta}$ and $\hat{\beta}$ using the observed Fisher information matrix are 269.8 and 1.787. In the case of the new step-up method, the maximum likelihood estimates are $\hat{\eta}=3157$ and $\hat{\beta}=8.625$ and their approximate standard errors are 170.6 and 1.486. The standard errors for the estimates in the new step-up method seem to be smaller than those in the conventional step-up method. This tendency is generally true as will be shown in the next section. This is the second beneficial property in the new step-up method.

4 OPTIMAL TEST PROCEDURE

Let us define, similarly to [3], the asymptotic errors, $s(\eta)$ and $s(\beta)$, for η and β by the square root of each diagonal element of the inverse matrix of I, where

$$I=-\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial\eta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\beta\partial\eta}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\eta\partial\beta}\right) & E\left(\frac{\partial^2 \log L}{\partial\beta^2}\right)\end{pmatrix} \quad (10)$$

If the insulation is not broken at level i, the expectation of surviving at level i for 1 test piece is

$$E(w_i^0)=\prod_{j=0}^{i}\{1-F(\nu_j)\}^m \quad (11)$$

If the insulation is broken at level i, the expectation of failure at level i for 1 test piece is

$$E(w_i^1)=F(\nu_i)\{1-F(\nu_i)\}^{m(i)-1}\prod_{j=0}^{i-1}\{1-F(\nu_j)\}^m \quad (12)$$

Therefore, each element of I for the conventional step-up test is expressed as

$$E\left(\frac{\partial^2 \log L^F}{\partial\theta_a\partial\theta_b}\right)=\sum_i E(w_i^0)E\left(\frac{\partial^2 \log(1-F(\nu_i))}{\partial\theta_a\partial\theta_b}\right)+\sum_i E(w_i^1)E\left(\frac{\partial^2 \log F(\nu_i)}{\partial\theta_a\partial\theta_b}\right) \quad (13)$$

and that for the new step-up test is expressed as

$$E\left(\frac{\partial^2 \log L^f}{\partial\theta_a\partial\theta_b}\right)=\sum_i E(w_i^0)E\left(\frac{\partial^2 \log(1-F(\nu_i))}{\partial\theta_a\partial\theta_b}\right)+\sum_i E(w_i^1)E\left(\frac{\partial^2 \log f(V_i)}{\partial\theta_a\partial\theta_b}\middle| V_i<\nu_i\right) \quad (14)$$

where θ_a or θ_b denotes η or β. More specifically

$$E\left(\frac{\partial^2 \log F(\nu_i)}{\partial\eta^2}\right)=-\frac{1}{p_i^2}\left(-\frac{\beta y_i z_i}{\eta}\right)^2+\frac{1}{p_i}\left[\frac{\beta}{\eta^2}y_i z_i(1-\beta(y_i-1))\right],$$

$$E\left(\frac{\partial^2 \log F(\nu_i)}{\partial\eta\partial\beta}\right)=-\frac{1}{p_i^2}\left(-\frac{\beta y_i z_i}{\eta}\right)(y_i z_i\log x_i)+\frac{1}{p_i}\left[-\frac{1}{\eta}y_i z_i(1-\beta(y_i-1)\log x_i)\right] \quad (15)$$

$$E\left(\frac{\partial^2 \log F(\nu_i)}{\partial\beta^2}\right)=-\frac{1}{p_i^2}(y_i z_i\log x_i)^2+\frac{1}{p_i}\left[-y_i z_i(y_i-1)(\log x_i)^2\right],$$

$$E\left(\frac{\partial^2 \log(1-F(\nu_i))}{\partial\eta^2}\right)=-\frac{1}{q_i^2}\left(-\frac{\beta y_i z_i}{\eta}\right)^2-\frac{1}{q_i}\left[\frac{\beta}{\eta^2}y_i z_i(1-\beta(y_i-1))\right],$$

