More Accurate Breakdown Voltage Estimation for the New Step-up Test Method

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ABSTRACT

The step-up method is used to estimate the impulse breakdown voltages when the electrical insulation is not usable after it is broken. This paper analyses the reliability of the estimates of the underlying breakdown probability distribution in the step-up method, when (1) the observed breakdown voltage itself is available and (2) it is not available. The former case has many advantages compared to the latter case such that (i) the confidence intervals of the estimates become smaller and (ii) the estimates can be obtained with higher probability. Consequently, this paper recommends using the estimates of the underlying distribution for the breakdown voltages instead of the nominal breakdown voltages. Some illustrative examples are given.

Index Terms — **Impulse breakdown voltage, nominal breakdown voltage, step-up test method, optimal test, electrical insulation, normal distribution.**

1 INTRODUCTION

IN impulse voltage tests, various test methods are used according to the purpose of the test; (1) withstand test, (2) 50% flashover test, (3) impulse test by increasing voltage and (4) *V-t* (voltage-time) test are among them. To estimate the impulse breakdown voltage (or impulse flashover voltage) for electrical insulation, which has a self-restoring property such as air and SF_6 gas, multiple-level tests and the up-and-down test methods are used [1-3]. If the insulation does not have a self-restoring property, e.g., the insulation will not be able to be used when it is broken, such as epoxy resin, impulse test by increasing voltage is used; we call this *the step-up method* [4] in this paper.

The up-and-down test method is as follows: (1) the initial voltage is set around the mean breakdown voltage level, say v_0, and (2) if the insulation is not broken at stress level v_0, then the stress level will be set to a higher level, $v_1 = v_0 + d$, otherwise the stress level will be set to a lower level, $v_{-1} = v_0 - d$, and this up-and-down procedure continues for prescribed number of times. If the impulse breakdown voltage follows a normal distribution with mean μ and standard deviation σ, ($N(\mu,\sigma)$), Dixon and Mood [1] recommend to test under the condition that d is set to around σ and the number of up-and-down repetition times is larger than 40.

The step-up test method is as follows: (1) the initial voltage is set to a sufficiently low stress level (e.g., v_0) where the insulation will hardly be broken, and (2) the stress level will be set to a higher level, $v_1 = v_0 + d$ if the insulation is not broken at stress level v_0 in m times impulse tests, this procedure continues until the insulation is finally broken. If the impulse breakdown voltage follows $N(\mu,\sigma)$, Hirose [4] recommends testing under the condition that (1) d is set to around σ, (2) $m = 1$, and (3) the number of test specimens is larger than 20.

Both test methods explained above use only the information that the breakdown occurred (indicator is 1) or did not (indicator is 0) at stress level v_j. Hirose and Kato [5] recently proposed the *new up-and-down method* in which the observed breakdown voltage itself is incorporated into the estimation procedure because of recent improvements of high speed voltage measuring instruments; Komori and Hirose [6] showed an easy parameter estimation method for the test. Using the proposed method, the estimated error of $\hat{\sigma}$ is shown to be dramatically improved. In addition, we do not need to take care of the optimal value of d unlike the conventional up-and-down method. When the insulation does not have a self-restoring property, incorporating the observed breakdown voltage values into the

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estimation procedure in the step-up method may reduce the estimated errors of the estimates similarly to the new up-and-down method.

This paper proposes a new method to estimate the breakdown voltages more accurately in the step-up method not only using the information of breakdown occurrence at a given stress level, but also using the observed breakdown voltage values themselves. The breakdown voltage is, here, assumed to follow a normal distribution $N(\mu,\sigma)$. The proposed method is, here, referred to as the *new step-up method*.

2 NOMINAL BREAKDOWN VOLTAGE

Suppose that an impulse voltage with peak value v is applied to the insulation. We can assume that the insulation will be broken if the random variable V of the failure of the insulation is smaller than v. That is, the breakdown probability p by a single impulse strike is denoted by $P(V \leq v)$. By the assumption of the normal distribution for the underlying distribution

$$p = P(V \leq v) = F(v) = \int_{-\infty}^{v} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\} dx. \tag{1}$$

The impulse breakdown test by the step-up method starts at a very low stress level v_0 and continues until the insulation is broken at some stress level $v_i = v_0 + id$. If each test piece is numbered as $1,2,\ldots,n$, we obtain n sampled values of $v_i(k)$, $(k = 1,\ldots,n)$.

Some electrical engineers use the mean and standard deviation of $v_i(k)$ as the impulse breakdown voltage index. It can, however, easily be seen that the frequency distribution of $v_i(k)$ depends on d; e.g., the smaller the d, the smaller the distribution of $v_i(k)$. Figure 1 shows the histograms of $v_i(k)$ with $d = 0.1\sigma - 1.0\sigma$ when the underlying distribution is $N(\mu,\sigma)$. For instance, if we draw random variables from $N(3000,500)$ and simulate a step-up test as shown in Table 1 (the step-up distance is 50), the nominal mean and the standard deviation of the breakdown voltage are obtained to be 2440 and 390, respectively. These values are apparently smaller than the parameter values of 3000 and 500, and this corresponds to Figure 1. Thus, the use of the nominal estimates obtained directly by $v_i(k)$ is not recommended (when the step-up test is done) as the breakdown voltage index. Even if the breakdown test is done by the new step-up method, this tendency remains unchanged.

