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Statistical Tests and Analysis Related to Disruptive Discharge

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Abstract

Concerning disruptive discharge in insulation, some statistical methods with the step-up test procedure have been recently proposed. The present paper aims to clarify the differences between the methods and the conventional methods provided by International Electrotechnical Commission, and fairly evaluate their performance in the mathematical or practical aspect. As a result, it is shown that depending on the assumption of statistical independence, the successive discharge method or the modified up-and-down method is the most practical.

1 Introduction

We are concerned with statistical methods dealing with disruptive discharge voltages in insulation. The conventional methods are given by International Electrotechnical Commission (IEC) [1]. On the other hand, two different types of methods have been recently introduced or proposed in a series of papers [2–4]. Although only the two types of methods have been compared with in the papers, it is not clear whether the methods are superior to the conventional methods provided by IEC or not. Hence, the present paper aims to clarify the differences among them and fairly evaluate their performance in the mathematical or practical aspect.

Three classes of disruptive discharge tests are provided by IEC (Appendix A in [1]). Of them, the classes in which the test voltage level may increase until a disruptive discharge occurs are up-and-down tests and successive discharge tests. One of the most important differences between them is what is supposed to be statistically independent.

Incidentally, the two types of methods mentioned above are the step-up method (SM), which has been originally proposed in [5], and the new step-up method (NSM). In the methods, each result in voltage stress applications is supposed to be statistically independent regardless of whether a disruptive discharge occurs or not. This assumption is the same as that in the up-and-down tests. In spite of that, the test procedure for obtaining data is very similar to that in the successive discharge tests, where each result in voltage stress applications is supposed to be statistically independent only if a disruptive discharge occurs. Now, a question arises: "is the test procedure really appropriate on the assumption?" The present paper considers this question.

In the sequel we will use the word "method" to mean a pair of a test procedure and the way of analysis of test results, such as the up-and-down method (UM).

In Sections 2 and 3 we will introduce the UM, the successive discharge method and the SM, and explain their differences clearly. In Sections 4 and 5 the SM and the UM or the NSM and the new up-and-down method (NUM) will be compared in the asymptotic or empirical errors. The modification of the UM and the SM will be considered in Section 6, and conclusions will be given in Section 7. An iterative formula to get estimates will be shown in Appendix A.

2 Methods provided by IEC

Concerning the UM and the successive discharge method, we introduce the test procedures, the likelihood methods and remarks.

2.1 Up-and-down method

The up-and-down 50% disruptive discharge voltage test is defined as follows ([1], p. 91).

i) Decide the first voltage level U_1, a small amount ΔU and the total number N of voltage stress applications.

ii) When a voltage stress is applied at the voltage level U_1, if no disruptive discharge occurs, set the next voltage level U_2 at $U_1 + \Delta U$, otherwise set that at $U_1 - \Delta U$.

iii) Perform similar tests at the succeeding voltage levels $U_2, U_3, \ldots, U_N$.

The likelihood function for the test is given as follows ([1], p. 99). Denote by d_i the number of discharge found in a voltage application at a voltage level U_i. Since $d_i = 1$ or 0, the number of withstand is given by $1 - d_i$ at U_i. Hence, if $F(U;\boldsymbol{\theta})$ is the discharge probability distribution function ($\boldsymbol{\theta}$ denotes a vector of parameters), the likelihood function L becomes:

$$L = \prod_{i=1}^{N} \Big(F(U_i;\boldsymbol{\theta})\Big)^{d_i} \Big(1 - F(U_i;\boldsymbol{\theta})\Big)^{1-d_i}. \tag{1}$$

All the test results are expressed by U_1, ΔU and $\{d_i\}_{i=1}^{N}$. As seen in (1), each result d_i in voltage stress applications is dealt with statistically independently.

2.2 Successive discharge method

The successive discharge test is defined as follows ([1], p. 91).

i) Continuously increase the test voltage on a test object until a disruptive discharge occurs.

ii) Perform similar tests n times.

The likelihood function for the test is given as follows ([1], p. 101). Denote by $\tilde{U}_j$ the voltage level at which a disruptive discharge occurs. If $f(\tilde{U};\boldsymbol{\theta})$ is the discharge probability density function, the likelihood function becomes:

$$L = \prod_{j=1}^{n} f(\tilde{U}_j;\boldsymbol{\theta}). \tag{2}$$

All the test results are expressed by $\{\tilde{U}_j\}_{j=1}^{n}$. As seen in (2), only voltage levels $\tilde{U}_i$'s at which disruptive discharges occur are dealt with statistically independently. The other voltage levels U_i's before disruptive discharges occur do not appear in (2). In other words, it is assumed that disruptive discharge voltages on test objects are expressed by a independent random variable $\tilde{U}$, and $\tilde{U}_j$ is an approximation to a value that $\tilde{U}$ takes on.