$$E\left(\frac{\partial^2 \log(1-F(\nu_i))}{\partial\eta\partial\beta}\right)=-\frac{1}{q_i^2}\left(-\frac{\beta y_i z_i}{\eta}\right)(y_i z_i\log x_i)-\frac{1}{q_i}\left[-\frac{1}{\eta}y_i z_i(1-\beta(y_i-1)\log x_i)\right],$$

$$E\left(\frac{\partial^2 \log(1-F(\nu_i))}{\partial\beta^2}\right)=-\frac{1}{q_i^2}(y_i z_i\log x_i)^2-\frac{1}{q_i}\left[-y_i z_i(y_i-1)(\log x_i)^2\right] \quad (16)$$

$$E\left(\frac{\partial^2 \log f(V_i)}{\partial\beta^2}\middle| V_i<\nu_i\right)=\frac{1}{p_i}\int_{-\infty}^{\nu_i}\frac{\partial^2 \log f(\nu)}{\partial\beta^2}f(\nu)d\nu,$$

$$E\left(\frac{\partial^2 \log f(V_i)}{\partial\eta\partial\eta}\middle| V_i<\nu_i\right)=\frac{1}{p_i}\int_{-\infty}^{\nu_i}\frac{\partial^2 \log f(\nu)}{\partial\eta\partial\beta}f(\nu)d\nu \quad (17)$$

$$E\left(\frac{\partial^2 \log f(V_i)}{\partial\eta^2}\middle| V_i<\nu_i\right)=\frac{1}{p_i}\int_{-\infty}^{\nu_i}\frac{\partial^2 \log f(\nu)}{\partial\eta^2}f(\nu)d\nu,$$

$$\frac{\partial^2 \log f(\nu)}{\partial\eta^2}=\frac{\beta(1-(1+\beta)y_i)}{\eta^2},$$

$$\frac{\partial^2 \log f(\nu)}{\partial\eta\partial\beta}=\frac{-1+y_i(1+\beta\log x_i)}{\eta} \quad (18)$$

$$\frac{\partial^2 \log f(\nu)}{\partial\beta^2}=\left(-\frac{1}{\beta^2}-y_i(\log x_i)^2\right),$$

where $x_i=\nu_i/\eta$, $y_i=x_i^\beta$, $z_i=\exp(-y_i)$, and $p_i=F(\nu_i)=1-q_i$.

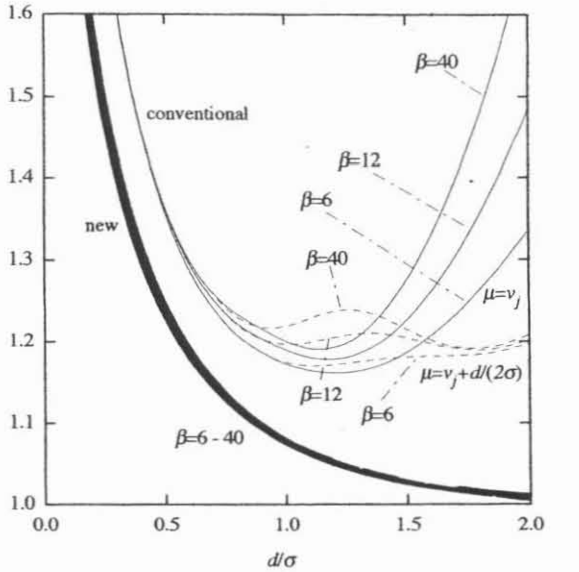

Figure 1. Asymptotic unit error, $e(\eta)$, in the new and conventional step-up method. Thick line, $\exists j$, s.t. $\nu_j = \mu$; Thin line, $\exists j$, s.t. $(\nu_{j-1} + \nu_j)/2 = \mu$.