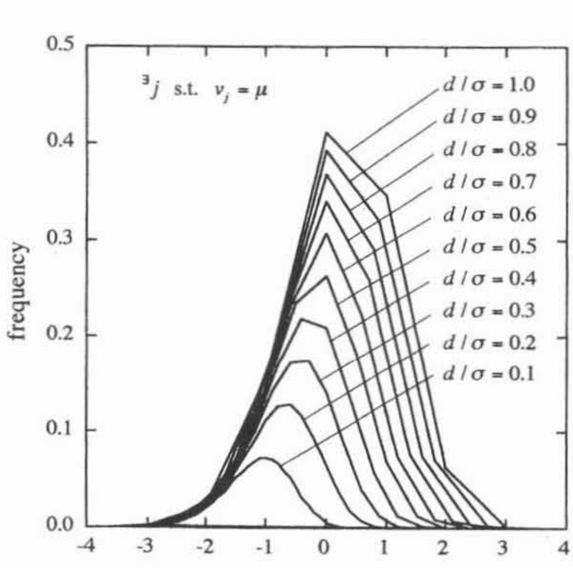


Figure 1. Histogram of nominal breakdown voltage by the step-up method.

Table 1. Simulated breakdown voltages by step-up method.

test piece number	final breakdown stress	final setup stress
1	2184	2650
2	2267	2600
3	2343	2950
4	2363	2600
5	2253	2400
6	2020	2950
7	2371	2450
8	1621	1900
9	2228	2600
10	1428	1450
11	2328	2350
12	2244	2350
13	2614	2700
14	2099	2100
15	2460	2550

The first stress is 500, and the step-up distance is 50.

3 ESTIMATION METHOD

Suppose that the breakdown voltage test is done by the conventional step-up method. Then, the likelihood function for the test sequence is denoted as

$$L^F = \prod_{k=1}^{n} l_k^F, \tag{2}$$

and

$$l_k^F = F(v_{i(k)})\{1 - F(v_{i(k)})\}^{m(k)-1} \prod_{j=0}^{i(k)-1} \{1 - F(v_j)\}^m, \tag{3}$$

where $m(k)$ denotes the number of strikes until the insulation is broken at the final stage $i(k)$ for sample k. The expression l_k^F can be considered as the probability of an extended geometric distribution that the insulation is first broken at stress level $v_{i(k)}$.

Suppose that the breakdown voltage test is done by the new step-up method. Then, the likelihood function for the test sequence is denoted as

$$L^f = \prod_{k=1}^{n} l_k^f, \tag{4}$$

where

$$l_k^f = f(V_{i(k)})\{1 - F(v_{i(k)})\}^{m(k)-1} \prod_{j=0}^{i(k)-1} \{1 - F(v_j)\}^m, \tag{5}$$

and

$$f(V) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left\{-\frac{(V-\mu)^2}{2\sigma^2}\right\}. \quad (6)$$

The random variable $V_{i(k)}$ is obtained under the condition that $V_{i(k)} \le v_{i(k)}$.

The estimates, $\hat{\mu}$ and $\hat{\sigma}$ for the conventional step-up method, can be obtained by solving the log-likelihood equations

$$\partial \log L^F/\partial\mu = 0, \quad \partial \log L^F/\partial\sigma = 0. \quad (7)$$

Some iterative methods, e.g., the Newton method, can be used to obtain the estimates of the parameters. Their confidence intervals are computed using the observed Fisher information matrix. However, (7) may not have solutions in a mathematical sense when *B-level* < 2, Here, *B-level* is defined as the number of stress levels such that breakdowns and no-breakdowns are mixed at the same stress level.

For the new step-up method, the solution can be obtained by solving the log-likelihood equations

$$\partial \log L^f/\partial\mu = 0, \quad \partial \log L^f/\partial\sigma = 0. \quad (8)$$

It should be noted that (8) has the solutions with probability 1, unlike the log-likelihood equations in the conventional step-up method. This is one of the beneficial properties in the new step-up test procedure.

3.1 EXAMPLE 1

Suppose that the breakdown voltages are obtained as shown in Table 1. The starting stress level is 500, the step-up stress is 50, and $m = 1$. Thus, test piece 1 is broken after $44(=\{(2650-500)/50\}+1)$ impulse strikes. We assume that the breakdown voltage follows a normal distribution $N(3000,500)$.

