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ACCURATE STRESS INTENSITY FACTORS FOR KINKED INTERFACE CRACK IN BONDED DISSIMILAR HALF-PLANE

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In this study, the stress intensity factor (SIF) of an interface kinked crack is analyzed by the singular integral equation of the body force method. The problem can be expressed by distributing the body force doublets of the tension and shear types along all the boundaries of the kinked and interface crack parts. The SIFs can be obtained directly from the densities of the body force doublets at the crack tips. Although the problem has already been calculated using the crack connection model, the accuracy of the analysis has not been clarified. From the analysis results in this study, it can be seen that the SIFs calculated by the crack connection model have a non-negligible error, and the present method gives more accurate results. The advantage of the present method is that the SIFs of the kinked and the interface crack tips can be obtained at the same time with high accuracy.

Keywords: Stress intensity factor; Kinked interface crack; Singular integral equation.

1. Introduction

In recent years, composites and adhesively bonded materials have been used in a wide range of engineering applications. The problems associated with interfacial cracks are of great interest because defects and cracks along the interface can reduce the strength of the structure.[1-6] However, there are few detailed studies on the stress intensity factor (SIF) of kinked interface cracks,[3,5] which is of practical importance, and the accuracy of the analysis has not been clarified.

We have previously reported that the SIFs for kinked cracks in a homogeneous plane [7] and collinear interface cracks in a bonded dissimilar plane [6] can be accurately calculated by the numerical solution of singular integral equation of the body force method (BFM).

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In this paper, the method is applied to the kinked interface crack problem. By comparing the present results with those of other studies, the accuracy of the proposed method is demonstrated. The effects of kinked angle and material combination on the SIF are also discussed.

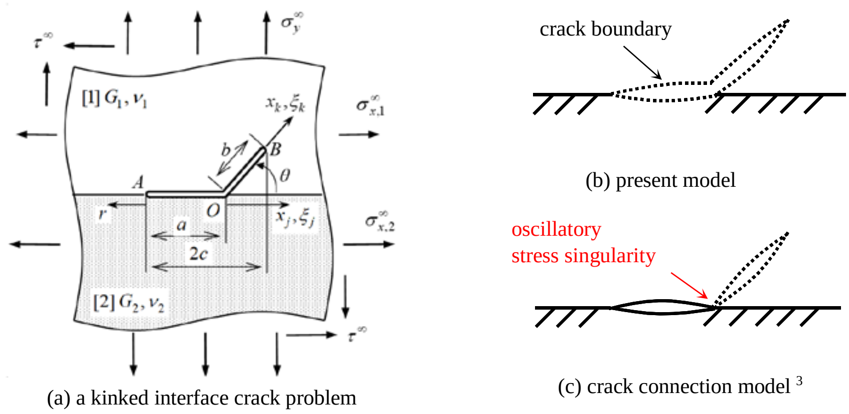

Fig. 1. Treated problem and crack boundary distributed force doublets, (a) a kinked interface crack problem, (b) present model and (c) crack connection model in Ref. 4 to satisfy the boundary condition of crack surface.

2. Numerical Solution of Singular Integral Equation

In this study, the problem of a kinked interface crack in dissimilar materials is treated as shown in Fig. 1. The elastic constants are given as shear modulus and Poisson's ratios for the upper (material 1) and the lower (material 2) half-planes, that is, (G_1, ν_1) and (G_2, ν_2). From the perfect bonding condition of interface, the remote stresses have the following relation,[3]

$$\sigma_{x,2}^{\infty} = \frac{G_2}{G_1}(1+\kappa_1)\sigma_{x,1}^{\infty} + \left\{3-\kappa_2-\frac{G_2}{G_1}(3-\kappa_1)\right\}\sigma_y^{\infty}. \tag{1}$$

Here, $\kappa_m=(3-\nu_m)/(1+\nu_m)$ for plane stress, $\kappa_m=3-4\nu_m$ for plane strain (m=1, 2).