For a normal distribution, for example, since $f(\tilde{U};\boldsymbol{\theta}) = \exp[-(\tilde{U} - \mu)^2/2\sigma^2]/\sqrt{2\pi}\sigma$ in (2), the estimates of the parameters μ and σ are the sample mean and the sample standard deviation (Appendix A.3.3 in [1]).

3 Step-up method

Concerning the SM [2], we introduce the test procedure, the likelihood function and remarks.

The test procedure proceeds as follows.

i) Decide the first voltage level U_1, a small amount ΔU and a maximum number m of voltage stress applications at a voltage level. Here, U_1 should be sufficiently low such that almost no disruptive discharge occurs at the level.

ii) If no disruptive discharge occurs when substantially equal voltage stresses are applied m times at the voltage level U_1, set the next voltage level U_2 at $U_1 + \Delta U$. Similarly increase the succeeding voltage levels $U_3, U_4, \ldots$ until a disruptive discharge occurs.

iii) Perform ii) n times.

The likelihood function is given as follows. For the jth disruptive discharge ($1 \le j \le n$), denote by δ_j and m_j an integer for which a disruptive discharge occurs at U_{δ_j} and the number of voltage stress applications at the level. The likelihood function becomes:

$$L = \prod_{j=1}^{n} \left\{ F(U_{\delta_j};\boldsymbol{\theta})\Big(1 - F(U_{\delta_j};\boldsymbol{\theta})\Big)^{m_j-1} \prod_{k=1}^{\delta_j-1} \Big(1 - F(U_k;\boldsymbol{\theta})\Big)^{m} \right\}. \tag{3}$$

All the test results are expressed by U_1, ΔU and $\{(\delta_j, m_j)\}_{j=1}^{n}$. As seen in (3), each result (discharge or withstand) in voltage stress applications is dealt with statistically independently. Hence, the assumption concerning statistical independence is the same as that in the UM, not that in the successive discharge method. For this, the SM and the successive discharge method are not comparable (Section 2 in [2]), while the SM and the UM are comparable.

The UM has been proposed by Dixon and Mood [6] and they have given the following explanation concerning U_1: if U_1 is poorly chosen, the early observations from U_1 to some U_i will be spent in getting from U_1 to the region of the mean; they will obviously contribute little to the more precise location of the mean.

According to this, the SM spends n times the labor of testing in the UM to obtain almost valueless observations. Hence, our question in Section 1 arises here again. To seek an answer, in the next section we will evaluate their performance.

4 Comparison of the methods

We first compare the SM and the UM in the asymptotic errors of parameter estimators, and second investigate how many times an experimenter needs to test to obtain good estimates whose errors are close to the asymptotic errors. In the sequel the discharge probability distribution is supposed to be a normal distribution with mean μ and standard deviation σ.

4.1 Asymptotic unit errors

We define the asymptotic unit errors of the maximum likelihood (ML) estimators of μ and σ by $\sqrt{n(\mathcal{I}^{-1})_{11}}$ and $\sqrt{n(\mathcal{I}^{-1})_{22}}$, where $\mathcal{I}$ stands for the Fisher information matrix. In order to seek the matrix, let us rewrite (3). Denote by λ_i and ν_i the total numbers of discharges and withstands in the voltage applications at a voltage level U_i, respectively. These are expressed by

$$\lambda_i = \sum_{j=1}^{n} I_i(\delta_j), \qquad \nu_i = \sum_{j=1}^{n} \Big(m\tilde{I}_i(\delta_j) + (m_j - 1) I_i(\delta_j) \Big),$$

where $I_i(k) \overset{\text{def}}{=} 1$ if $i = k$ or 0 otherwise, and $\tilde{I}_i(k) \overset{\text{def}}{=} 1$ if $i < k$ or 0 otherwise. By utilizing these expressions, we can obtain

$$L = \prod_{i \ge 1} \Big(F(U_i;\mu,\sigma) \Big)^{\lambda_i} \Big(1 - F(U_i;\mu,\sigma) \Big)^{\nu_i} \tag{4}$$

from (3).

Next, let us seek the Fisher information matrix for (4). Denote $F(U_i; \mu, \sigma)$, $1 - p_i$ and $\prod_{k=1}^{i} q_k^m$ by p_i, q_i and r_i, respectively. In addition, introduce the following symbols: $x_i \stackrel{\text{def}}{=} (U_i - \mu)/\sigma$,

$$z_i \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x_i^2}{2}\right], \quad A_i \stackrel{\text{def}}{=} \begin{bmatrix} 1 & x_i \\ x_i & x_i^2 \end{bmatrix}.$$