The maximum likelihood estimates for the conventional step-up method are such that $\hat{\mu} = 3244$ and $\hat{\sigma} = 630$. The approximate 95% confidence intervals for $\hat{\mu}$ and $\hat{\sigma}$ using the observed Fisher information matrix are $2791 \le \mu \le 3698$ and $354 \le \sigma \le 906$. In the case of the new step-up method, the maximum likelihood estimates are $\hat{\mu} = 3090$ and $\hat{\sigma} = 524$ and their approximate 95% confidence intervals are $2790 \le \mu \le 3390$ and $353 \le \sigma \le 695$. The confidence intervals for the estimates in the new step-up method seem to be smaller than those in the conventional step-up method. This tendency is generally true as will be shown in the next section. This is the second beneficial property in the new step-up method.

4 OPTIMAL TEST PROCEDURE

We first define the asymptotic unit errors $e(\mu)$ and $e(\sigma)$ by the square root of each diagonal element of the inverse matrix of I, where

$$I = -\begin{pmatrix} E\left(\dfrac{\partial^2 \log L}{\partial\mu^2}\right) & E\left(\dfrac{\partial^2 \log L}{\partial\sigma\partial\mu}\right) \\ \left(E\dfrac{\partial^2 \log L}{\partial\mu\partial\sigma}\right) & E\left(\dfrac{\partial^2 \log L}{\partial\sigma^2}\right) \end{pmatrix}. \quad (9)$$

If the insulation is not broken at level i, the expectation of surviving at level i for 1 test piece is

$$E(w_i^0) = \prod_{j=0}^{i}\{1-F(v_j)\}^m. \quad (10)$$

If the insulation is broken at level i, the expectation of failure at level i for 1 test piece is

$$E(w_i^1) = F(v_i)\{1-F(v_i)\}^{m(i)-1}\prod_{j=0}^{i-1}\{1-F(v_j)\}^m. \quad (11)$$

Therefore, each element of I for the conventional step-up test is expressed as

$$E\left(\frac{\partial^2 \log L^F}{\partial\theta_a\partial\theta_b}\right) = \sum_i E(w_i^0)E\left(\frac{\partial^2 \log(1-F(v_i))}{\partial\theta_a\partial\theta_b}\right) + \sum_i E(w_i^1)E\left(\frac{\partial^2 \log F(v_i)}{\partial\theta_a\partial\theta_b}\right), \quad (12)$$

and that for the new step-up test is expressed as

$$E\left(\frac{\partial^2 \log L^f}{\partial\theta_a\partial\theta_b}\right) = \sum_i E(w_i^0)E\left(\frac{\partial^2 \log(1-F(v_i))}{\partial\theta_a\partial\theta_b}\right) + \sum_i E(w_i^1)E\left(\frac{\partial^2 \log f(V_i)}{\partial\theta_a\partial\theta_b}\middle| V_i < v_i\right), \quad (13)$$

where θ_a or θ_b denotes μ or σ. More specifically

$$E\left(\frac{\partial^2 \log(1-F(v_i))}{\partial\sigma^2}\right) = \frac{1}{\sigma^2}\left(\frac{-x_i^2 z_i^2}{q_i^2} + \frac{(x_i^3-2x_i)z_i}{q_i}\right),$$

$$E\left(\frac{\partial^2 \log(1-F(v_i))}{\partial\sigma\partial\mu}\right) = \frac{1}{\sigma^2}\left(\frac{-x_i z_i^2}{q_i^2} + \frac{(x_i^2-1)z_i}{q_i}\right),$$

$$E\left(\frac{\partial^2 \log(1-F(v_i))}{\partial\mu^2}\right) = \frac{1}{\sigma^2}\left(\frac{-z_i^2}{q_i^2} + \frac{x_i z_i}{q_i}\right), \quad (14a)$$

$$E\left(\frac{\partial^2 \log(F(v_i))}{\partial\sigma^2}\right) = \frac{1}{\sigma^2}\left(\frac{-x_i^2 z_i^2}{p_i^2} - \frac{(x_i^3-2x_i)z_i}{p_i}\right),$$

$$E\left(\frac{\partial^2 \log(F(v_i))}{\partial\sigma\partial\mu}\right) = \frac{1}{\sigma^2}\left(\frac{-x_i z_i^2}{p_i^2} - \frac{(x_i^2-1)z_i}{p_i}\right),$$

$$E\left(\frac{\partial^2 \log(F(v_i))}{\partial\mu^2}\right) = \frac{1}{\sigma^2}\left(\frac{-z_i^2}{p_i^2} - \frac{x_i z_i}{p_i}\right), \quad (14b)$$