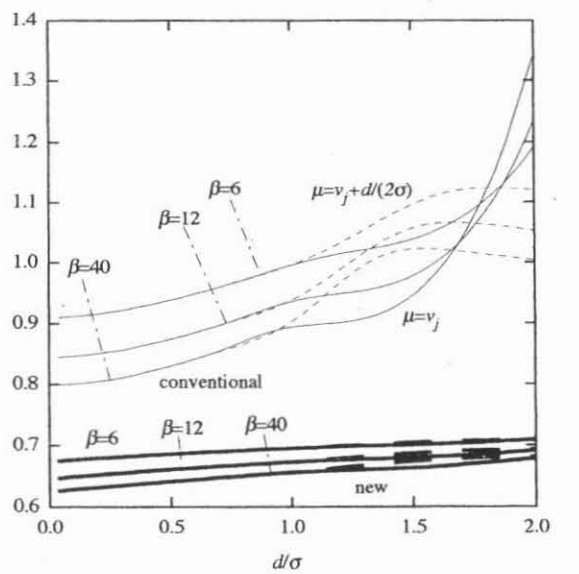

Figure 2. Asymptotic unit error, $e(\beta)$, in the new and conventional step-up method. Thick line, $\exists j$, s.t. $\nu_j = \mu$; Thin line, $\exists j$, s.t. $(\nu_{j-1} + \nu_j)/2 = \mu$.

Here, we define the asymptotic unit errors, $e(\eta)$ and $e(\beta)$ as

$$e(\eta) = \beta \cdot s(\eta)/\eta,\ e(\beta) = s(\beta)/\beta \qquad (19)$$

These errors are not affected much even if β varies. Figures 1 and 2 show $e(\eta)$ and $e(\beta)$ against d/σ when $m = 1$ for both the conventional and new step-up methods; the solid line expresses the case when some level ν_j is equal to μ, and the dotted line expresses the case when μ is located in the middle of ν_{j-1} and ν_j. These figures suggest the following.

(1) The asymptotic unit error in the new step-up method, $e_n(\eta)$, becomes smaller than that in the conventional step-up method, $e_c(\eta)$.

(2) The optimal test for $e_c(\eta)$ can be realized around $d/\sigma = 1.0$, but that for $e_n(\eta)$ can be realized for larger d/σ.

(3) The asymptotic unit error in the new step-up method, $e_n(\beta)$, becomes considerably smaller than that in the conventional step-up method, $e_c(\beta)$. For example, the difference between $e_n(\beta) = 0.6724$ and $e_c(\beta) = 0.9358$ means that the samples in the conventional step-up method requires samples about twice as large as in the new step-up method to obtain the equivalent confidence interval, when $\beta = 12$, some level ν_j is equal to μ, and $d/\sigma = 1.0$.

(4) The optimal test for $e_n(\beta)$ does not depend on d/σ, while the optimal test for $e_c(\beta)$ can be realized at smaller d/σ.

(5) The asymptotic unit errors, $e_n(\eta)$, $e_n(\beta)$, $e_c(\eta)$, and $e_c(\beta)$ do not depend on the starting point x_0, when $d/\sigma < 1.0$.

In short, the new step-up method markedly improves the reliability of the estimates of η and β compared to the conventional step-up method. The larger the d/σ, the smaller the asymptotic unit errors as long as $d/\sigma \le 2.0$. For the estimate of β, about a half of the sample size in the new step-up method is sufficient for obtaining the equivalent reliability to the conventional method. This is the result for the case of $m = 1$, but this tendency is also true for $m > 1$.

5 MONTE CARLO SIMULATION

A Monte Carlo simulation study is done in order to investigate the asymptotic properties of the estimates for the conventional and new step-up methods. The simulation conditions are as follows:

(1) The very first stress step, x_0, is set to around the point that satisfies $F(x_0) = 10^{-7}$, and some stress level is set just to $\eta = \nu_j$ because the errors are not affected by the starting point as long as $d/\sigma < 1$.

(2) The number of samples, n, is 100, 50, 20, 10.