From (4) the Fisher information matrix becomes:

$$\mathcal{I} = \frac{n}{\sigma^2} \sum_{i \geq 1} r_{i-1} \left(\sum_{k=0}^{m-1} q_i^k \right) \frac{z_i^2}{p_i q_i} A_i \tag{5}$$

since the expectations $E[\lambda_i]$ and $E[\nu_i]$ are expressed by

$$\sum_{j=1}^{n} \left(r_{i-1} \sum_{k=0}^{m-1} q_i^k p_i \right) = n r_{i-1} (1 - q_i^m) \tag{6}$$

and

$$\sum_{j=1}^{n} \left(m r_i + r_{i-1} \sum_{k=0}^{m-1} k q_i^k p_i \right) = n r_{i-1} \sum_{k=1}^{m} q_i^k, \tag{7}$$

respectively. Concerning (5), let $e_s(\mu)$ be the asymptotic unit error of the ML estimator of μ and $e_s(\sigma)$ that of σ.

On the other hand, the Fisher information matrix for (1) becomes:

$$\mathcal{I} = \frac{1}{\sigma^2} \sum_{i=1}^{N} \sum_{k=-i}^{i} P[\bar{I}_i(k) = 1] \frac{z_k^2}{p_k q_k} A_k, \tag{8}$$

where $\bar{I}_i(k) \stackrel{\text{def}}{=} 1$ if $U_i = U_1 + k\Delta U$ or 0 otherwise. (For details, see [7].) Similarly, let us denote by $e_u(\mu)$ and $e_u(\sigma)$ the asymptotic unit errors of the ML estimators concerning (8).

Suppose that $\mu = 0$ and $\sigma = 1$. We investigate the two cases, Case A: the mean falls on a voltage level and Case B: the mean is midway between two voltage levels. In each case, each graph of $e_s(\mu)$ and $e_s(\sigma)$ in the interval of $\Delta U/\sigma$ from 0.2 to 4 keeps the same shape if $U_1 \leq \mu - 3.5\sigma$ and $m = 1$. For this, we set $m = 1$ and $U_1 = \max_i(\mu - i\Delta U)$ or $\max_i(\mu - (i + 0.5)\Delta U)$ under the condition that $U_1 \leq \mu - 3.5\sigma$. On the other hand, each graph of $e_u(\mu)$ and $e_u(\sigma)$ also keeps the same shape if $N \geq 40$. This means the sample size 40 is large enough for us to know how the errors asymptotically behave in the UM. Thus, we set $N = 40$.

Finally, we obtain $e_s(\mu)$, $e_u(\mu)$, $e_s(\sigma)$ and $e_u(\sigma)$ in Figure 1. The thick or normal lines correspond to the SM or the UM, respectively. Throughout the present paper, the solid or dotted lines correspond to Case A or B, respectively.

The figure tells us that the SM is quite superior to the UM in the asymptotic unit errors. Especially, it is remarkable that $e_s(\mu)$ and $e_s(\sigma)$ are small, and in the interval of $\Delta U/\sigma$ from 0.2 to 1.8, each of them is almost the same in Cases A and B.

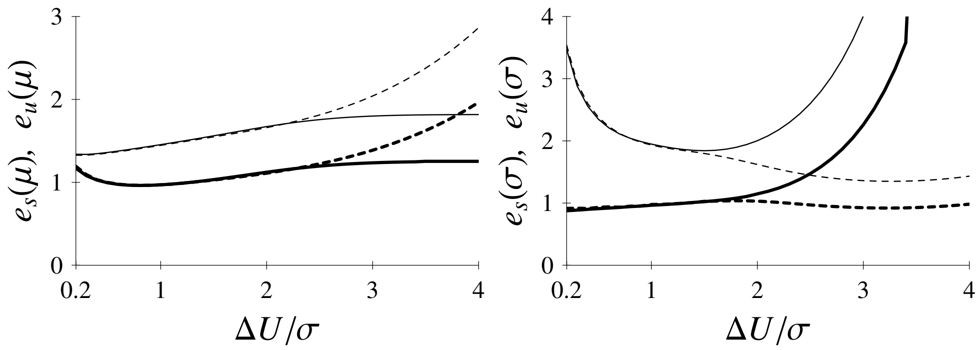

Figure 1: Asymptotic unit errors of the SM or the UM

4.2 Evaluation in small samples

We have seen that the SM can provide asymptotically good estimations in the interval [0.2, 1.8]. This is not, however, sufficient to approve the performance of the method. In this subsection, let us evaluate the method in small samples by means of Monte Carlo simulation.

We define the empirical unit error of an ML estimator by $\sqrt{n}$ times its root mean square error, and denote by $\bar{e}_s(\mu)$ and $\bar{e}_s(\sigma)$ the empirical unit errors of μ and σ in the SM. When $n = 20$ or 80, 10000 sets of independent pseudo-random samples are considered for each value of $\Delta U/\sigma (= 0.2, 0.3, \ldots 2.0)$.