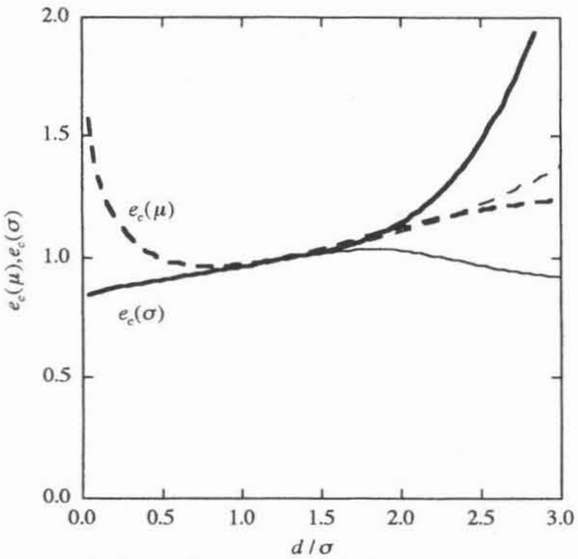


Figure 2. $e_c(\mu)$, $e_c(\sigma)$ against d/σ in the conventional step-up method. Thick line, $^\exists j$ s.t. $v_j = \mu$; Thin line, $^\exists j$ s.t. $(v_{j-1}+v_j)/2 = \mu$.

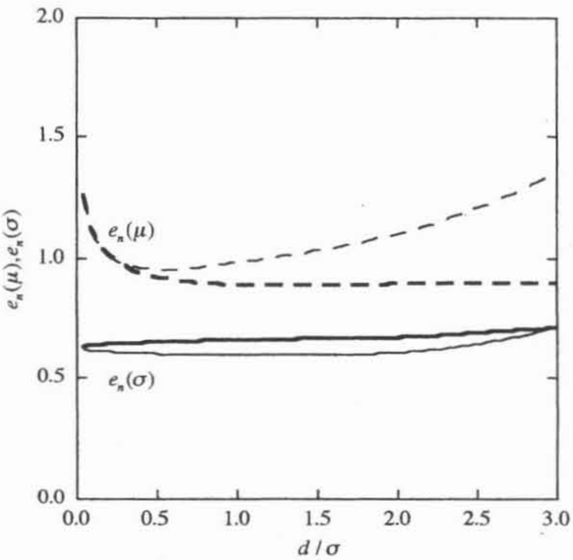


Figure 3. $e_n(\mu)$, $e_n(\sigma)$ against d/σ in the new step-up method. Thick line, $^\exists j$ s.t. $v_j = \mu$; Thin line, $^\exists j$ s.t. $(v_{j-1}+v_j)/2 = \mu$.

and

$$E\left(\frac{\partial^2 \log f(V_i)}{\partial\sigma^2}\middle| V_i < v_i\right) = \frac{1}{p_i}\int_{-\infty}^{v_i}\frac{\partial^2 \log f(v)}{\partial\sigma^2} f(v)\,dv = \frac{1}{\sigma^2 p_i}(3x_i z_i - 2p_i),$$

$$E\left(\frac{\partial^2 \log f(V_i)}{\partial\sigma\partial\mu}\middle| V_i < v_i\right) = \frac{1}{p_i}\int_{-\infty}^{v_i}\frac{\partial^2 \log f(v)}{\partial\sigma\partial\mu} f(v)\,dv = \frac{2z_i}{\sigma^2 p_i},$$

$$E\left(\frac{\partial^2 \log f(V_i)}{\partial\mu^2}\middle| V_i < v_i\right) = \frac{1}{p_i}\int_{-\infty}^{v_i}\frac{\partial^2 \log f(v)}{\partial\mu^2} f(v)\,dv = -\frac{1}{\sigma^2}, \qquad (15)$$

where $x_i = (v_i - \mu)/\sigma$, $z_i = (1/\sqrt{2\pi})e^{-x_i^2/2}$, and $p_i = F(v_i) = 1 - q_i$.

Figures 2 and 3 show $e(\mu)$ and $e(\sigma)$ against d/σ when $m = 1$; the thick line expresses the case when some level v_j is equal to μ, and the thin line expresses the case when μ is located in the middle of v_{j-1} and v_j. These figures suggest the following.

(1) The asymptotic unit error $e_n(\sigma)$ in the new step-up method becomes smaller than $e_c(\mu)$ in the conventional step-up method.

(2) The optimal test for $e(\sigma)$ can be realized around $0.5 \le d/\sigma \le 1.0$.

(3) The asymptotic unit error $e_n(\sigma)$ in the new step-up method becomes considerably smaller than $e_c(\sigma)$ in the conventional step-up method. For example, the difference between $e_n(\sigma) = 0.6586$ and $e_c(\sigma) = 0.9619$ at $d/\sigma = 1.0$ means that the samples in the conventional step-up method requires samples about twice as large as in the new step-up method to obtain the equivalent confidence interval.

(4) The optimal test for $e_n(\sigma)$ does not depend on d/σ, while the optimal test for $e_c(\sigma)$ can be realized at smaller d/σ.

In short, the new step-up method markedly improves the reliability of the estimates of μ and σ compared to the conventional step-up method. It is desirable to set d/σ such that $0.5 \le d/\sigma \le 1.0$ to obtain the smaller errors of the estimates. For the estimate of σ, about a half of the sample size in the new step-up method is sufficient for obtaining the equivalent reliability to the conventional method. This is the result for the case of $m = 1$, but this tendency is also true for $m > 1$.