(3) The step-up distance to σ, d/σ, is 0.1, 0.2, 0.5, 1.0.

(4) The parameter values are $\eta = 1$ and $\beta = 6,8,12,25,40$.

(5) The number of repetition times of strikes at the same stage is $m = 1$.

(6) The number of trial times is 1000.

Here, we define the biases and standardized unit errors as

$$\bar{\theta} = \left(\frac{1}{M} \sum_{i=1}^{M} \hat{\theta}_i \right), \quad bias(\hat{\theta}) = \bar{\theta} - \theta \qquad (20)$$

$$S(\hat{\theta}) = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(\hat{\theta}_i - \theta\right)^2},$$

$$sue(\hat{\eta}) = \frac{\sqrt{n}\,\beta S(\hat{\eta})}{\eta},\ sue(\hat{\beta}) = \frac{\sqrt{n}\,S(\hat{\beta})}{\beta} \quad (21)$$

where, M denotes the number of successful estimation cases. Table 1 and 2 show the *bias*($\hat{\theta}$) and *sue*($\hat{\theta}$) for the estimates $\hat{\eta}$ and $\hat{\beta}$ when $\beta = 12$. Comparing the unit asymptotic errors in Figures 1 and 2 with the simulation results, mean and standardized error in the simulation agree well with the asymptotic values as long as n is larger than 20. It can be seen that the cases in which we cannot obtain the estimates (not numerically, but mathematically) are not rare in the conventional step-up method when n is small such as $n \leq 10$. When $d/\sigma = 1.0$ and $n = 10$ in the conventional step-up test, $sue(\hat{\eta})$ differs far from $e(\eta)$ due to very little information at $\mu - \sigma$ or $\mu - 2\sigma$; this lack of information can be compensated for by using the explicit breakdown voltage values in the new step-up method, and thereby $sue(\hat{\eta})$ is approximately the same as $e(\eta)$.

To reduce the *bias*($\hat{\theta}$) and *sue*($\hat{\theta}$) when n is small, the bootstrap method (Efron [5]) may be applicable. Table 3 shows the bootstrapped results for the *bias*($\hat{\theta}$) and *sue*($\hat{\theta}$) when $n = 10$ and $d/\sigma = 1.0$. The number of drawings in the bootstrap procedure is set to 100 to each estimate. The bootstrap procedure is as follows: To each estimate $\hat{\theta}_i$, we obtain $\hat{\theta}_{i,j}$, $(j = 1, \ldots, B)$, using the random variables generated from $F(\nu; \hat{\eta}_i, \hat{\beta}_i)$. If we define

$$\theta_i^* = \frac{1}{B}\sum_{j=1}^{B}\hat{\theta}_{i,j} \quad (22)$$

then, the bias reduced estimate $\tilde{\theta}_i$ is computed by

$$\tilde{\theta}_i = 2\hat{\theta}_i - \theta_i^* \quad (23)$$

The bootstrapped $sue(\tilde{\theta})$ is computed by substituting $\hat{\theta}$ to $\tilde{\theta}$ in (21). The bootstrap method works well.

The properties described above are also true when $\beta =$ 6, 8, 25, 45.

6 DISCUSSION

6.1 THREE PARAMETER WEIBULL MODEL

Hirose [4] treats the three-parameter Weibull model which includes the location parameter, γ, and he concludes that the three-parameter case cannot be used in general because of the substantial estimation errors. If we define the asymptotic unit errors for the three-parameter model as,

$$e(\eta) = s(\eta)/(\eta\beta),\ e(\beta) = s(\beta)/\beta^2,\ e(\gamma) = s(\gamma)/(\eta\beta), \quad (24)$$

Table 2. Bias ($\hat{\theta}$) and *sue*($\hat{\theta}$) of the estimates in the new step-up method.