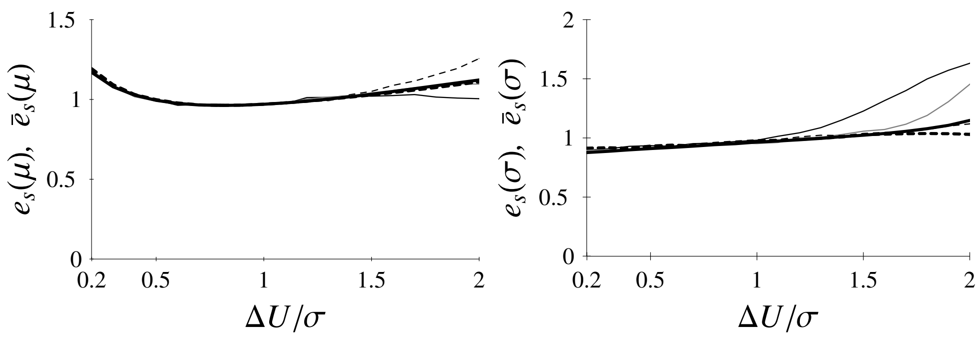

Figure 2: Empirical or asymptotic unit errors of the SM

Figure 2 gives $\bar{e}_s(\mu)$ and $\bar{e}_s(\sigma)$ for $n = 20$, 80 as well as $e_s(\mu)$ and $e_s(\sigma)$ for comparison. The thick lines correspond to the asymptotical unit errors, and normal or gray lines correspond to the empirical unit errors for $n = 20$ or 80, respectively. From the figure we can see the following: when $n = 20$, the empirical unit errors go away from the asymptotic unit errors as $\Delta U/\sigma$ becomes close to 2; when $n = 80$, the empirical unit errors are almost the same as the asymptotic unit errors except $\bar{e}_s(\sigma)$ in Case A.

Next, let us seek the average of the number of voltage stress applications necessary to obtain one disruptive discharge. By adding (6) and (7), substituting $n = 1$ into it and taking a summation over possible values of i, we obtain the average: $\sum_{i\geq 1} r_{i-1}(\sum_{k=0}^{m-1} q_i^k)$. Table 1 shows its values when $m = 1$. From the table, for example, we can see that it may be necessary to apply voltage stresses 272 times when $n = 80$ and $\Delta U/\sigma = 1.6$ in Case A.

Table 1: Average number of voltage stress applications

$\Delta U/\sigma$	0.4	0.8	1.2	1.6	2.0
Case A	9.0	5.1	4.4	3.4	2.5
Case B	7.5	4.6	3.9	2.9	3.0

Summarizing what we have seen in this section, we can say that the SM is superior to the UM in the asymptotic errors of the ML estimators, but it requires a much larger number of voltage stress applications than the UM for a good estimation. Consequently, the SM is less practical than the UM.

5 New step-up and new up-and-down methods

We have so far discussed the methods that deal with censored data ([8], p. 24) only. If some data are given by observation values, however, more precise estimation can be expected. Now, let us consider methods dealing with such data, which are called complete data.

A counterpart of the SM or the UM that deals with complete data is the NSM or the NUM [9]. In the test procedures, the difference between the NSM and the SM or the NUM and the UM is only whether a voltage, say u_j, at the moment when a disruptive discharge occurs is supposed to be recorded or not.

5.1 Asymptotic unit errors

First, we introduce the likelihood function and seek the Fisher information matrix in the NSM.

The likelihood function is given as follows [2]:

$$L = \prod_{j=1}^{n} \left\{ f(u_j;\boldsymbol{\theta})\Big(1 - F(U_{\delta_j};\boldsymbol{\theta})\Big)^{m_j-1} \prod_{k=1}^{\delta_j-1} \Big(1 - F(U_k;\boldsymbol{\theta})\Big)^{m} \right\}. \tag{9}$$

All the test results are expressed by U_1, ΔU and $\{(\delta_j, m_j, u_j)\}_{j=1}^{n}$.

Let us seek the Fisher information matrix in the usual way. By remembering that F is a normal distribution function with mean μ and standard deviation σ and rewriting (9) similarly to (4), we obtain

$$L = \prod_{i\geq 1} \left\{ \prod_{j=1}^{n} \Big(f(u_j;\mu,\sigma)\Big)^{I_i(\delta_j)} \Big(1 - F(U_i;\mu,\sigma)\Big)^{\nu_i} \right\}. \tag{10}$$