5 MONTE CARLO SIMULATION

A Monte Carlo simulation study is done in order to investigate the asymptotic properties of the estimates for the conventional and new step-up methods. The simulation conditions are as follows:

(1) The very first stress step is set to around $\mu - 6\sigma$, and some stress level is set just to (a) $\mu = v_j$ and (b) $\mu = (v_{j-1} + v_j)/2$.

(2) The number of samples, n, is 100, 50, 20, 10.

(3) The step-up distance to σ, d/σ, is 0.1, 0.2, 0.5, 1.0.

(4) The parameter values are $\mu = 0$ and $\sigma = 1$.

(5) The number of repetition times of strikes at the same stage is $m = 1$.

(6) The number of trial times is 1000.

Figures 4 and 5 show the scatter plots of the estimates $\hat{\mu}$ and $\hat{\sigma}$ for the conventional and new step-up methods, respectively, when $n = 100$, $d/\sigma = 1$, $\mu = v_j$ for some j. We can see the reduction of the confidence region of the

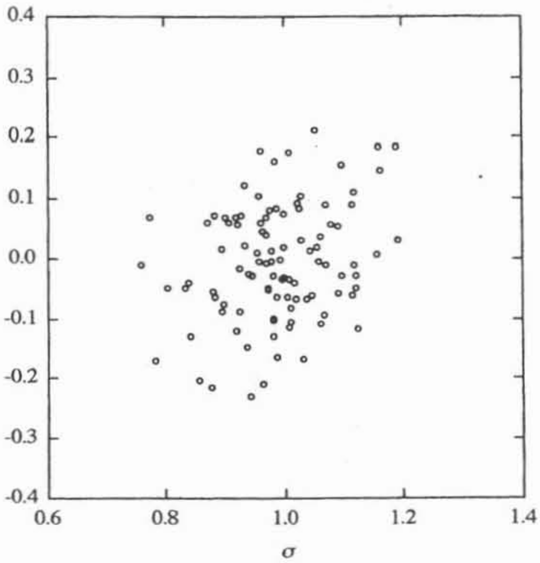


Figure 4. Scatter plot of estimates in the conventional step-up method.

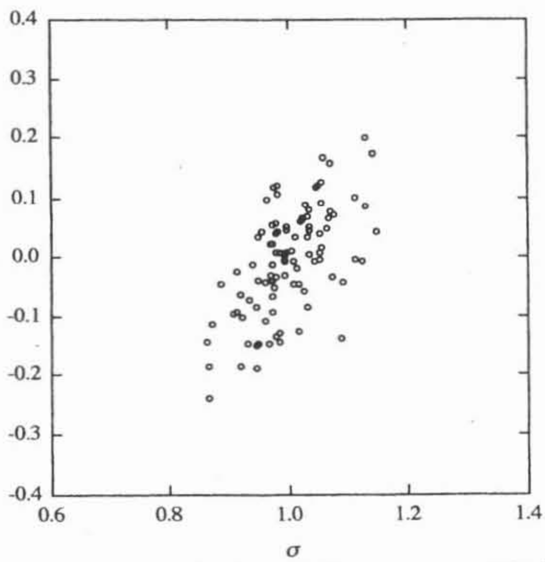


Figure 5. Scatter plot of estimates in the new step-up method.

new step-up method compared to that of the conventional one. In addition, a correlation between $\hat{\mu}$ and $\hat{\sigma}$ is observed to be much stronger in the new step-up method than in the conventional step-up method. Tables 2 and 3 show the bias and $\sqrt{n}\cdot$RMSE of the estimates $\hat{\mu}$ and $\hat{\sigma}$, where RMSE denotes root mean square error. The value $\sqrt{n}\cdot$RMSE corresponds to the unit asymptotic error $e(\mu)$ or $e(\sigma)$. Comparing the unit asymptotic errors in Figures 2 and 3, the mean and standard error in the simulation correspond to the asymptotic values.

It should be noted that the cases in which we cannot obtain the estimates (not numerically, but mathematically) are not rare in the conventional step-up method when n is small such as $n \le 10$. In the tables, M denotes the number of successfully computed estimates. This is one of the reasons we recommend the use of the new step-up method.

Table 2. Bias and $\sqrt{n}\cdot$RMSE of the estimates in the conventional step-up method.