The problem can be formulated in terms of singular integral equations by means of the BFM.[6,7] In the interface crack problem, the stress fields induced by two kinds of standard set of force doublets, tension type and shear type, in dissimilar materials without the crack are used as the fundamental solution.[6,7] The singular integral equations can be obtained by the force doublets along the imaginary boundaries of the crack as shown in Fig.1(b). The SIFs can be directly evaluated from the densities of body force doublets distributed on the imaginary interface crack part and kinked crack part, respectively.[6,7]

This problem has been already analyzed by using the crack connection model as shown in Fig.1(c).[3] In this model, the boundary condition of the interface crack part is completely satisfied because the fundamental solution of bonded half planes with an interface crack is used. However, the accuracy of the obtained SIFs has not yet been clarified. As shown in later, the present method gives more accurate results than using the crack connection model.

3. Numerical Results and Discussion

The problem of the kinked interface crack is analyzed when the relative kinked crack length b/a and the material combination G_2/G_1 are changed systematically. In this analysis, Poisson's ratio $\nu_1=\nu_2=0.3$ and the plane stress condition is assumed. The normalized SIFs, $F_{1,A}$, $F_{2,A}$, $F_{I,B}$ and $F_{II,B}$, are defined by the following expressions:

$$K_{1,A} + iK_{2,A} = \{F_{1,A} + iF_{2,A}\}\sigma\sqrt{\pi a}(1+2i\varepsilon), \tag{2}$$

$$K_{I,B} = F_{I,B}\sigma\sqrt{\pi b},\ K_{II,B} = F_{II,B}\sigma\sqrt{\pi b}. \tag{3}$$

Here, $K_{1,A}$ and $K_{2,A}$ are the SIFs of the interface crack defined by [2]

$$\sigma_y + i\tau_{xy} = \frac{K_{1,A} + iK_{2,A}}{\sqrt{2\pi r}}\left(\frac{r}{2a}\right)^{i\varepsilon}, \qquad \varepsilon = \frac{1}{2\pi}\ln\left(\frac{G_2\kappa_1 + G_1}{G_1\kappa_2 + G_2}\right) \tag{4}$$

where 'r' is the distance from the interface crack tip shown in Fig.1(a).

Table 1 shows the comparison of the numerical results between the present model and the crack connection model illustrated in Fig.1. From Table 1, when G_2/G_1=1.0, both results are in good agreement with each other. However, for G_2/G_1=0.25 and 4.0, there is a maximum difference of about 10 % between both results. The values in parentheses are the results analyzed by the finite element method (FEM) for b/a=1. The FEM values are close to the present results. Therefore, the numerical results obtained in this study are more accurate than the values calculated by the crack connection model.

Table 1. Comparison of normalized SIF, $F_{I,B}$ ($\sigma_y^{\infty} = 1.0, \theta = 45°$).

G_2/G_1	0.25		1.0		4.0	
b/a	Present [Fig.1(b)]	Ref.3 [Fig.1(c)]	Present [Fig.1(b)]	Ref.3 [Fig.1(c)]	Present [Fig.1(b)]	Ref.3 [Fig.1(c)]
0.1	0.743	0.703	0.655	0.655	0.634	0.705
0.5	0.800	0.770	0.663	0.663	0.620	0.668
1.0	0.902	0.877	0.743	0.744	0.690	0.732
1.0 (FEM)	(0.899)		(0.743)		(0.687)	
1.5	1.000	0.977	0.824	0.824	0760	0.799

Figures 2 and 3 show the normalized SIFs of the interface crack tip A and the kinked crack tip B under three types of loading conditions, respectively. The analysis is performed by changing the kinked angle θ and G_2/G_1. As shown in Fig. 2, $F_{1,A}$, $F_{2,A}$ of interface crack tip A under tension in y-direction are 0.75 < ($F_{1,A}$, $F_{2,A}$) < 1.2 even when G_2/G_1 changes significantly. From Fig.3, $F_{I,B}$ and $F_{II,B}$ for kinked crack tip B are less affected by the stiffness ratio when $G_2/G_1 \geq 0.5$, regardless of the loading conditions.