d/σ	n	M	bias ($\hat{\eta}$)	$sue(\hat{\eta})$	bias($\hat{\beta}$)	$sue(\hat{\beta})$
0.1	∞			1.89451		0.64887
0.1	100	1000	−0.00108	1.88359	0.08180	0.66970
0.1	50	1000	−0.00131	1.84142	0.16056	0.63958
0.1	20	1000	−0.00481	1.89206	0.51491	0.71882
0.1	10	1000	−0.01263	1.89336	1.10730	0.80784
0.2	∞			1.57761		0.65199
0.2	100	1000	−0.00142	1.57974	0.10897	0.68525
0.2	50	1000	−0.00304	1.58756	0.28375	0.70040
0.2	20	1000	−0.00620	1.62220	0.65963	0.76104
0.2	10	1000	−0.01067	1.64503	1.28692	0.89442
0.5	∞			1.24063		0.66040
0.5	100	1000	−0.00048	1.25926	0.11384	0.66092
0.5	50	1000	−0.00125	1.25380	0.15058	0.69824
0.5	20	1000	−0.00319	1.24325	0.52642	0.75465
0.5	10	1000	−0.00928	1.27175	1.12535	0.81002
1.0	∞			1.07840		0.67237
1.0	100	1000	0.00017	1.09685	0.10127	0.65522
1.0	50	1000	−0.00125	1.10419	0.24846	0.71033
1.0	20	1000	−0.00282	1.08290	0.64356	0.77980
1.0	10	1000	−0.00536	1.14768	1.38034	0.96501

$\exists j$, s.t. $\nu_j = \mu$, M: number of estimates successfully computed.

Table 3. Bias ($\hat{\theta}$) and *sue*($\hat{\theta}$) of the bootstrapped estimates in the new step-up Method.

d/σ	n	M	bias ($\hat{\eta}$)	$sue(\hat{\eta})$	bias($\hat{\beta}$)	$sue(\hat{\beta})$
0.1	10	1000	0.00135	2.03216	−0.29660	0.704733
0.2	10	1000	−0.00143	1.69528	−0.18206	0.709945
0.5	10	1000	−0.00090	1.30094	−0.00705	0.747083
1.0	10	1000	0.00110	1.12561	−0.23508	0.747230

$\exists j$, s.t. $\nu_j = \mu$, M: number of estimates successfully computed.

these errors are not affected much when β varies in the conventional step-up test. As for the numerical example, $e(\beta) \approx 0.8$ when $\beta = 8$, which means that more than 4000 samples are needed if we require the 10% standard error for $\hat{\beta}$ ([4]). This tendency is considered to be true in the new step-up test method. Thus, we do not go deeply into the three-parameter case in this paper.

6.2 IN THE CASE OF FIXED LOCATION PARAMETER

This paper deals with the case that $\gamma = 0$, which is often discussed in fitting the probability distribution model to the data obtained from increasing voltage tests. If $c\nu$ is located in $0.03 < c\nu < 0.20$, the shape parameter varies $6 < \beta < 40$ as stated above. However, if we know the strictly positive location value *a priori*, the value of β may vary even when $0.03 < c\nu < 0.20$. However, the reliability of the estimate in such a case can be guessed in general by using the results obtained by this paper and [3] in which the case of shape parameter of around 3.4 is dealt with.

7 CONCLUDING REMARKS

TO estimate the impulse breakdown voltages accurately for non-self-restoring electrical insulation, the new step-up test method is recommended when the underlying probability distribution is assumed to be a Weibull model. This paper first recommends the use of the parameters of the underlying probability distribution, e.g., the scale and shape parameters. Second, it is advantageous to use the new step-up method if the observed breakdown voltage itself rather than the two-valued information of breakdown and non-breakdown is available. Using the new step-up method, the number of test specimens can be substantially reduced comparing to that in the conventional step-up method for the estimate of shape parameter. The optimal test procedure is obtained with larger d/σ. When sample size is small, the bootstrap method reduces the bias of the estimate, particularly for the shape parameter.

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