(a) $\mu = v_j$						
d/σ	n	M	bias(μ)	$\sqrt{n}\cdot$RMSE(μ)	bias(σ)	$\sqrt{n}\cdot$RMSE(σ)
0.1	∞			1.33883		0.85820
0.1	100	1000	−0.01565	1.31505	−0.01033	0.83696
0.1	50	1000	−0.02535	1.35425	−0.01754	0.86715
0.1	20	1000	−0.06068	1.32464	−0.04674	0.86408
0.1	10	999	−0.09605	1.40147	−0.07132	0.92080
0.2	∞			1.16763		0.87334
0.2	100	1000	−0.00541	1.19667	−0.00452	0.90923
0.2	50	1000	−0.00972	1.14082	−0.00750	0.86883
0.2	20	1000	−0.06702	1.17483	−0.05297	0.88114
0.2	10	1000	−0.09592	1.18525	−0.08995	0.90371
0.5	∞			0.99371		0.90737
0.5	100	1000	−0.00311	0.99546	−0.00580	0.92966
0.5	50	1000	−0.00547	0.96530	−0.01449	0.91845
0.5	20	1000	−0.03734	0.98349	−0.05845	0.89573
0.5	10	998	−0.05822	0.96757	−0.08851	0.92603
1.0	∞			0.96975		0.96190
1.0	100	1000	−0.00888	0.97953	−0.00974	0.95370
1.0	50	1000	−0.01172	0.98726	−0.02458	0.97271
1.0	20	997	−0.02043	0.97285	−0.04165	1.00539
1.0	10	934	−0.02356	0.97229	−0.05966	1.21542
(b) $\mu = (v_{j-1} + v_j)/2$						
d/σ	n	M	bias(μ)	$\sqrt{n}\cdot$RMSE(μ)	bias(σ)	$\sqrt{n}\cdot$RMSE(σ)
0.1	∞			1.33883		0.85820
0.1	100	1000	−0.01068	1.38688	−0.00822	0.87317
0.1	50	1000	−0.02565	1.32665	−0.01832	0.85871
0.1	20	1000	−0.05886	1.32150	−0.04839	0.87245
0.1	10	1000	−0.1211	1.31681	−0.07708	0.86031
0.2	∞			1.16763		0.87334
0.2	100	1000	−0.00126	1.16286	−0.00272	0.86909
0.2	50	1000	−0.02148	1.16130	−0.01592	0.87124
0.2	20	1000	−0.04787	1.12645	−0.03977	0.84833
0.2	10	1000	−0.08908	1.13111	−0.08386	0.90046
0.5	∞			0.99371		0.90737
0.5	100	1000	−0.00128	0.97771	−0.00611	0.90194
0.5	50	1000	−0.00706	0.97784	−0.01595	0.91662
0.5	20	1000	−0.02518	0.92641	−0.05540	0.87286
0.5	10	994	−0.06459	0.99400	−0.08936	0.95084
1.0	∞			0.96985		0.96225
1.0	100	1000	−0.00498	1.00257	−0.01004	0.98712
1.0	50	1000	−0.01005	0.99450	−0.02239	0.96821
1.0	20	997	−0.02182	0.97482	−0.04150	0.99532
1.0	10	926	−0.01451	0.94294	−0.07614	1.26010

M: number of estimates successfully computed.

6 DISCUSSION

6.1 WEIBULL UNDERLYING PROBABILITY DISTRIBUTION

In fitting the probability distribution model to the data obtained from increasing voltage tests, the Weibull model is often applied. It would, thus, be natural to assume a Weibull distribution for the breakdown voltage probability distribution. However, the results obtained in this paper will also be useful in some Weibull models, since the Weibull distribution in which the shape parameter is set to around 3.4 can be approximated to a normal distribution. I will continue the research for the optimal test method in the case of the Weibull distribution in the future.

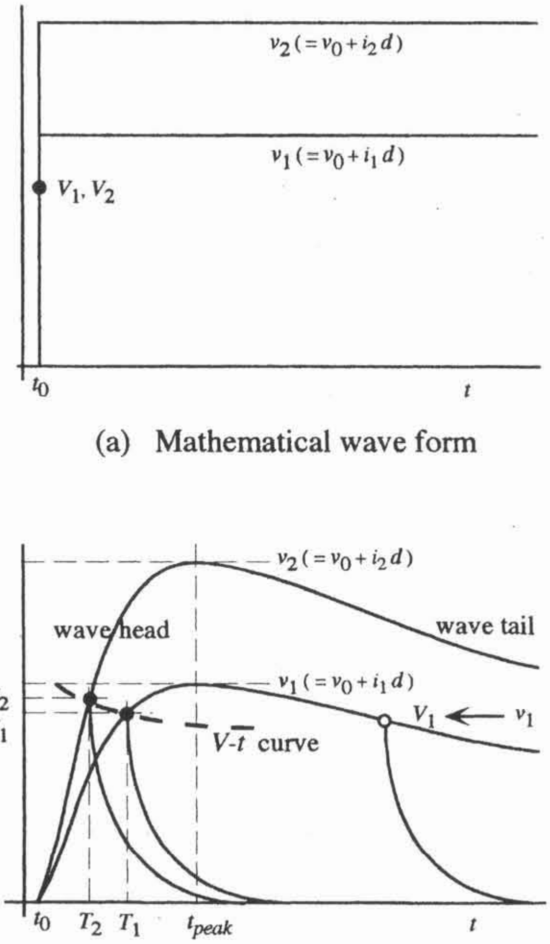
Table 3. Bias and $\sqrt{n}\cdot$RMSE of the estimates in the new step-up method.