Here, note that the replacement of $f(u_j;\mu,\sigma)$ with $F(U_i;\mu,\sigma)$ yields (4). When we introduce

$$B_i \stackrel{\text{def}}{=} \begin{bmatrix} -x_i z_i + p_i & -z_i - x_i^2 z_i \\ -z_i - x_i^2 z_i & -x_i z_i - x_i^3 z_i + 2p_i \end{bmatrix},$$

the Fisher information matrix becomes:

$$\mathcal{I} = \frac{n}{\sigma^2} \sum_{i\geq 1} r_{i-1} \left(\sum_{k=0}^{m-1} q_i^k \right) \left(\frac{z_i^2}{q_i} A_i + B_i \right) \tag{11}$$

since for $\theta_1, \theta_2 \in \{\mu, \sigma\}$

$$\begin{aligned} E\left[I_i(\delta_j)\frac{\partial^2 \ln f(u_j;\mu,\sigma)}{\partial\theta_1\partial\theta_2}\right] &= P[I_i(\delta_j)=1]E\left[\frac{\partial^2 \ln f(u_j;\mu,\sigma)}{\partial\theta_1\partial\theta_2}\middle| I_i(\delta_j)=1\right] \\ &= E[I_i(\delta_j)]E\left[\frac{\partial^2 \ln f(u_j;\mu,\sigma)}{\partial\theta_1\partial\theta_2}\middle| u_j \le U_i\right], \end{aligned}$$

where $P[\cdot]$ and $E[\cdot|\cdot]$ means a probability and a conditional expectation. We denote by $e_{nu}(\mu)$ and $e_{nu}(\sigma)$ the asymptotic unit errors of the ML estimators concerning (11).

Next, we show the likelihood function and the Fisher information matrix in the NUM. The likelihood function is given as follows [9]:

$$L = \prod_{i=1}^{N}\Big(f(u_i;\boldsymbol{\theta})\Big)^{d_i}\Big(1-F(U_i;\boldsymbol{\theta})\Big)^{1-d_i}. \tag{12}$$

The Fisher information matrix for this becomes:

$$\mathcal{I} = \frac{1}{\sigma^2}\sum_{i=1}^{N}\sum_{k=-i}^{i} P[\bar{I}_i(k)=1]\left(\frac{z_k^2}{q_k}A_k + B_k\right). \tag{13}$$

(For details, see [7].) Similarly, let us denote by $e_{nu}(\mu)$ and $e_{nu}(\sigma)$ the asymptotic unit errors concerning (13).

It is remarkable that the expressions in the right-hand side of (5) and (11) or (8) and (13) are the same except the difference between $z_i^2A_i/p_iq_i$ and $z_i^2A_i/q_i + B_i$ or $z_k^2A_k/p_kq_k$ and $z_k^2A_k/q_k + B_k$.

Finally, when we set μ, σ, m, U_1 and N as in Subsection 4.1, we obtain $e_{ns}(\mu)$, $e_{nu}(\mu)$, $e_{ns}(\sigma)$ and $e_{nu}(\sigma)$ in Figure 3. The thick or normal lines correspond to the NSM or the NUM, respectively.

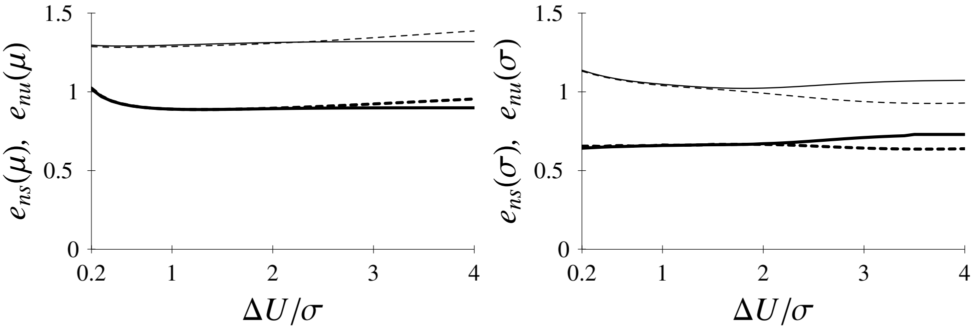

Figure 3: Asymptotic unit errors of the NSM or the NUM

The figure tells us that the NSM is superior to the NUM in the asymptotic unit errors, and in the interval of $\Delta U/\sigma$ from 0.2 to 2.0, each of $e_{ns}(\mu)$ and $e_{ns}(\sigma)$ is almost the same in Cases A and B.

5.2 Evaluation in small samples

Let us evaluate the performance of the NSM in small samples by means of Monte Carlo simulation. We denote by $\bar{e}_{ns}(\mu)$ and $\bar{e}_{ns}(\sigma)$ the empirical unit errors of the ML estimators of μ and σ in the NSM. When $n = 20$, we consider 10000 sets of independent pseudo-random samples for each simulation.

Figure 4 gives $\bar{e}_{ns}(\mu)$ and $\bar{e}_{ns}(\sigma)$ as well as $e_{ns}(\mu)$ and $e_{ns}(\sigma)$ for comparison. The thick or normal lines correspond to the asymptotical or empirical unit errors, respectively. From this we can see that the empirical unit errors are almost the same as the asymptotic unit errors in the interval $[0.2, 2.0]$. Since $n = 20$, Table 1 tells us that it is necessary to apply voltage stresses 50 or 60 times when $\Delta U/\sigma = 2.0$.