4. Conclusion

In this paper, the SIFs for kinked interface crack were calculated accurately by using the singular integral equations of the BFM. By comparing the present results with those of other methods, it was found that the present method gives more accurate results for the kinked interface crack. The normalized SIFs at the kinked crack tip are less affected by the stiffness ratio when $G_2/G_1 \geq 0.5$, regardless of the loading conditions.

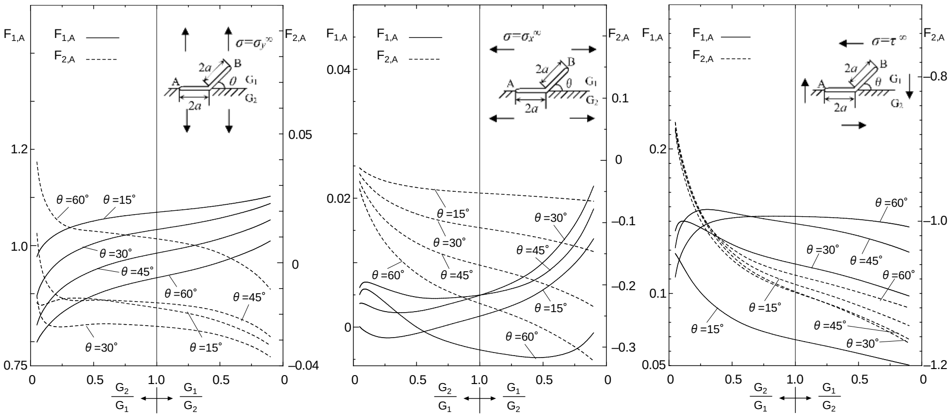

Fig. 2. Normalized SIFs $F_{1,A}$, $F_{2,A}$ for kinked interface crack at the interface crack tip A under three different loads, (a) tension in y-direction, (b) tension in x-direction and (c) shear when b/a=1.0, $\nu_1 = \nu_2$=0.3, Plane stress.

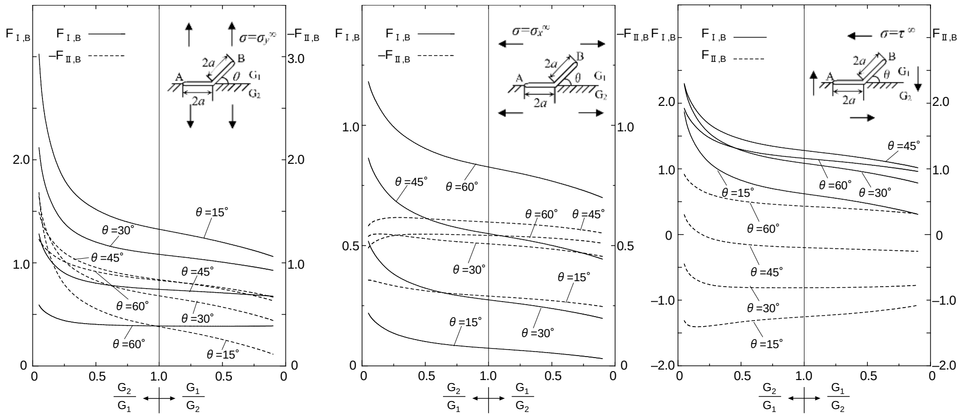

Fig. 3. Normalized SIFs $F_{I,B}$, $F_{II,B}$ for kinked interface crack at the kinked crack tip B under three different loads, (a) tension in y-direction, (b) tension in x-direction and (c) shear when b/a=1.0, $\nu_1 = \nu_2$=0.3, Plane stress.

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