d/σ	n	M	bias(μ)	$\sqrt{n}\cdot$RMSE(μ)	bias(σ)	$\sqrt{n}\cdot$RMSE(σ)
(a) $\mu = \upsilon_j$						
0.1	∞			1.11440		0.63546
0.1	100	1000	−0.00963	1.08275	−0.00511	0.61369
0.1	50	1000	−0.01643	1.10707	−0.00972	0.62618
0.1	20	1000	−0.03647	1.11234	−0.02567	0.64478
0.1	10	999	−0.06839	1.18494	−0.04311	0.68755
0.2	∞			1.01717		0.64114
0.2	100	1000	−0.00670	1.01013	−0.00494	0.65019
0.2	50	1000	−0.01044	0.99752	−0.00596	0.64280
0.2	20	1000	−0.05309	1.02456	−0.03327	0.64341
0.2	10	1000	−0.06653	1.02672	−0.05054	0.65316
0.5	∞			0.92269		0.65030
0.5	100	1000	−0.00531	0.91557	−0.00665	0.66281
0.5	50	1000	−0.03231	0.92322	−0.03398	0.63499
0.5	20	1000	−0.03231	0.92322	−0.03398	0.63499
0.5	10	998	−0.05540	0.91112	−0.06020	0.65536
1.0	∞			0.89007		0.65857
1.0	100	1000	−0.00838	0.89254	−0.00606	0.67670
1.0	50	1000	−0.01235	0.88715	−0.01421	0.64960
1.0	20	997	−0.02139	0.91663	−0.02264	0.67348
1.0	10	934	−0.03615	0.89258	−0.05351	0.67783
(b) $\mu = (\upsilon_{j-1} + \upsilon_j)/2$						
d/σ	n	M	bias(μ)	$\sqrt{n}\cdot$RMSE(μ)	bias(σ)	$\sqrt{n}\cdot$RMSE(σ)
0.1	∞			1.10144		0.61693
0.1	100	1000	−0.00783	1.13302	−0.00535	0.63344
0.1	50	1000	−0.01647	1.12203	−0.01031	0.64193
0.1	20	1000	−0.05886	−0.03365	1.09740	−0.02638
0.1	10	1000	−0.1211	−0.09275	1.12715	−0.05077
0.2	∞			1.01277		0.61210
0.2	100	1000	−0.00216	0.99116	−0.00270	0.62303
0.2	50	1000	−0.02148	−0.01606	1.01032	−0.00827
0.2	20	1000	−0.04787	−0.03792	1.01108	−0.02440
0.2	10	1000	−0.08908	−0.06174	0.98009	−0.04642
0.5	∞			0.95738		0.60375
0.5	100	1000	−0.00305	0.91099	−0.00529	0.90194
0.5	50	1000	−0.00706	−0.00563	0.90486	−0.00850
0.5	20	1000	−0.02518	−0.01991	0.87475	−0.03076
0.5	10	994	−0.06459	−0.06115	0.93542	−0.05844
1.0	∞			0.98486		0.59879
1.0	100	1000	−0.00673	0.91309	−0.00504	0.98712
1.0	50	1000	−0.01005	−0.01158	0.89719	−0.01170
1.0	20	997	−0.02182	−0.02360	0.88801	−0.02532
1.0	10	926	−0.01451	−0.04143	0.91529	−0.06435

M: number of estimates successfully computed.

6.2 MATHEMATICAL AND PHYSICAL MODELS

This paper considers an impulse breakdown voltage estimation method from a mathematical point of view. The physical impulse waveform is not taken into account of, i.e., the impulse waveform is assumed to simply be a rectangular form (see Figure 6a). In actual tests, a time to reach the peak value (t_{peak} in Figure 6b), e.g., 1.2 μ sec, is required. It is known that the breakdown voltages depend on the tangent of the wave rise even if t_{peak} is the same, i.e., the steeper the tangent, the higher the breakdown voltage. This phenomenon is known to as the *V-t* curve (dashed curve in Figure 6b). The *V-t* curve consists of the points of (V_b, t_b) where $V_b = V_1$ in Figure 6b if the breakdown occurs at the wave head, i.e., $T_1 < t_{peak}$, and $V_b \leftarrow \upsilon_1$ if the breakdown occurs at the wave tail, i.e., $T_1 > t_{peak}$.



(a) Mathematical wave form

(b) Physical wave form

Figure 6. Physical and mathematical impulse voltage breakdown wave forms. a, mathematical wave form; b, physical wave form.

We assume in this paper that μ and σ are independent of impulse voltage value in the mathematical model, but actually they depend on it. Because of this, the estimated $\hat{\mu}$ (50% impulse breakdown voltage) in the mathematical model is smaller than actual $\mu(\upsilon)$ in the physical model; $\hat{\mu}$, therefore, can be used in a safety side, since $\hat{\mu}$ becomes conservative.