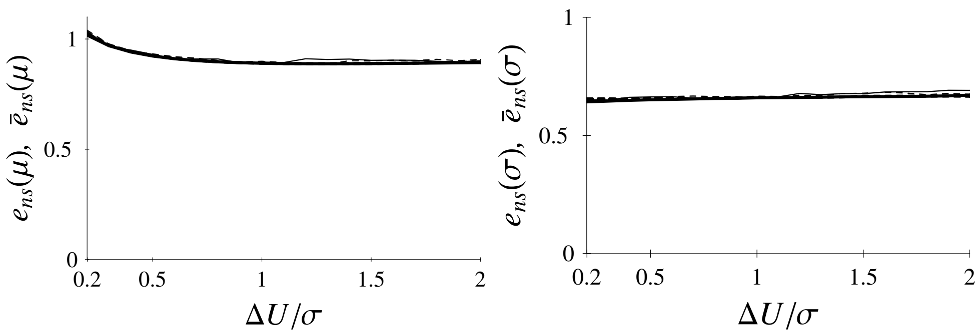

Figure 4: Empirical or asymptotic unit errors of the NSM

The summary of this section is as follows: the NSM is superior to the NUM in the asymptotic errors; compared with the SM, it can reduce the number of voltage stress applications necessary to attain the empirical unit errors as small as the asymptotic unit errors. This is one of the virtues of including complete data.

In a realistic situation, however, it may be impossible to obtain all data as complete data when disruptive discharges occur. This is because disruptive discharges do not always occur on the front of the impulse, but often occur on the tail of that ([1], pp. 119-120). When a disruptive discharge occurs on the tail, the data should not be regarded as a complete datum. Consequently, the NSM and the NUM are not very practical.

6 Discussion

As expected, we have seen that complete data can lead to increasing the precision of parameter estimation. In this section, thus, let us modify the UM and the SM by adopting a disruptive discharge voltage as a complete datum only if a disruptive discharge occurs on the head of the impulse. Then, let us call these methods in this case the modified up-and-down method (MUM) and the modified step-up method (MSM).

6.1 Asymptotic unit errors

First, we seek the likelihood function and the Fisher information matrix in the MSM. The likelihood function and the Fisher information matrix are sought as follows. For the jth disruptive discharge ($1 \leq j \leq n$), define

$$\tau_j \stackrel{\text{def}}{=} \begin{cases} 1 & \text{(it occurs on the tail)}, \\ 0 & \text{(it occurs on the head)}. \end{cases}$$

The likelihood function becomes:

$$L = \prod_{j=1}^{n} \left\{ \Big(F(U_{\delta_j};\boldsymbol{\theta})\Big)^{\tau_j} \Big(f(u_j;\boldsymbol{\theta})\Big)^{1-\tau_j} \Big(1 - F(U_{\delta_j};\boldsymbol{\theta})\Big)^{m_j-1} \prod_{k=1}^{\delta_j-1} \Big(1 - F(U_k;\boldsymbol{\theta})\Big)^{m} \right\}.$$

As in Subsection 5.1, this can be rewritten into

$$L = \prod_{i\geq 1} \left\{ \prod_{j=1}^{n} \left(\Big(F(U_{\delta_j};\mu,\sigma)\Big)^{\tau_j} \Big(f(u_j;\mu,\sigma)\Big)^{1-\tau_j} \right)^{I_i(\delta_j)} \Big(1 - F(U_i;\mu,\sigma)\Big)^{\nu_i} \right\}. \tag{14}$$

When we suppose that disruptive discharges occur on the tail independently of voltage stress levels, we can set

$$P[\tau_j = 1 | I_i(\delta_j) = 1] = \gamma,$$

where γ is a constant. By similar calculations to those in Subsection 5.1, the Fisher information matrix for (14) is given as follows:

$$\mathcal{I} = \frac{n}{\sigma^2} \sum_{i\geq 1} r_{i-1} \left(\sum_{k=0}^{m-1} q_i^k \right) C_i, \tag{15}$$

where

$$C_i \stackrel{\text{def}}{=} \left(\frac{(p_i + \gamma q_i) z_i^2}{p_i q_i} A_i + (1-\gamma) B_i \right).$$

We denote by $e_{mu}(\mu)$ and $e_{mu}(\sigma)$ the asymptotic unit errors of the ML estimators concerning (15).

On the other hand, by similar calculations to those in the UM and the MSM, the Fisher information matrix for the MUM becomes:

$$\mathcal{I} = \frac{1}{\sigma^2} \sum_{i=1}^{N} \sum_{k=-i}^{i} P[\bar{I}_i(k) = 1] C_k. \tag{16}$$

Let us denote by $e_{mu}(\mu)$ and $e_{mu}(\sigma)$ the asymptotic unit errors concerning (16).

The following are remarkable:

- The expressions in the right-hand side of (5) and (15) or (8) and (16) are the same except the difference between $z_i^2 A_i / p_i q_i$ and C_i or $z_k^2 A_k / p_k q_k$ and C_k.
- When $\gamma = 1$ or 0, (15) is equivalent to (5) or (11) while (16) is equivalent to (8) or (13).

In addition to the setting of μ, σ, m, U_1 and N in Subsection 4.1, we set $\gamma = 0.3$. Then, Figure 5 gives $e_{ms}(\mu)$, $e_{mu}(\mu)$, $e_{ms}(\sigma)$ and $e_{mu}(\sigma)$. The thick or normal lines correspond to the MSM or the MUM, respectively. The figure shows that the MSM is superior to the MUM in the asymptotic unit errors, and in the interval of $\Delta U/\sigma$ from 0.2 to 2.0, each of $e_{ms}(\mu)$ and $e_{ms}(\sigma)$ is almost the same in Cases A and B.

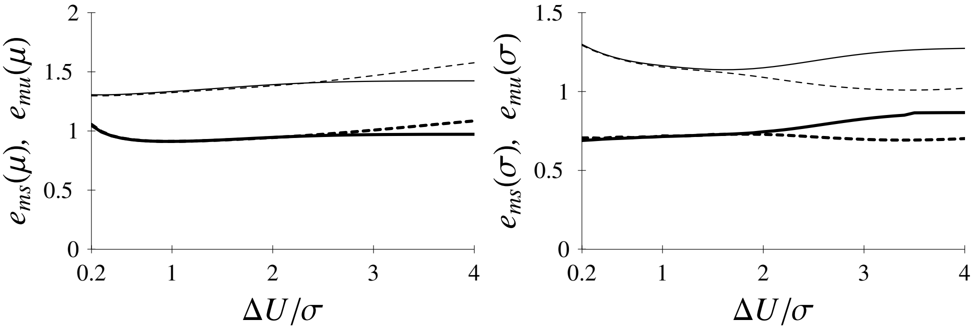

Figure 5: Asymptotic unit errors of the MSM or the MUM when $\gamma = 0.3$

6.2 Evaluation in small samples

We evaluate the performance of the MSM in small samples by means of Monte Carlo simulation. We denote by $\bar{e}_{ms}(\mu)$ and $\bar{e}_{ms}(\sigma)$ the empirical unit errors of the ML estimators of μ and σ in the MSM. Under the setting that $n = 20$ and $\gamma = 0.3$, 10000 sets of independent pseudo-random samples are considered for each simulation.

Figure 6 gives $\bar{e}_{ms}(\mu)$ and $\bar{e}_{ms}(\sigma)$ as well as $e_{ms}(\mu)$ and $e_{ms}(\sigma)$ for comparison. The thick or normal lines correspond to the asymptotical or empirical unit errors, respectively. From this we can see that the empirical unit errors are very close to the asymptotic unit errors throughout the interval $[0.2, 2.0]$. Since $n = 20$, Table 1 tells us that it is necessary to apply voltage stresses 50 or 60 times when $\Delta U/\sigma = 2.0$.

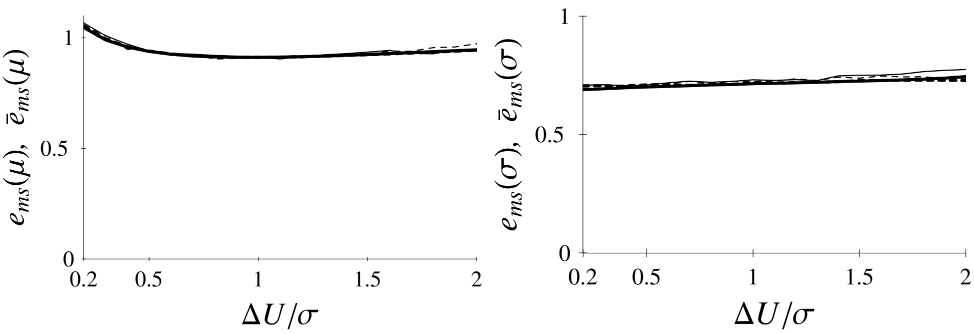

Figure 6: Empirical or asymptotic unit errors of the MSM when $\gamma = 0.3$

In this section we have considered a more realistic situation, that is, the case that some disruptive discharges occur on the tail of of the impulse. As a result, we have seen the following: the MSM is superior to the MUM in the asymptotic errors; compared with the SM, it can reduce the number of voltage stress applications necessary to attain the empirical unit errors as small as the asymptotic unit errors.