6.3 BATCH PURITY AND STANDARDIZATION FOR INSULATION SAMPLES

Since the test samples cannot be made exactly the same size, the breakdown voltage itself cannot be used as a standard index. Instead, the breakdown strength of each sample can be used for standardization. The electric field of the insulation in each sample is computed in advance, and is taken into account in the estimation procedure.

6.4 INSTRUMENTATION ERROR EFFECTS

Even if the impulse system is set to give the same waveform, we cannot avoid the fluctuation of the waveform caused by the test instrument system; peak value v varies each time. However, recent improvements of the high-speed voltage-measuring instrument enable us to provide the exact peak value v when no breakdown occurs, and the maximum likelihood method enables us to utilize these exact values in the estimation procedure; we only have to change v_j in equations (3) and (5) to $v_{k,j}$ which depends on test samples and test times.

6.5 EXAMPLE 2

To understand the actual estimation procedure more precisely, we provide a second example. The testing material is a solid insulation. The final impulse voltage v_k, corresponding strength u_k, and breakdown voltage V_k ($k = 1,...,1.5$) are shown in Table 4. The thickness of each insulation is calculated by $d_k = v_k/u_k$. For example, $d_1 = 0.0869$ for test sample 1. The very first impulse voltage is $v_{1,0} = 45$ and the step-up voltage is 1.5; the impulses applied three times at each stage ($m = 3$). The insulation is broken by the first impulse at the final stage $v_{1,63} = 139.5$ with $V_{1,63} = 138.9$; then, $u_{1,63} = 1647.28$ and $U_{1,63} = 1637.84$. We denote U_k as the breakdown strength. Therefore, the impulses are applied 190 times by the step-up method ($190 = 63 \times 3 + 1$). In this example, we did not use each observed $v_{k,j}$.

In Table 4, there are samples that have the same values of impulse and breakdown voltages; these samples are broken at wave tail or peak (we say group TP), while others at wave head (we say group H); see Figure 6b and the reference [7]).

If we regard all the data of group TP as the wave tail broken data, the corresponding likelihood part for these data may be treated as a case of incomplete data as is described in [7] (i.e., $F(v_{i(k)})$ is used for the likelihood function like equation (3)), unlike the likelihood part for the wave head broken data in which they are treated as complete data (i.e., $f(V_{i(k)})$ is used for the likelihood function like equation (5)). It looks as if equations (3) and (5) are mixed in the mathematical formulation. Using this estimation procedure, μ and σ are estimated to be $\hat{\mu} = 2841.3$ and $\hat{\sigma} = 470.5$ in breakdown strength. The nominal mean of the breakdown strength is 1906.86 from a direct calculation using Table 4. If a single impulse of $u = 1906.86$ is applied to the insulation, then the probability that the insulation is broken is 0.02351 which is far smaller than 0.5, if a normal distribution is assumed as the underlying probability distribution function.

If we regard all the data of group TP as the wave peak broken data, $\hat{\mu} = 2678.0$ and $\hat{\sigma} = 382.8$ are estimated in the breakdown strength. If a single impulse of $u = 1906.86$ is applied to the insulation, then the probability that the insulation is broken is 0.02197.

Table 4. Actual step-up test data.

test piece number	final impulse voltage	final impulse strength	final breakdown voltage	Breakdown point
1	139.5	1647.28	138.8	head
2	135.0	1725.98	135.0	tail/peak
3	127.5	1596.82	123.8	head
4	148.5	2234.24	135.1	head
5	151.5	1957.32	142.8	head
6	148.5	1878.81	142.6	head
7	150.0	1980.18	131.1	head
8	121.5	1678.08	121.5	tail/peak
9	142.5	1766.56	142.5	tail/peak
10	157.5	2174.88	157.5	tail/peak
11	148.5	2074.78	148.5	tail/peak
12	145.5	1963.9	145.5	tail/peak
13	160.5	2383.86	160.5	tail/peak
14	123.0	1698.78	119.2	head
15	141.0	1841.36	141.0	tail/peak

The first impulse voltage is 45, and the step-up voltage is 1.5. The impulses are applied three times at each stage, and the insulation is broken at by the first impulse at the final stage.

7 CONCLUDING REMARKS

To estimate the impulse breakdown voltages for non-self-restoring electrical insulation, the step-up method has been used. Since the nominal breakdown voltages using the data of the step-up method do not express the consistent values or invariant values of the step-up distance, this paper first recommends the use of the parameters of the underlying probability distribution, e.g., mean and standard deviation in a normal distribution. Second, it is advantageous to use the new step-up method if the observed breakdown voltage itself rather than the two-valued information of breakdown and non-breakdown is available. Using the new step-up method, the number of test specimens can be reduced to half of that used in the conventional step-up method for the estimate of σ. The optimal test procedure is to set d/σ such that $0.5 \leq d/\sigma \leq 1.0$. Third, even if there exists cases in which the estimates cannot be obtained mathematically in the conventional step-up method, the new step-up method has the solutions with probability 1. Some examples are given for easy understanding of the estimation procedure.

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