Incidentally, in order to decrease the number of voltage stress applications, it is considered to set n at a smaller number than 20. This can, however, often cause failure in finding an ML estimates, even if we use the iterative formula in Appendix A, which has good convergence property.

7 Conclusions

First, we have stated that there are two kinds of assumptions in the statistical methods related to disruptive discharge: one supposes that all voltage stress applications are statistically independent; the other supposes that only the disruptive discharge voltage applications are statistically independent. We should note that only the successive discharge method is usable under the latter assumption, such as in the case of non-self-restoring insulation.

Second, we have investigated the performance of the statistical methods under the former assumption. The following facts have been disclosed.

- The asymptotical unit errors of the ML estimators in the SM are smaller than those in the UM. In order to attain the empirical unit errors as small as them, however, the SM demands a much larger number of voltage stress applications than 40, which is sufficient in the UM.

- The asymptotical unit errors of the ML estimators in the NSM or MSM are smaller than those in the NUM or MUM, too. For small empirical unit errors, the NSM or the MSM demands a little larger number of voltage stress applications than 40 only if $\Delta U/\sigma$ is chosen close to 2.0.

Hence, the step-up test procedure is not always vain even though it requires a start from a sufficiently low voltage level on every subject under the statistical independence of all voltage stress applications.

The test procedure is, however, very difficult to handle since U_1 and ΔU should be chosen properly to avoid increasing the number of voltage stress applications. In contrast, the up-and-down test procedure allows us to chose U_1 roughly since it automatically concentrates testing near the mean. In addition, the MUM makes it possible to chose ΔU roughly since the errors of the ML estimators are less sensitive to its value than the UM. Consequently, the MUM is the most useful method under the statistical independence of all voltage stress applications.

Appendix

A Iterative formula

We give an iterative formula to get the ML estimates of μ and σ in the MSM. Let n_t and w denote the number of disruptive discharges on the tails of impulses and the number of withstands, respectively. Of all the voltage levels $\{U_i\}_{i=1}^{w+n}$, pick up the voltage levels $\{U_{(i)}\}_{i=1}^{w}$ for withstands and $\{U_{(i)}\}_{i=w+1}^{w+n_t}$ for discharges on the tail. On the other hand, of all the discharge voltages $\{u_j\}_{j=1}^{n}$, pick up the discharge voltages on the head $\{u_{(j)}\}_{j=1}^{n-n_t}$. Then, we can obtain the iterative formula

$$
\begin{aligned}
\mu^{(k+1)} &= \frac{1}{w+n}\left\{(w+n_t)\mu^{(k)} + \sum_{j=1}^{n-n_t} u_{(j)}\right\} + \frac{\left(\sigma^{(k)}\right)^2}{w+n}\left\{\sum_{i=1}^{w} D_i^{(k)} - \sum_{i=w+1}^{w+n_t} E_i^{(k)}\right\}, \\
\sigma^{(k+1)} &= \left\{ \frac{1}{w+n}\left\{(w+n_t)\left((\sigma^{(k)})^2 + (\Delta\mu_1^{(k)})^2\right) + \sum_{j=1}^{n-n_t}(u_{(j)} - \mu^{(k+1)})^2\right\}\right.
\end{aligned}
$$

$$+\frac{\left(\sigma^{(k)}\right)^2}{w+n}\left(\sum_{i=1}^{w}(U_{(i)}+\Delta\mu_2^{(k)})D_i^{(k)}-\sum_{i=w+1}^{w+n_t}(U_{(i)}+\Delta\mu_2^{(k)})E_i^{(k)}\right)\Bigg\}^{1/2}$$

by utilizing the expectation-maximization algorithm [7, 8]. Here,

$$D_i^{(k)}\stackrel{\text{def}}{=}\frac{f(U_{(i)};\mu^{(k)},\sigma^{(k)})}{1-F(U_{(i)};\mu^{(k)},\sigma^{(k)})},\qquad E_i^{(k)}\stackrel{\text{def}}{=}\frac{f(U_{(i)};\mu^{(k)},\sigma^{(k)})}{F(U_{(i)};\mu^{(k)},\sigma^{(k)})},$$
$$\Delta\mu_1^{(k)}\stackrel{\text{def}}{=}\mu^{(k)}-\mu^{(k+1)},\qquad \Delta\mu_2^{(k)}\stackrel{\text{def}}{=}\mu^{(k)}-2\mu^{(k+1)}.$$

When $\mu^{(0)}$ and $\sigma^{(0)}$ are properly given, the formula provides two series of approximates $\{\mu^{(k)}\}_{k\geq 1}$ and $\{\sigma^{(k)})\}_{k\geq 1}$ to the ML estimates of μ and σ. Note that the formula gives ML estimates in the SM if $n = n_t$, and it gives ML estimates in the NSM if $n_t = 0$.

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